
**اسم التدريسي : زهراء هاشم كريم**

**اسم المادة : رياضيات**

**عنوان المحاضرة: Determinants**

**رقم المحاضرة:5**

**الايم يل الجامعي للتدريسي:** zahraa.hashim@mustaqbal-college.edu.iq

**AL-Mustaqbal university college**
**Class: 1st**
**Subject: mathmatic**

**Lecturer: zahraa hashim kareem**

**Lecture :5**

**E-mail : zahraa.hashim@mustaqbal-college.edu.iq**

# Determinants

**Determinants :-**

Determinants are arrays which are very useful in the analysis and solution of systems of linear algebraic equations.

- ***The determinant of a 2×2 matrix*** $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Det(A) or |A| is defined to be determinant

$|A| = a*d - b*c$

- ***The determinant of a 3×3 matrix*** $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$|A| = c_{11}\, a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + c_{12}\, a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + c_{13}\, a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$c_{ij} = (-1)^{i+j}$

$|A| = a_{11}(a_{22}*a_{33} - a_{23}*a_{32}) - a_{12}\,(a_{21}*a_{33}) + a_{13}(a_{21}*a_{32} - a_{22}*a_{31})$

**AL-Mustaqbal university college**
**Class: 1st**
**Subject: mathmatic**

**Lecturer: zahraa hashim kareem**

**Lecture :5**

**E-mail : zahraa.hashim@mustaqbal-college.edu.iq**

***Example 8:***. Find determinants for the following:

1- $\begin{vmatrix} 3 & -2 \\ 7 & 4 \end{vmatrix} = (3*4) - (-2*7) = 12 - (-14) = 26$

2- $\begin{vmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{vmatrix} = 1\begin{vmatrix} 4 & -1 \\ -2 & 0 \end{vmatrix} - 5\begin{vmatrix} 2 & -1 \\ 0 & 0 \end{vmatrix} + 0\begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix}$

$= 1(-2) - 5(0) + 0(-2) = -2$

3- $\begin{vmatrix} 4 & -7 & 6 \\ -2 & 4 & 0 \\ 5 & 7 & -4 \end{vmatrix} = 4\begin{vmatrix} 4 & 0 \\ 7 & -4 \end{vmatrix} - (-7)\begin{vmatrix} -2 & 0 \\ 5 & -4 \end{vmatrix} + 6\begin{vmatrix} -2 & 4 \\ 5 & 7 \end{vmatrix}$

$= 4(-16-0) + 7(8-0) + 6(-14-20) = -212$

4- $\begin{vmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{-1}{3} & \frac{-3}{5} \end{vmatrix} = \frac{1}{2} * \frac{-3}{5} - \left(\frac{2}{3} * \frac{-1}{3}\right) = \frac{-3}{10} + \frac{2}{9} = \frac{-7}{90}$

**AL-Mustaqbal university college**
**Class: 1st**
**Subject: mathmatic**

**Lecturer: zahraa hashim kareem**

**Lecture :5**

**E-mail : zahraa.hashim@mustaqbal-college.edu.iq**

**Notes:.**

The value of det. Of any matrix is the same if the expansion is done by using any row or column.

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

***Example 9:.*** $\begin{vmatrix} 1 & 2 & 1 \\ -1 & 3 & 2 \\ 2 & -1 & 1 \end{vmatrix}$

- If we take the first row.

$$= 1\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix} = 5+10-5= 10$$

- If we take the second row.

$$= -(-1)\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = 3-3+10= 10$$

- If we take the first collumn.

$$= 1\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 5+3+2= 10$$

- ***The determinant of a 4×4 matrix*** $A = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{vmatrix}$

$$= 0\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= -1(0\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}) = -1$$

**AL-Mustaqbal university college**
**Class: 1st**
**Subject: mathmatic**

**Lecturer: zahraa hashim kareem**

**Lecture :5**

**E-mail : zahraa.hashim@mustaqbal-college.edu.iq**

***H.W:.*** Evaluate the following determinants:.

1. $\begin{vmatrix} 2 & 3 & 1 \\ 4 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix}$ ans. (-5)

2. $\begin{vmatrix} 2 & -1 & -2 \\ -1 & 2 & 1 \\ 3 & 0 & -3 \end{vmatrix}$ ans. (0)

3. $\begin{vmatrix} 2 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 2 & 1 \end{vmatrix}$ ans. (-7) [ by used the third row ]

4. $\begin{vmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 2 & 0 & 1 \end{vmatrix}$ ans. (6) [ by used the first column]

5. $\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & -1 & 0 & 7 \\ 3 & 0 & 2 & 1 \end{vmatrix}$ ans. (38)

6. $\begin{vmatrix} 1 & -1 & 2 & 3 \\ 2 & 1 & 2 & 6 \\ 1 & 0 & 2 & 3 \\ -2 & 2 & 0 & -5 \end{vmatrix}$ ans. (2)