Chapter 4: Beams

4.1 INTRODUCTION

Beams are structural members that support transverse loads and are therefore subjected primarily to flexure or bending.

Although some degree of axial load will be present in any structural member, in many practical situations this effect is negligible.

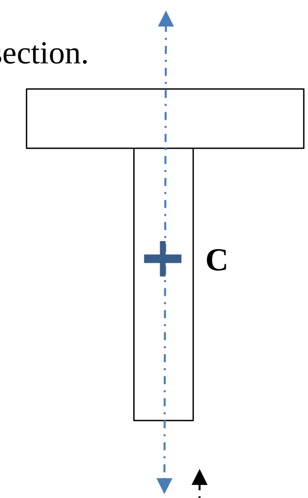
Commonly used cross-sectional shapes include the W, S, and M shapes.

Coverage of beams in the AISC Specification is spread over two chapters:

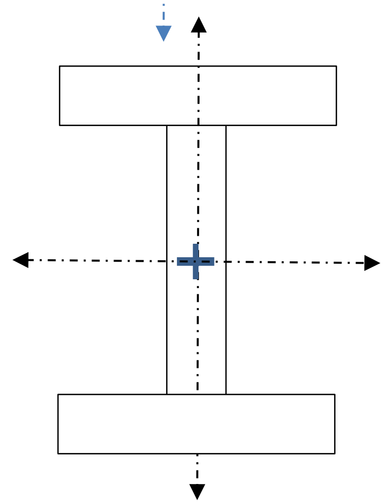
- Chapter F, “Design of Members for Flexure,” and
- Chapter G, “Design of Members for Shear.”

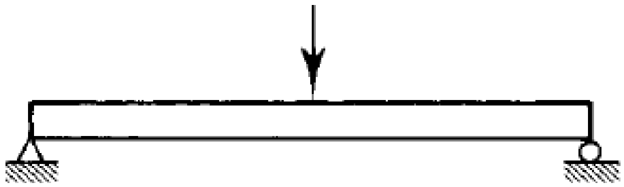
Brief review of preliminaries

Elastic neutral axis (ENA) passes through the centroid of the cross section.

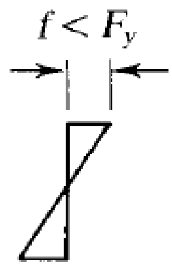


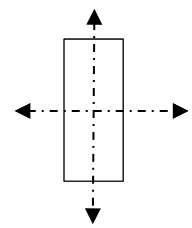
Centroid is located on an axis of symmetry





(a)





$$f_b = \frac{My}{I_x}$$

$$f_{\max} = \frac{Mc}{I_x} = \frac{M}{I_x/c} = \frac{M}{S_x}$$

$$I = \frac{1}{12}bh^3$$

$$S_x = \frac{I}{C} = elastic\ section\ modulus, in^3$$

$$Max\ \sigma_x = \frac{M}{\frac{I}{C}} = \frac{M}{S}$$

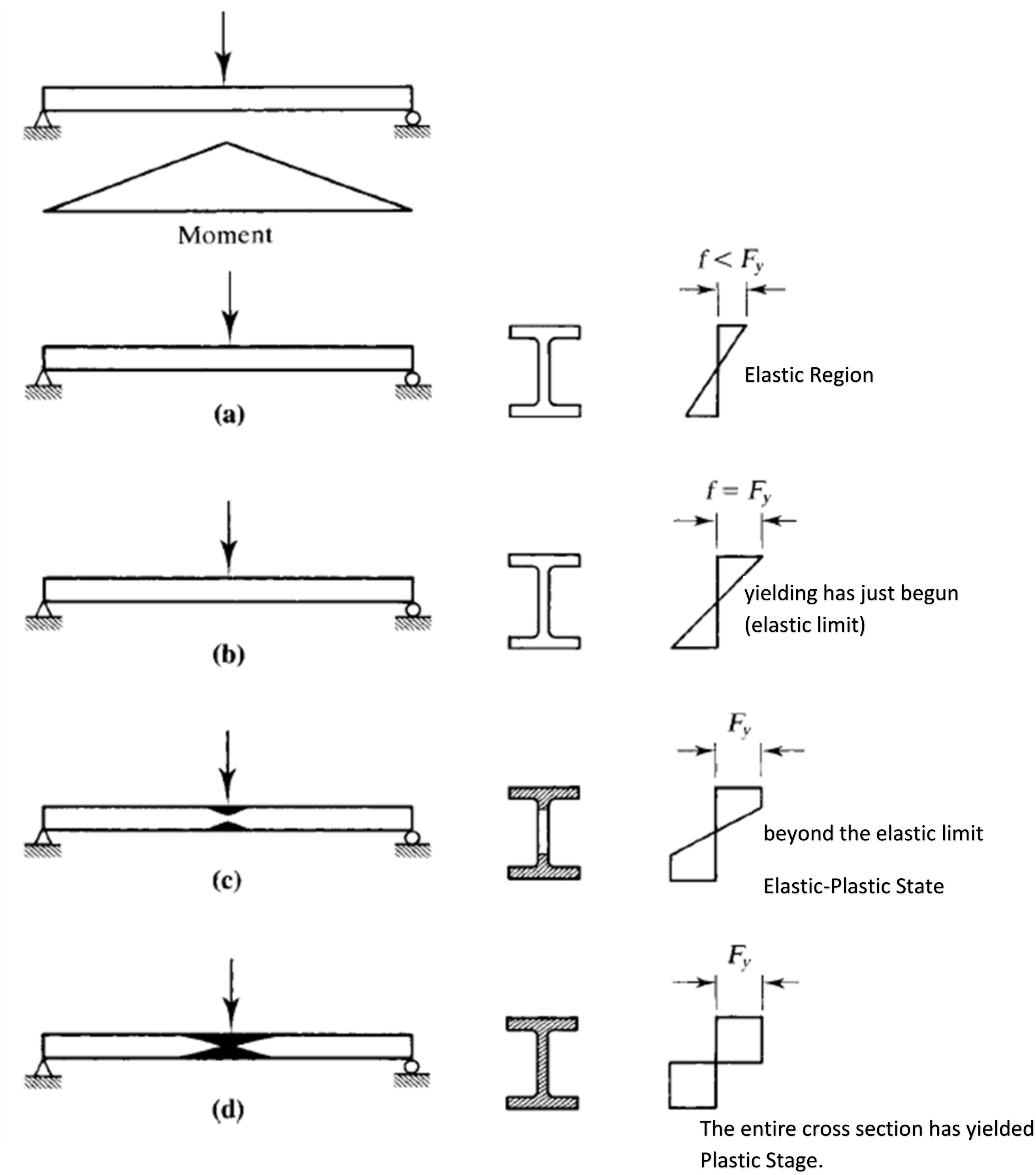
$$S = \frac{I}{C} = \frac{1}{12}bh^3\frac{2}{h} = \frac{bh^2}{6}$$

When $Max\ \sigma_x = F_y$

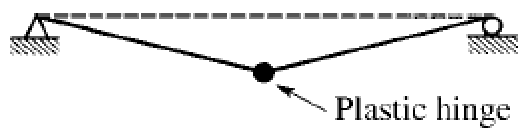
$$M = M_y = moment\ at\ the\ elastic\ liumit$$

$$F_y = \frac{M_y}{S}$$

$$M_y = F_y S$$

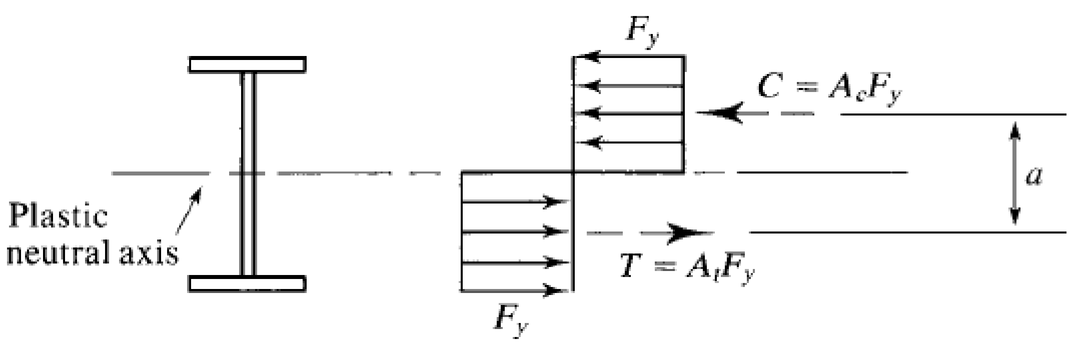

Moment
f < Fy
Elastic Region
(a)
f = Fy
yielding has just begun
(elastic limit)
(b)
Fy
beyond the elastic limit
Elastic-Plastic State
(c)
Fy
(d)
The entire cross section has yielded
Plastic Stage.

A plastic hinge is said to have formed at the center of the beam, and this hinge along with the actual hinges at the ends of the beam constitute an unstable mechanism. During plastic collapse, the mechanism motion will be as shown below.



The plastic moment capacity, which is the moment required to form the plastic hinge, can easily be computed from a consideration of the corresponding stress distribution.

The compressive and tensile stress resultants are shown below, where A_c is the cross-sectional area subjected to compression, and A_t is the area in tension.



$C = T$

$A_c\,F_y = A_t\,F_y$

$A_c = A_t$

Thus, the plastic neutral axis divides the cross section into two equal areas. For shapes that are symmetrical about the axis of bending, the elastic and plastic neutral axes are the same. The plastic moment, M_p, is the resisting couple formed by the two equal and opposite forces, or

$$M_p = F_y\,(A_c)\,a = F_y\,(A_t)\,a = F_y\,(A/2)\,a$$

$$M_p = F_y Z$$

Where,

A = total cross-sectional area

a = distance between the centroids of the two half-area

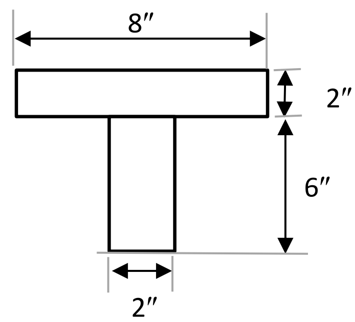
$Z = (A/2)\,a$ = plastic section modulus

Example

Given the cross section shown, F_y=36 ksi

Determine

1- Elastic section modulus (S)
2- M_y
3- Plastic section modulus, Z
4- Plastic moment, M_p



Solution:

1-) Locate the ENA, locate the centroid of the cross section

$$X^- = 0$$

$$y^- = \frac{\sum A.y}{A}$$

$$A = 2(8) + 2(6) = 28\ in^2$$

$$\sum A.y = 2(6)(3) + 2(8)(6+1) = 148\ in^3$$

$$y^- = \frac{148}{28} = 5.286''$$

$$I = \frac{8\times 12^3}{12} + 8\times 2(2.714-1)^2 + \frac{1}{12}(2)(6)^3 + 2(6)(5.286-3)^2 = 151\ in^4$$

$S = \frac{I}{C} = \frac{151}{5.286} = 28.57 in^3$

2-)

$M_y = F_y S = 36 \frac{28.57}{12} = 85.71\ kips - ft$

3-) First locate the PNA

Assume that PNA is in the flange. This assumption should be verified.

$8h = (2 - h)(8) + 2(6)$

$h = 1.75'' < 2''$ $\quad$ *The assumption is OK*

$Z = 8(1.75)\left(\frac{1.75}{2}\right) + 8(0.25)\left(\frac{0.25}{2)}\right) + 2(6)(3.25) = 51.5\ in^3$

4-) $M_p = F_y Z = 36(51.5)/12 = 154.5$ *kips-ft*