ALMUSTAQBAL UNIVERSITY COLLEGE

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

YIELD LINE THEORY SOLVED EXAMPLES II

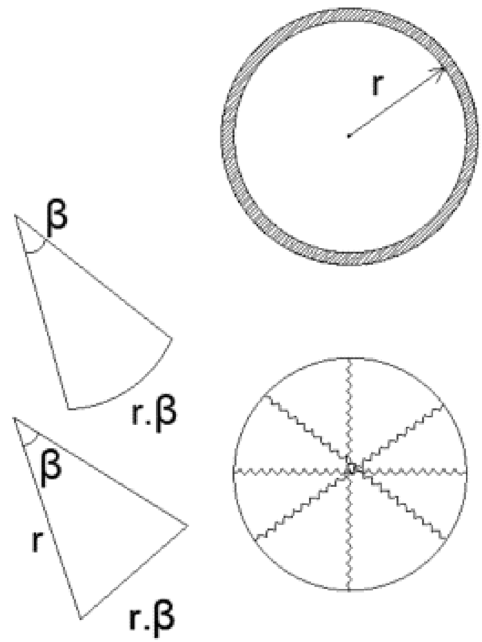
Dr. MOHAMMED ZUHEAR AL-MULALI

EXAMPLE 13: by using the yield line theory, determine the moment (m) for the simply supported two-way reinforced concrete slab shown in the figure below which is subjected to a uniformly distributed load (w) kPa.

SOLUTION:

$W_E = w \times A \times displacement$

$= \left(w \times \left(\frac{1}{2} \times r \times r\beta\right) \times \frac{1}{3}\right) \times \frac{2\pi}{\beta} = \frac{w\pi r^2}{3}$



$W_I = m \times l \times \theta$

$= \left(m \times r\beta \times \frac{1}{r}\right) \times \frac{2\pi}{\beta} = 2\pi m$

$W_E = W_I$

$\frac{w\pi r^2}{3} = 2\pi m \rightarrow \therefore m = \frac{w\pi r^2}{6\pi} = \frac{wr^2}{6}.$

EXAMPLE 14: the circular slab of a radius (r) is supported by 4 columns. Find the ultimate resisting moment (m) per linear meter required just to sustain a concentrated factored load of P kN.

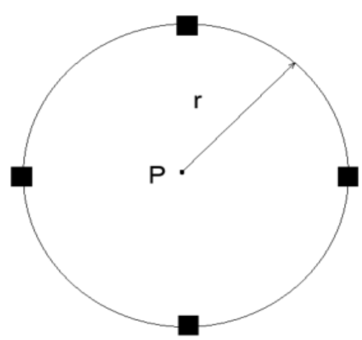
SOLUTION:

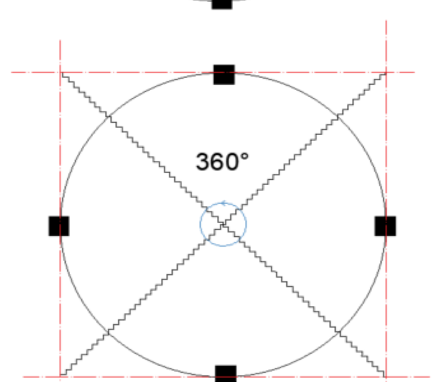
$W_E = P \times 1 = P$

$W_I = \left(m \times \frac{r\sqrt{2}}{2} \times 2 \times \frac{1}{r}\right) \times 4 = 4\sqrt{2}\, m$

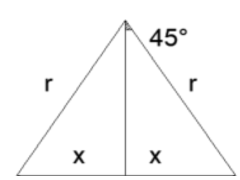
$W_E = W_I$

$\therefore P = 4\sqrt{2}m \quad \rightarrow \quad m = \frac{P}{4\sqrt{2}}$

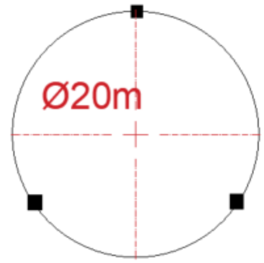


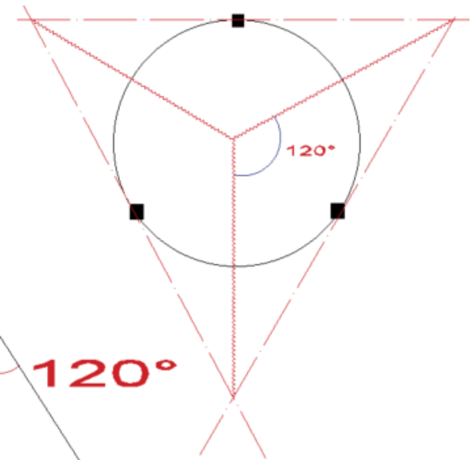


$\sin 45 = \frac{x}{r}$



EXAMPLE 15: the circular slab with a diameter of 20m is supported by three columns as shown in the figure below. Find the ultimate resisting moment per linear meter (m) if the slab is subjected to a concentrated load of P kN at the centre of the slab.





SOLUTION:

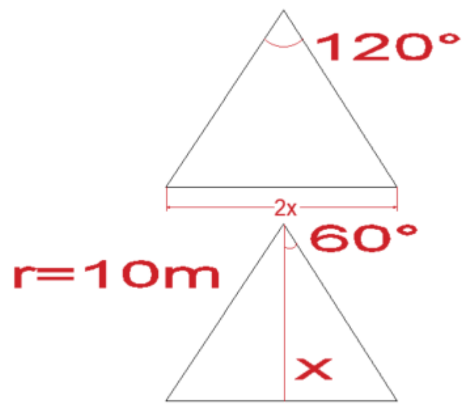
$W_E = P \times 1 = P$

$W_I = m \times l \times \theta$

$= \left(m \times 17.32 \times \frac{1}{10}\right) \times 3 = 5.196m$

$W_E = W_I$

$m = \dfrac{P}{5.196}$



$\sin 60 = \dfrac{x}{10}$

$= \dfrac{\sqrt{3}}{2} \times 10 = 8.66$

$2x = 2 \times 8.66$
$= \mathbf{17.32m}$

EXAMPLE 16: the same circular slab shown in example 15, but it is subjected to a uniformly distributed ultimate load of (20) kPa. Determine the ultimate resisting moment per linear meter (m). Assume that the displacement of a single segment is 0.5513m.

SOLUTION:

$W_E = w \times A \times displacement$

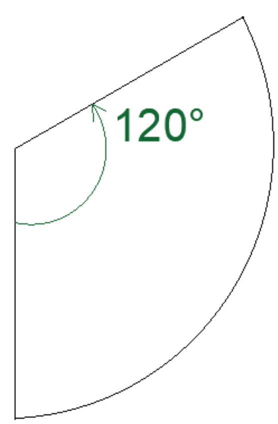
$= 20 \times \left(\frac{120}{360} \times \pi \times 10^2\right) \times 0.5513 \times 3$

$= 3464\ kN.m$

$W_I = \left(m \times 17.32 \times \dfrac{1}{10}\right) \times 3 = 5.196m$

$W_E = W_I$

$\therefore m = 667 kN.m$



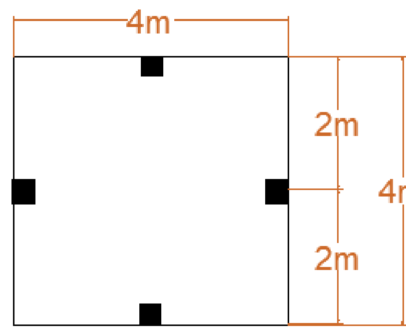
$A = \dfrac{120}{360} \times \pi \times r^2$

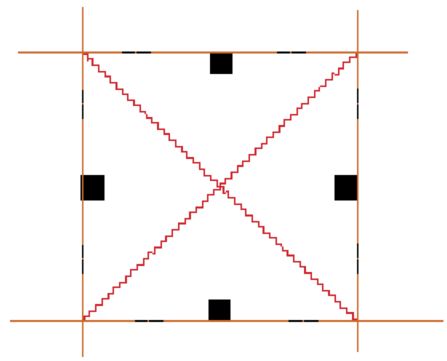
$= \dfrac{120}{360} \times \pi \times 10^2$

$= 104.72m^2$

EXAMPLE 17: The two-way slab shown in the figure below is supported by four columns. Determine the ultimate resisting moment per linear meter (m) if the slab was subjected to uniformly distributed load of (w) kPa.

SOLUTION:





$W_E = w \times A \times displacement.$

$= w \times \left(4 \times 2 \times \frac{1}{2}\right) \times \frac{1}{3} \times 4 = \frac{16}{3}w.$

$W_I = m \times l \times \theta$

$= m \times 4 \times \frac{1}{2} \times 4 = 8m.$

$W_E = W_I$

$\therefore m = \frac{16}{24}w.$