Measures of Central Tendency

It is the metrics that attempt to describe the point of data collection (observations) and its idea goes back to the English scholar Francis Galton. Central tendency measures are used to summarize data numerically since they are considered typical or ideal values for data. Also, these metrics are used to describe a data set or to compare it with other data sets. There are many types of measures of central tendency as follows:

The Mean, The Median and The Mode

1- The Mean

The mean is the most value used to express the central location of the data and is divided into several types, including:

A- The Arithmetic Mean

It is a value which a set of data are collected around it. It is denoted by the symbol $(\bar{X})$ and calculated as follows:

a) Arithmetic Mean for not tabulated Data

If there is a set of data $(x_1, x_2, x_3, x_4, \dots \dots \dots, x_n)$, The Arithmetic Mean is calculated as :

$$\bar{X} = \frac{x_1 + x_2 + x_3 + x_4 + \cdots \dots \dots \dots + x_n}{n}$$

$$\bar{X} = \frac{\sum x_i}{n}$$

Example 1: Find the arithmetic mean for this data:
500, 20, 40, 60, 100, 200, 50
Solution:

$$\bar{X} = \frac{\sum x_i}{n}$$

$$\bar{X} = \frac{500 + 20 + 40 + 60 + 100 + 200 + 50}{7} = \frac{970}{7} = 138.571$$

b) Arithmetic Mean for Tabulated Data

Tabulated data are the data set in frequency distribution table, and for each class there is an upper and lower limit:

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i}$$

$$x_i = \frac{upper\ limit + lower\ limit}{2}$$

f_i: *frequency of class*

Example: Find the arithmetic mean for this data:

class	frequency
20-25	4
25-30	8
30-35	16
35-40	8
40-45	4

Solution:

class	f_i	x_i	$f_i\, x_i$
20-25	4	22.5	90
25-30	8	27.5	220
30-35	16	32.5	520
35-40	8	37.5	300
40-45	4	42.5	170
summatio	$\sum f_i = 4$		$\sum f_i x_i = 130$

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1300}{40} = 32.5$$

B- The Geometric Mean

The geometric mean is used to calculate average values in case data are ratios, as is the case with population growth rates, and it is calculated in two cases:

a) The Geometric Mean for not tabulated Data

The geometric mean of a set of values $(x_1, x_2, x_3, x_4, \dots \dots \dots, x_n)$ is defined as the square root of the product of these values, and is denoted by the symbol $\bar{X}_g$

$$\log \bar{X}_g = \frac{1}{n} \sum \log x_i$$

Example: Find the value of the geometric mean for these values:
6, 4.2, 6, 4.2, 3.7, 4.8

Solution:

$$\text{Log } \bar{X}_g = \frac{1}{6} \Sigma \; (Log\ 6 + \ Log\ 4.2 + Log 6 + Log\ 4.2 + \ Log\ 3.7 + Log\ 4.8)$$

$$\text{Log } \bar{X}_g = \frac{1}{6} (\ 0.77 + 0.62 + 0.77 + 0.62 + 0.56 + 0.68)$$

$$\text{Log } \bar{X}_g = 0.67$$

$$\bar{X}_g = 10^{0.67} = 4.677$$

b) The Geometric Mean for Tabulated Data

To calculate the value of the geometric mean for tabulated data used this equation:

$$\log \bar{X}_g = \frac{1}{\sum f_i} \sum f_i \log x_i$$

x_i: central of class

f_i: frequency of class

Example: Find the geometric mean for data:

Class	80-109	110-139	140-169	170-199	200-229	230 and more
Frequency	26	78	122	34	14	8

Solution :

Class	fi	Xi	Log xi	fi Log xi
80-109	26	94.5	1.975	51.35
110-139	78	124.5	2.095	163.41
140-169	122	154.5	2.189	267.058
170-199	34	184.5	2.266	77.044
200-229	14	214.5	2.331	32.634
230 -….	8	244.5	2.388	19.104
Summation	282			610.6

$$\text{Log } \bar{X}_g = \frac{1}{\Sigma fi} \sum_{i=1}^{n} fi \text{ Log } xi$$

$$\text{Log } \bar{X}_g = \frac{1}{282} (610.6)$$

$$\text{Log } \bar{X}_g = 2.165$$

$$\bar{X}_g = 10^{2.165} = 146.218$$

C-The Harmonic Mean

The harmonic mean is calculated in two cases:

a) The Harmonic Mean for not tabulated Data

Its calculated by using this equation:

$$\overline{X_h} = \frac{1}{\frac{1}{n}\sum \frac{1}{x_i}} = \frac{n}{\sum \frac{1}{x_i}} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots \ldots \ldots + \frac{1}{x_n}}$$

Example: Find the Harmonic Mean for data

18, 37, 25, 46, 77, 20

Solution:

$$\overline{X_h} = = \frac{n}{\sum \frac{1}{x_i}} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots \ldots \ldots + \frac{1}{x_n}}$$

$$= \frac{7}{\frac{1}{18} + \frac{1}{37} + \frac{1}{25} + \frac{1}{46} + \frac{1}{77} + \frac{1}{20}} = 31.1315$$

b) The Harmonic Mean for tabulated Data

Its calculated by using this equation:

$$\overline{X_h} = = \frac{\sum f_i}{\sum \frac{f_i}{x_i}}$$

Example : Find the Harmonic Mean for data :

Class	60-69	70-79	80-89	90-99	100-109	110-119	120-129
Frequency	5	15	20	15	10	5	30

Solution:

Class	fi	xi	$\frac{fi}{xi}$
60-69	5	64.5	0.078
70-79	15	74.5	0.201
80-89	20	84.5	0.237
90-99	15	94.5	0.159
100-109	10	104.5	0.096
110-119	5	114.5	0.044
120-129	5	125	0.04
Summation	75		0.855

$$\bar{X}_h = \frac{\sum fi}{\sum \frac{fi}{xi}}$$

$$\bar{X}_h = \frac{75}{0.855} = 87.719$$

2- The Median

It is the value in which a set of values is mediated after ascending or descending order and symbolized by the symbol (M). It is calculated in two cases:

- **Case of not tabulated data**

The group values are arranged in ascending or descending order, and the median is the value that falls exactly in the middle.

- If n is odd number then order of median $\frac{n+1}{2}$
- If n is even number then order of median $\frac{value\,\frac{n}{2} + (\frac{n}{2}+1)}{2}$

Example: Find the value of the Median from the data:
13, 50, 7, 15, 47, 12, 5

Solution: The data is arranged in ascending or descending order:
In descending order: 50, 47, 15, 13, 12, 7, 5
In ascending order: 5, 7, 12, 13, 15, 47, 50

median	5	7	12	13	15	47	50
n	1	2	3	4	5	6	7

$n = 7$, order of median $\frac{n+1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$

The median is (13)

Example: Find the value of the Median from the data:

20, 12, 42, 15, 65, 30

Solution:

In ascending order: 12, 15, 20, 30, 42, 65

$$n = 6\,, \frac{n}{2} = \frac{6}{2} = 3$$

$$\frac{\boldsymbol{n}}{\boldsymbol{2}} = 3$$

$$\left(\frac{n}{2} + 1\right) = 3 + 1 = 4$$

median	12	15	20	30	42	65
n	1	2	3	4	5	6

$$M = \frac{30 + 20}{2} = 25$$