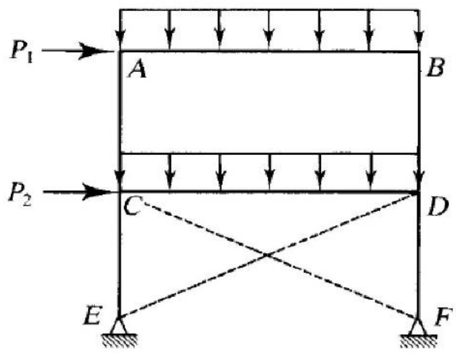
Chapter Five

BEAM – COLUMN

(Combined Axial and Bending)

DEFINITION

- For many structural members bending and axial effect are acts at the same times. For example if the beam is statistically indeterminate axial forces developed in the beam. Also the column in frame is subjected to both axial and bending effects.
- Beam-columns are structural members which combine the beam function of transmitting transverse forces or moments (uniaxial bending or biaxial bending) with the compression (or tension) member function of transmitting axial forces.



INTERACTION FORMULA

- The relationship between required and the available strength is defined as:
- $\frac{required\ strength}{avialable\ strength} \leq 1.0$
- For compression members the strength are axial force
- $\frac{P_u}{\emptyset P_n} \leq 1.0$ for (LRFD) and $\frac{P_a}{\frac{P_n}{\Omega}} \leq 1.0$ for (ASD)
- In general form: $\frac{P_r}{P_c} \leq 1.0$
- For combined bending and axial forces: $\frac{P_r}{P_c} + \frac{M_r}{M_c} \leq 1.0$
- Here M_r= required moment = M_u(LRFD) or = M_a(ASD)
- For combined biaxial bending moment and axial force:

$$\frac{P_r}{P_c} + \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \leq 1.0$$

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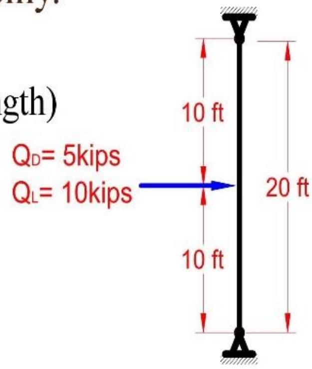
AISC-Specifications Chapter H Page(70)

- Two formulas are given in the specification: one for large axial loads in which the bending moment is reduced by 1/9 . The other equation is for small axial loads, in which it reduced by ½ ;
- $\frac{P_r}{P_c} \geq 0.2$ H1-1a
- $\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$
- $\frac{P_r}{P_c} < 0.2$ H1-1b
- $\frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \leq 1.0$
- These expressions are expected for both LRFD and ASD.

Example1: A 50 ksi $W12 \times 40$ column is subjected to compressive load (PD= 25kips, PL= 55kips) and lateral Load as shown in Figure. Determine whether this member is satisfied with AISC Specification, or not. Lateral bracing provided at the ends only.



- Solution:(LRFD)
- 1) compute ultimate loads (required strength)
- $P_u = 1.2 \times 25 + 1.6 * 55 = 118\ kips$
- $Q_u = 1.2 \times 5 + 1.6 \times 10 = 22\ kips$
- $M_u = \frac{Q_u L}{4} = \frac{22*20}{4} = 110\ k-ft$
- 2)compute available strength:
- 2-1 compressive strength. $K_y L = 1.0 * 20 = 20ft$
- Use column Tables (4-18) $\emptyset_c P_n = 172\ kips$
- 2-2 bending strength.
- $Z_x = 57\ in^3$
  , $\emptyset_b M_{px} = 214\ k-ft$, BF= 5.5 kips, $L_P = 6.85\ ft$, $L_r = 21.1\ ft$
  $> L_b = 20$ the beam is laterally unsupported Zone 2

- $\emptyset_b M_n = C_b[\emptyset_b M_{px} - BF(L_b - L_P)] \leq \emptyset_b M_{px}$
- $C_b = 1.32\ Table\ (3-1)$
- $\emptyset_b M_n = 1.32[214 - 5.5(20 - 6.85)] = 187\ k - ft \leq 214$
- 3) use interaction formula
- $\frac{P_r}{P_c} = \frac{118}{172} = 0.686 \geq 0.2$
- $\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.686 + \frac{8}{9}\left(\frac{110}{187}\right) = 1.208 > 1.0$ N.G.
- The section does not satisfy the AISC Specification.
- **H.W.**
- A) for the previous example compute the maximum allowable load P_u?
- B) for the previous example compute the maximum allowable load Q_u?

Design Column Tables (pages 4-10 to 4-23)

- The AISC manual introduces column tables for the most popular sections, based on the effective length along the y-axis (KL_y). When the buckling occur along x-axis $\left(i.e. \frac{KL_y}{r_y} < \frac{KL_x}{r_x}\right)$, the tables can be used by introducing $\left(\frac{KL_x}{\frac{r_x}{r_y}}\right)$, as illustrated in the following example.

- Example2: Using column Tables Compute the ultimate strength capacity $\emptyset_c P_n$ according to LRFD, also the allowable compressive strength $\left(\frac{P_n}{\Omega_c}\right)$ according to ASD. Data: (W12x96, $F_y = 50 ksi$, $KL_x = 1.2 * 15 = 18 ft$, and $KL_y = 0.8 * 15 = 12 ft$)
- Solution:
- $KL_y = 12 ft > \left(\frac{KL_x}{\frac{r_x}{r_y}}\right) = \frac{18}{1.76} = 10.23\ ft$, then buckling along y-axis. Then use $KL_y = 12 ft$
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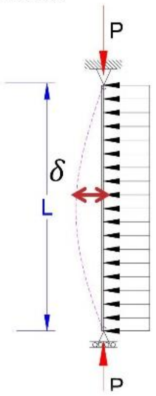
Shape		W12×									
Wt/ft		96		87		79		72		65	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
d...s of gyration r_y	0	844	1270	766	1150	694	1040	633	951	571	859
	6	811	1220	735	1110	667	1000	607	913	548	824
	7	800	1200	725	1090	657	987	598	899	540	811
	8	787	1180	713	1070	646	971	588	884	531	798
	9	772	1160	699	1050	634	952	577	867	520	782
	10	756	1140	685	1030	620	932	565	849	509	765
	11	739	1110	669	1010	606	910	551	828	497	747
	12	720	1080	652	980	590	887	537	807	484	727

- $\emptyset_c P_n$ =1080 kips (LRFD) and $\left(\frac{P_n}{\Omega_c}\right)$= 720 kips (ASD).
- H.W. repeat Example2 and select the lightest W12 if the applied compressive load :
- 1- P_a =900 kips.
- 2- P_a =600 kips.
- Use KLy= 12 ft KLx= 18 ft

Second- order- analysis (Stability Analysis) chapter C

- For steel structures the inclusion of secondary effect due to the stability analysis is very important.
- Additional moment results from $(P - \delta)$ analysis, is $M = P.\delta$. To account for this effect the following factor must be considered.

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}}$$



- Axial compression interaction with bending moment.
- (Modification Factor B1)
- Where C_m moment modification factor.
- α= 1.0 for (LRFD) and α= 1.6 for (ASD)
- P_r= required axial strength.
- P_{e1}= elastic critical buckling resistance about axis of bending.
- $P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2}$

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C_m moment modification factor

- It is a ($C_m \leq 1.0$) reduction factor for the condition of nonuniform moment distribution between the beam-column joints.
- **A**-Assuming **no joint translation**.

1. For no lateral load acting beween joints

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right)$$

- Where $\left(\frac{M_1}{M_2}\right)$ is positive for reverse bending curvature and it is negative for single curvature. Always $M_1 \leq M_2$

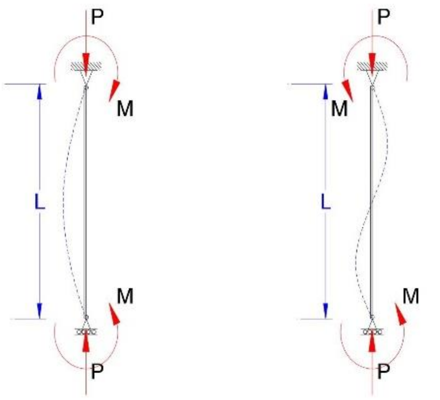
2. For lateral load acting beween joints

$$C_m = 1.0 + \psi\left(\frac{\alpha P_r}{P_{e1}}\right)$$

As defined in Table C-C2.1 Page 237

Conservatively:

Use $C_m = 1.0$ for Simple ends and $C_m = 0.85$ for fixed ends.



Single curveture $\left(\frac{M_1}{M_2}\right)$ -ve *Reverse curveture* $\left(\frac{M_1}{M_2}\right)$ +ve

- Then required bending moment strength is: $M_r = B_1 M_{nt}$
- Where M_{nt} = first-order moment using LRFD or ASD Load combinations (assuming no joint translations)

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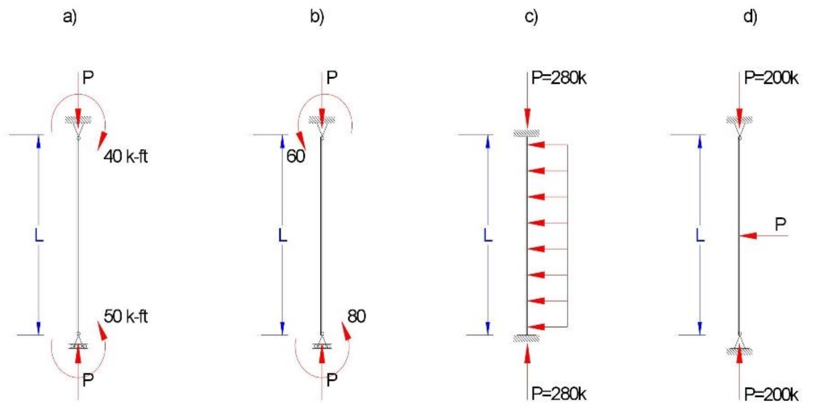
- Example 3: Compute C_m for the following conditions:



$Data$:			$W10 \times 49$ $KL_x = 20ft$	$W12 \times 58$ $KL_x = 20ft$

$Answers$

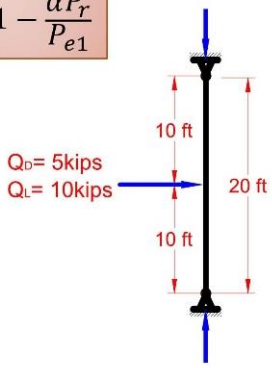
$C_m = 0.92$	$C_m = 0.30$	$C_m = 0.92$	$C_m = 0.98$

Example 4: For Example 1. . (use LRFD) if $P_r = 90\ kips$, check the provided section using the second order effect.
Data from Example 1: $P_c = 172\ kips$, $M_{rx} = 110\ k-ft$, $M_{cx} = 187\ k-ft$

- Solution:

$$M_r = B_1 M_{nt}$$

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}}$$



- Here $C_m = 1.0 + \psi\left(\frac{\alpha P_r}{P_{e1}}\right)$
- From table C-C2.1
- ψ= -0.2
- α= 1.0 for LRFD
- $P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 * 29000 * 307}{(20*12)^2} = 1525.5\ kips$
- $C_m = 1.0 - 0.2\left(\frac{1.0*90}{1525.5}\right) = 0.99 \cong 1.00$
- $B_1 = \frac{1.0}{1-\left(\frac{1.0*90}{1525.5}\right)} = 1.062$
- $M_r = B_1 M_{nt} = 1.062 * 110 = 116.6\ k-ft$
- $\frac{P_r}{P_c} = \frac{90}{172} = 0.523 \geq 0.2$
- $\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.523 + \frac{8}{9}\left(\frac{116.6}{187}\right) = 1.077 > 1.0$ N.G.

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Design Beam-Columns

Since there are many variables in the design of beam-column, a method of trial and error will be used.

Assume that equation (H1-1a) governs, then:

$$\frac{P_r}{P_c}+\frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0$$

For a certain section with effective length (KL_y), the required strength can be determined, and the following constants can be introduced:

$$p=\frac{1}{P_c}$$

$$b_x=\frac{8}{9M_{cx}}$$

$$b_y=\frac{8}{9M_{cy}}$$

Equations (H1-1) can be rewritten as:

$$\frac{P_r}{P_c}\geq 0.2$$

$$pP_r+b_xM_{rx}+b_yM_{ry}\leq 1.0$$

$$\frac{P_r}{P_c}<0.2$$

$$\frac{1}{2}pP_r+\frac{9}{8}\left(b_xM_{rx}+b_yM_{ry}\right)\leq 1.0$$

The values of these constants $(p, b_x\ and\ b_y)$ are calculated for each section for $C_b = 1.0$ in the AISC manual Table (6-1)

Notes:

1. Use the maximum value of $(KL_y\ or\ \frac{KL_x}{\frac{r_x}{r_y}})$
2. Correct the values of $b_x and\ b_y$ according to the values of C_b
3. The design usually started from trial section.

F_y = 50 ksi

Table 6–1 (continued)
Combined Axial and Bending
W Shapes

W40

Shape		W40×											
		431[h]				397[h]				392[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length KL (ft) with respect to least radius of gyration r_y or Unbraced Length L_b (ft) for X-X axis bending	0	0.263	0.175	0.182	0.121	0.285	0.190	0.198	0.132	0.290	0.193	0.208	0.139
	11	0.289	0.193	0.182	0.121	0.314	0.209	0.198	0.132	0.349	0.232	0.213	0.142
	12	0.295	0.196	0.182	0.121	0.320	0.213	0.198	0.132	0.361	0.240	0.217	0.144
	13	0.301	0.200	0.182	0.121	0.327	0.217	0.198	0.132	0.375	0.249	0.220	0.146
	14	0.307	0.204	0.184	0.122	0.334	0.222	0.201	0.133	0.391	0.260	0.223	0.148
	15	0.314	0.209	0.186	0.124	0.341	0.227	0.203	0.135	0.408	0.271	0.227	0.151
	16	0.322	0.214	0.188	0.125	0.350	0.233	0.205	0.137	0.428	0.284	0.230	0.153
	17	0.330	0.220	0.190	0.127	0.359	0.239	0.208	0.138	0.449	0.299	0.234	0.156
	18	0.340	0.226	0.193	0.128	0.369	0.246	0.211	0.140	0.474	0.315	0.238	0.158
	19	0.350	0.233	0.195	0.130	0.380	0.253	0.213	0.142	0.501	0.333	0.241	0.161
	20	0.361	0.240	0.197	0.131	0.392	0.261	0.216	0.144	0.531	0.354	0.245	0.163
	22	0.386	0.257	0.202	0.134	0.419	0.279	0.222	0.147	0.603	0.401	0.254	0.169
	24	0.415	0.276	0.207	0.138	0.451	0.300	0.227	0.151	0.693	0.461	0.263	0.175
	26	0.449	0.299	0.212	0.141	0.488	0.325	0.234	0.156	0.808	0.538	0.273	0.181
	28	0.489	0.325	0.218	0.145	0.532	0.354	0.240	0.160	0.937	0.623	0.283	0.188
	30	0.536	0.356	0.224	0.149	0.584	0.388	0.247	0.165	1.08	0.716	0.295	0.196
	32	0.591	0.393	0.230	0.153	0.644	0.429	0.255	0.169	1.22	0.814	0.307	0.204
	34	0.656	0.436	0.237	0.157	0.715	0.476	0.263	0.175	1.38	0.919	0.320	0.213
	36	0.734	0.488	0.243	0.162	0.801	0.533	0.271	0.180	1.55	1.03	0.335	0.223
	38	0.818	0.544	0.251	0.167	0.892	0.594	0.280	0.186	1.73	1.15	0.351	0.233
	40	0.906	0.603	0.259	0.172	0.989	0.658	0.289	0.193	1.91	1.27	0.372	0.247
	42	0.999	0.665	0.267	0.178	1.09	0.725	0.300	0.199	2.11	1.40	0.393	0.262
	44	1.10	0.729	0.276	0.184	1.20	0.796	0.311	0.207	2.31	1.54	0.415	0.276

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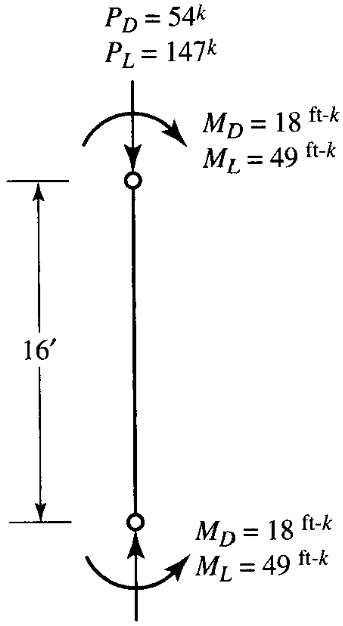
Example: starting from $\boldsymbol{W10 \times 60}$, Select a **W10** shape of **A992** steel for the beam-column shown in Figure. This member is part of a braced frame and is subjected to the service load axial force and bending moments as shown. Bending about the strong axis, and $K_y = K_x = 1.0$ Lateral support is provided only at the ends. Assume that $B_1 = 1.0$

Solution:



$P_u = 1.2 \times 54 + 1.6 \times 147 = 300\ kips$

$M_{ntx} = 1.2 \times 18 + 1.6 * 49 = 100\ kips - ft$

$M_{rx} = B_1 M_{ntx} = 1.0 * 100 = 100\ k - ft$

For $W10 \times 60$ ($p = 1.89 \times 10^{-3}, b_x = 3.51 \times 10^{-3}$)

Check $pP_u = 1.89 \times 10^{-3} \times 300 = 0.567 > 0.2$

Use (H1-1a) $pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0$

$0.567 + 3.51 \times 10^{-3} \times 100 + 0 = 0.918 < 1.0$ O.K.

The section is safe. For economy, check lighter section.

Try $W10 \times 54$ ($p = 2.12 \times 10^{-3}, b_x = 3.97 \times 10^{-3}$)

Check $pP_u = 2.12 \times 10^{-3} \times 300 = 0.636 > 0.2$

Use (H1-1a) $pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0$

$0.636 + 3.97 \times 10^{-3} \times 100 + 0 = 1.033 > 1.0$ N.G.

Then use $\boldsymbol{W10 \times 60}$

H.W.

1. repeat the previous example and select the lightest **W12** start from $\boldsymbol{W12 \times 65}$
2. repeat the previous example and select the lightest **W14** start from $\boldsymbol{W14 \times 68}$