Application of the integral

Volume

The volume of the solid is

$$V = \int_a^b \pi[f(x)]^2\, dx$$

Because the cross section is disk shaped the application of this formula is called the method of disk

Example

Find the volume of the solid that is obtained when the region under the cure $f(x) = \sqrt{x}$ over the interval [1,4] is revolved about x-axis.

Sol:

$$V = \int_a^b \pi[f(x)]^2\, dx$$

$$V = \int_1^4 \pi\left[\sqrt{x}\right]^2 dx$$

$$V = \int_1^4 \pi x\, dx$$

$$V = \pi \frac{x^2}{2}\Big|_1^4$$

$$= \pi \frac{4^2}{2} - \pi \frac{1^2}{2}$$

$$= \frac{15}{2}\pi$$

Work

$$W = \int_a^b F(x)\,dx$$

Example: A spring exerts a force of 5N when stretched 1m beyond its natural length how much work is required to stretch the spring 1.8 m beyond its natural length?

Hint $f(x) = 5x$

Solution:

$$W = \int_a^b F(x)\,dx$$

$$W = \int_0^{1.8} 5x\,dx$$

$$= \frac{5x^2}{2}\Big|_0^{1.8}$$

$$= \frac{5}{2}[1.8^2 - 0^2]$$

$$= \frac{5}{2}[3.24]$$

$$= 8.1 \quad J$$