Application of the integral

Area between two curve

If f and g are continuous on the interval $[a.b]$. and if $f(x) \geq g(x)$ for all x in $[a.b]$, then the area of the region bounded above by $y = f(x)$, below $y = g(x)$

On the left by the line $x = a$ and on the right by the line $x = b$ is

$$\boldsymbol{A = \int_a^b [f(x) - g(x)]\, dx}$$

Example: find the area of the region bounded above by $y = x + 6$, bounded below by $y = x^2$ and bounded on the sides by the lines $x = 0$ and $x = 2$

Sol.

$$A = \int_a^b [f(x) - g(x)]\, dx$$

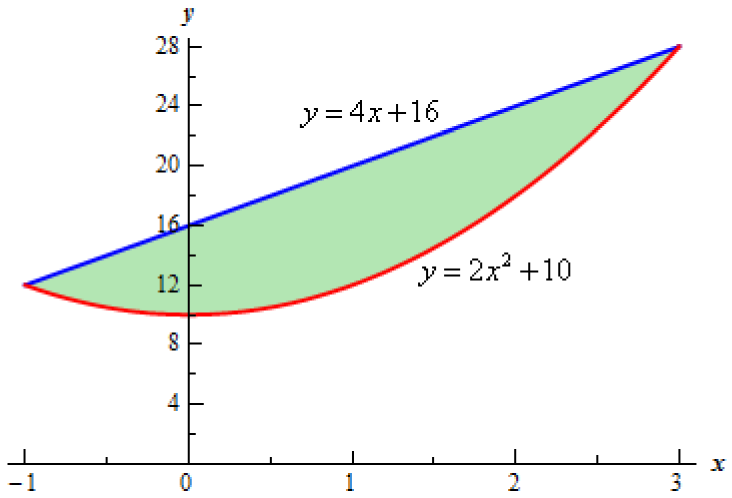
$$A = \int_0^2 [(x + 6) - (x^2)]\, dx$$

$$= \left| \frac{x^2}{2} + 6x - \frac{x^3}{3} \right|_0^2$$

$$= \frac{2^2}{2} + 6(2) - \frac{2^3}{3} - 0$$

$$= \frac{34}{3} \text{unit}^2$$

Determine the area of the region bounded by $y = 2x^2 + 10$ and $y = 4x + 16$.



solution

$$A = \int_a^b \begin{pmatrix} \text{upper} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{lower} \\ \text{function} \end{pmatrix} \, dx$$

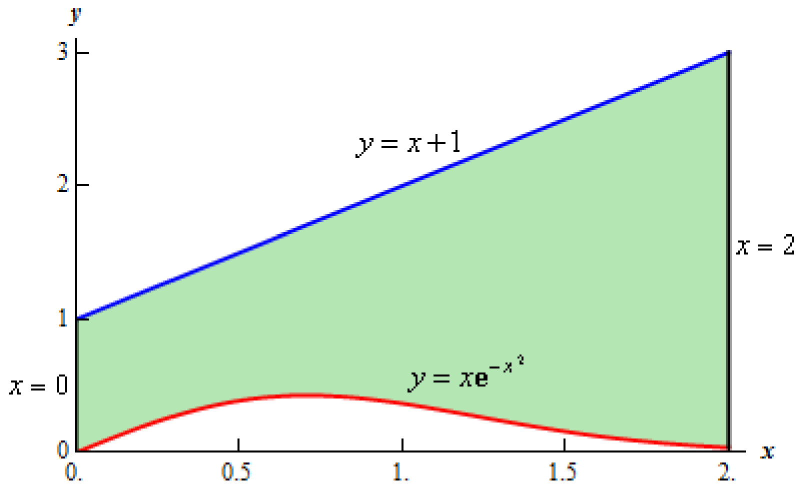
$$= \int_{-1}^{3} 4x + 16 - \left(2x^2 + 10\right) \, dx$$

$$= \int_{-1}^{3} -2x^2 + 4x + 6 \, dx$$

$$= \left(-\frac{2}{3}x^3 + 2x^2 + 6x\right)\Bigg|_{-1}^{3}$$

$$= \frac{64}{3}$$

Determine the area of the region bounded by $y = x\mathbf{e}^{-x^2}$, $y = x + 1$, $x = 2$, and the y-axis.



solution

$$A = \int_a^b \begin{pmatrix} \text{upper} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{lower} \\ \text{function} \end{pmatrix} dx$$

$$= \int_0^2 x + 1 - x\mathbf{e}^{-x^2}\, dx$$

$$= \left(\frac{1}{2}x^2 + x + \frac{1}{2}\mathbf{e}^{-x^2} \right) \Big|_0^2$$

$$= \frac{7}{2} + \frac{\mathbf{e}^{-4}}{2} = 3.5092$$