



Lecter 7

Application of the integral

Area between two curve

If f and g are continuous on the interval [a, b], and if $f(x) \ge g(x)$ for all x in [a, b], then the area of the region bounded above by y = f(x), below y = g(x)

On the left by the line x = a and on the right by the line x = b is

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

Example: find the area of the region bounded above by y = x + 6, bounded below by $y = x^2$ and bounded on the sides by the lines x = 0 and x = 2

Sol.

$$A = \int_{a}^{b} [f(x) - g(x)] dx$$
$$A = \int_{0}^{2} [(x+6) - (x^{2})] dx$$
$$= \left| \frac{x^{2}}{2} + 6x - \frac{x^{3}}{3} \right|_{0}^{2}$$
$$= \frac{2^{2}}{2} + 6(2) - \frac{2^{3}}{3} - 0$$
$$= \frac{34}{3} \text{unit}^{2}$$



Mathematical



MSC. Sarai Hamza

Lecter 7

Determine the area of the region bounded by $y=2x^2+10$ and y=4x+16



solution

$$egin{aligned} A &= \int_{a}^{b} igg(egin{aligned} ext{upper} \ ext{function} igg) - igg(egin{aligned} ext{lower} \ ext{function} igg) \, dx \ &= \int_{-1}^{3} 4x + 16 - ig(2x^2 + 10 ig) \, dx \ &= \int_{-1}^{3} -2x^2 + 4x + 6 \, dx \ &= igg(-rac{2}{3}x^3 + 2x^2 + 6x igg) \Big|_{-1}^{3} \ &= rac{64}{3} \end{aligned}$$



Mathematical



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Determine the area of the region bounded by $y=x{f e}^{-x^2}$, y=x+1, x=2, and the y-axis.



solution

$$egin{aligned} A &= \int_a^b \left(egin{aligned} ext{upper} \ ext{function} \end{array}
ight) - \left(egin{aligned} ext{lower} \ ext{function} \end{array}
ight) \, dx \ &= \int_0^{-2} x + 1 - x \mathbf{e}^{-x^2} \, dx \ &= \left(rac{1}{2} x^2 + x + rac{1}{2} \mathbf{e}^{-x^2}
ight) \Big|_0^2 \ &= rac{7}{2} + rac{\mathbf{e}^{-4}}{2} = 3.5092 \end{aligned}$$