Class: 2nd
Subject: Strength of Materials
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Strength of Materials

Second Stage

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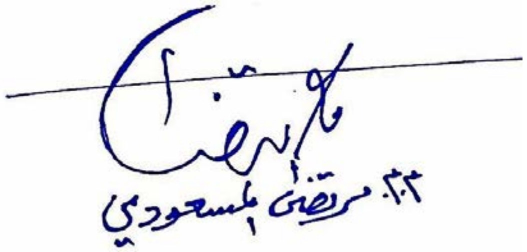


Simple Stresses

Concept of stress

Let us introduce the concept of stress as we know that the main problem of engineering mechanics of material is the investigation of the internal resistance of the body, i.e. the nature of forces set up within a body to balance the effect of the externally applied forces.

- The externally applied forces are termed as loads.
- These externally applied forces may be due to any one of the reason.
- due to service conditions
- due to environment in which the component works
- through contact with other members
- due to fluid pressures
- due to gravity or inertia forces.

As we know that in mechanics of deformable solids, externally applied forces acts on a body and body suffers a deformation. From equilibrium point of view, this action should be opposed or reacted by internal forces which are set up within the particles of material due to cohesion.

- These internal forces give rise to a concept of stress. Therefore, let us define a stress. Let us consider a rectangular bar of some cross – sectional area and subjected to some load or force (in Newton) as shown in Figure (1.1a).

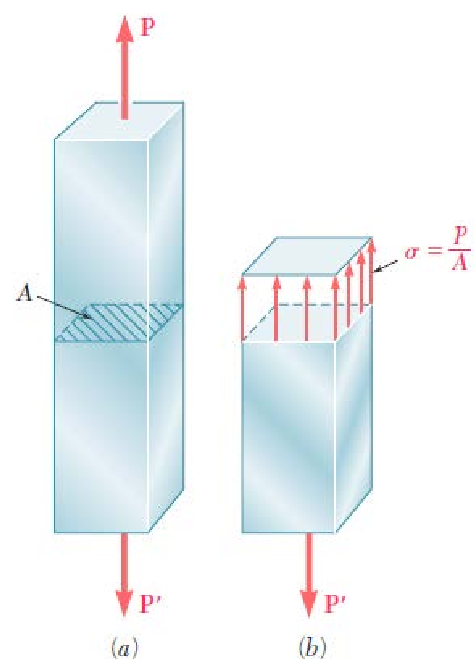


Figure (1.1): Member with an Axial Load

- Let us imagine that the same rectangular bar is assumed to be cut into two halves.
- The each portion of this rectangular bar is in equilibrium under the action of load **P** and the internal forces acting at the section has been shown in Figure (1.1b).
- Now stress is defined as the force intensity or force per unit area. Here we use a symbol (**σ**) to represent the stress.

$$\sigma = \frac{P}{A}$$

Where **A** is the area of cross section.

- Here we are using an assumption that the total force or total load carried by the rectangular bar is uniformly distributed over its cross – section.
- But the stress distributions may be for from uniform, with local regions of high stress known as stress concentrations.
- A positive sign will be used to indicate a tensile stress (member in tension) and a negative sign to indicate a compressive stress (member in compression).

Units:

The basic units of stress in S.I units i.e. (International system) are N / m^2 (or Pa)

- KPa = 10^3 Pa
- MPa = 10^6 Pa
- GPa = 10^9 Pa

Sometimes N / mm^2 units are also used, because this is an equivalent to MPa. While US customary unit is pound per square inch psi.

TYPES OF STRESSES

- Only two basic stresses exists :

(1) Normal stress and

(2) Shear stress.

- Other stresses either are similar to these basic stresses or are a combination of these e.g.
- Bending stress is a combination tensile, compressive.

- Shear stresses.
- Torsional stress, as encountered in twisting of a shaft is a shearing stress.
- Let us define the normal stresses and shear stresses in the following sections.

NORMAL STRESSES

Normal stresses: We have defined stress as force per unit area. If the stresses are normal to the areas concerned (Figure 1.2), then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter (**σ**)

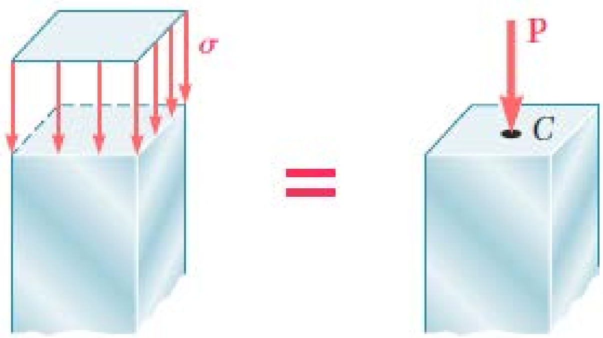
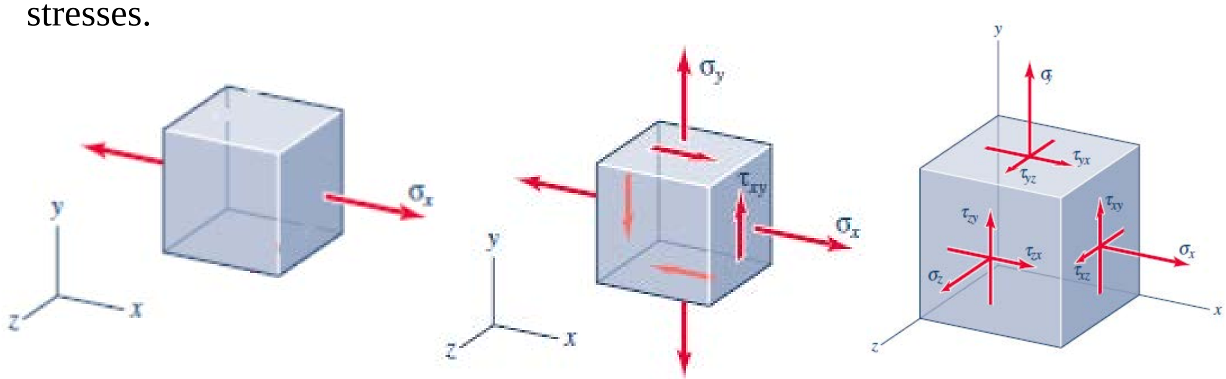


Figure (1.2)

- This is also known as uniaxial state of stress (Figure 1.3a), because the stresses acts only in one direction.
- However, such a state rarely exists, therefore we have biaxial (Figure 1.3b) and triaxial (Figure 1.3c) state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses.



(a) Uniaxial State of Stress **(b) Biaxial State of Stress** **(c) Triaxial State of Stress**

Figure (1.3): State of Stress Referred to as Rectangular Cartesian axes.

Tensile or compressive stresses

- The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area as shown in Figure (1.4).

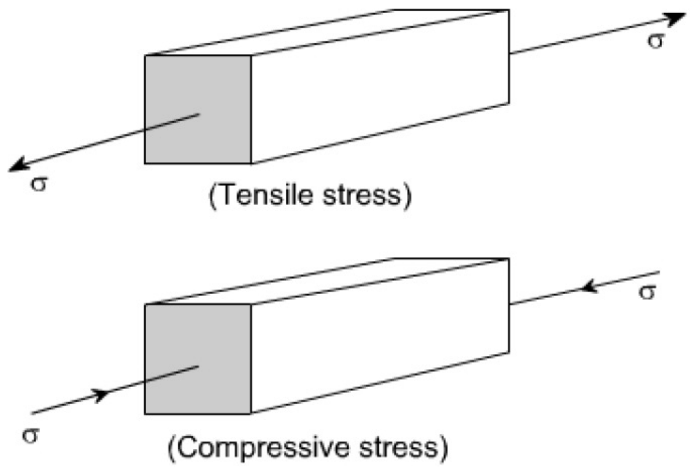


Figure (1.4)

Bearing Stress

- When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses) as shown in Figure (1.5).

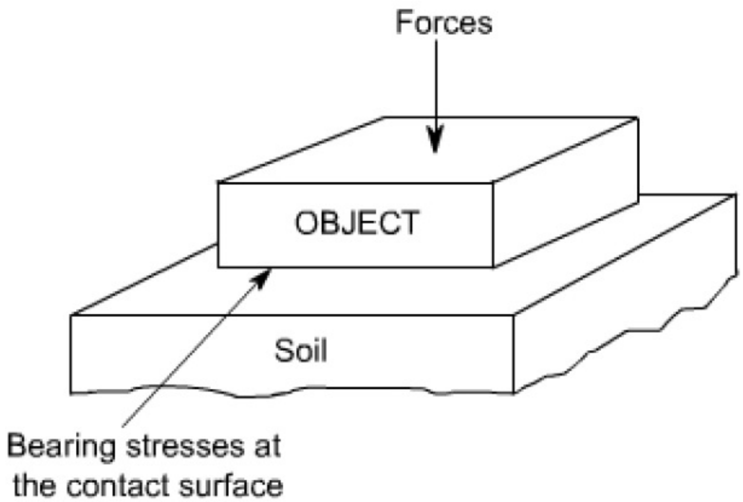


Figure (1.5)

BEARING STRESS IN CONNECTIONS

- Bolts, pins, and rivets create stresses in the members they connect, along the bearing surface, or surface of contact.
- The bolt exerts on plate A a force P equal and opposite to the force F exerted by the plate on the bolt (Figure 1.6).

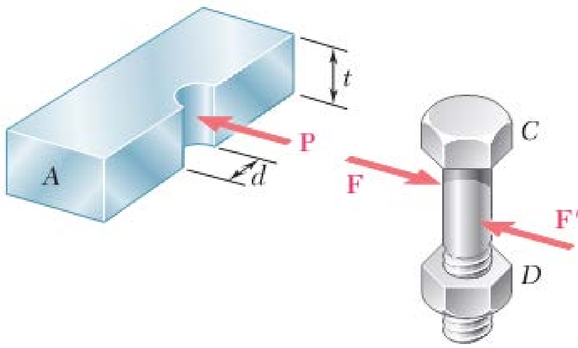


Figure (1.6)

- The force P represents the resultant of elementary forces distributed on the inside surface of a half- cylinder of diameter **d** and of length **t** equal to the thickness of the plate.
- An average nominal value σ_b of the stress, called the bearing stress, obtained by dividing the load P by the area of the rectangle representing the projection of the bolt on the plate section (Figure 1.7)

$$\sigma_b = \frac{P}{A} = \frac{P}{t.d}$$

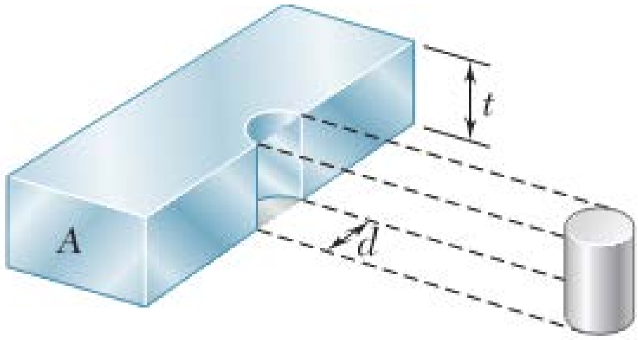
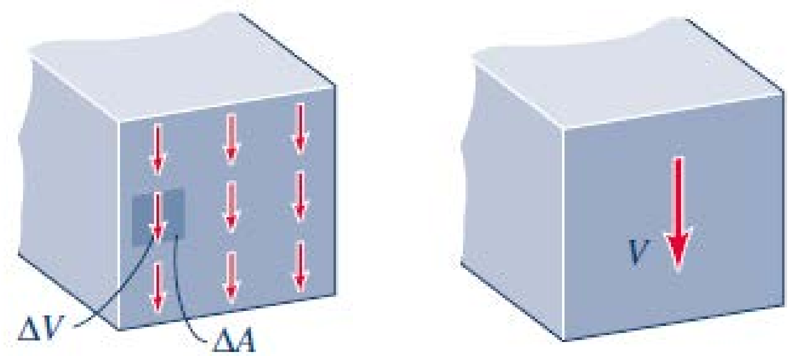


Figure (1.7)

Shear stresses

- Let us consider now the situation, where the cross – sectional area of a block of material is subject to a distribution of forces which are parallel, rather than normal, to the area concerned as shown in Figure (1.8a). Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force interested (Figure 1.8b) are known as shear stresses. The Greek symbol **τ** (tau)(suggesting tangential) is used to denote shear stress.



(a) **(b)**

Figure (1.8)

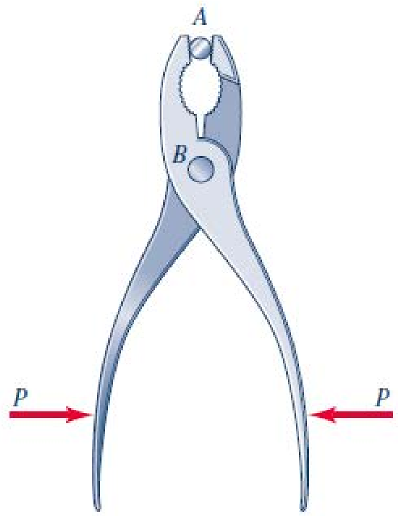
- The mean shear stress being equal to:

$$\tau = \frac{P}{A}$$

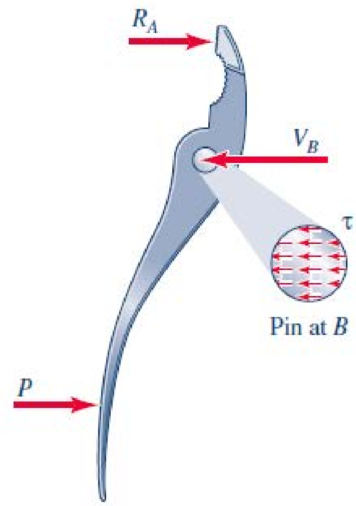
Where **P** is the total force and **A** the area over which it acts.

Single Shear and Double Shear

- Many of the circumstances that can be characterized as direct shear may be further classified as single shear or as double shear.
- This applies particularly to connections such as pinned, bolted, or welded joints.
- A single-shear connection is one where there is a single plane on which shear stress acts to transfer load from one member to the adjacent member.
- The pin of the pliers in Figure (1.9b) is one example of a single-shear connection.



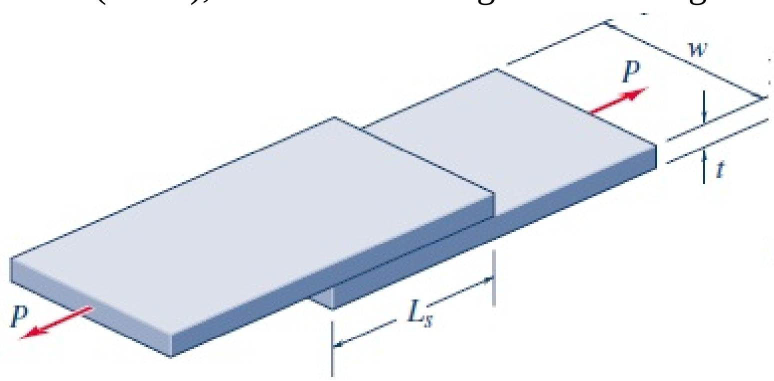
(a) A pair of pliers



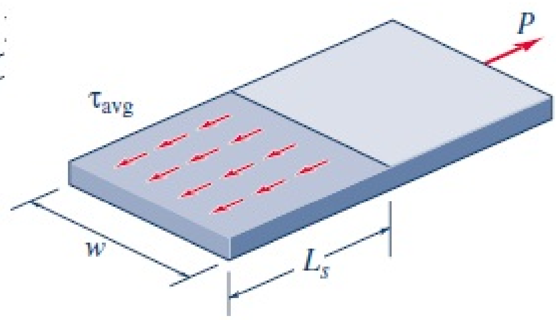
(b) Direct shear of pin

Figure (1.9): Example of Direct Shear

- As another example of single shear, consider the ***lap joint***, or lap splice, in Figure (1.10*a*), where two rectangular bars are glued together to form a tension member.



(a) A lap Splice



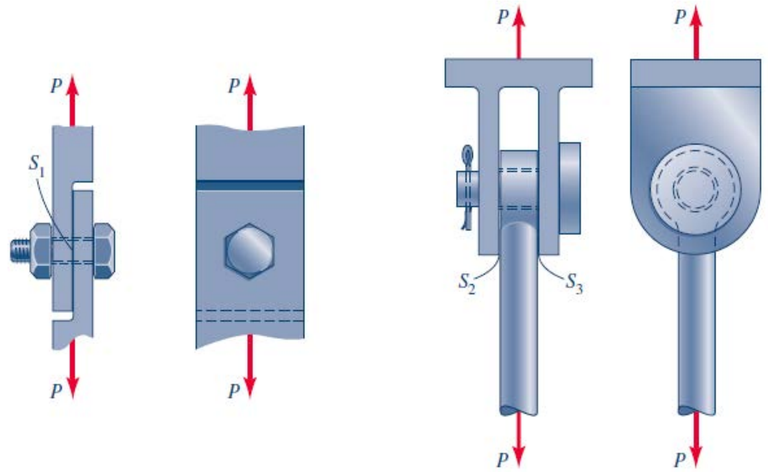
(b) Average Shear Stress Distribution

Figure (1.10): An illustration of direct shear—a lap splice.

- The average shear stress on the splice area is:

$$\tau_{avg.} = \frac{V}{A} = \frac{P}{L_s w}$$

- Figure (1.11) illustrate example of single and double shear.



(a) Single Shear **(b) Double Shear**

Figure (1.11)

Examples

Example (1-1): A steel rod (1m) long and (20mmx20mm) in cross – section is subjected to a tensile force of (40 kN). Determine the stress in the rod.

Solution:

$$A = 20 \times 20 = 400\ mm^2$$

$$\sigma_t = \frac{P}{A} = \frac{40 \times 10^3}{400} = 100\ \frac{N}{mm^2}\ (100\ MPa)$$

Example (1-2): A hollow cylinder (2m) long has an outer diameter of (50mm) and inside diameter of (30mm). If the cylinder is carrying a load of (25 kN), find the stress in the cylinder.

Solution:

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(50)^2 - (30)^2] = 1257 mm^2$$

▶ The stress in the cylinder is:

$$\sigma_t = \frac{P}{A} = \frac{25 \times 10^3}{1257} = 19.9\ MPa$$

Example (1-3): A load of (5kN) is to be raised with the help of a steel wire. Find the minimum diameter of the steel wire, if the stress is not to exceed (100 MPa).

Solution:

$$\sigma = \frac{P}{A} \rightarrow 100 = \frac{5 \times 10^3}{\frac{\pi}{4}(d^2)} \rightarrow d^2 = \frac{5 \times 10^3}{\frac{\pi}{4}(100)} = 63.66\,mm^2$$

$\therefore d = 7.98$ (8)

Example (1-4): A steel bar ABCD (4m) long is subjected to forces as shown in Figure (1.12). Find the stresses in each part.

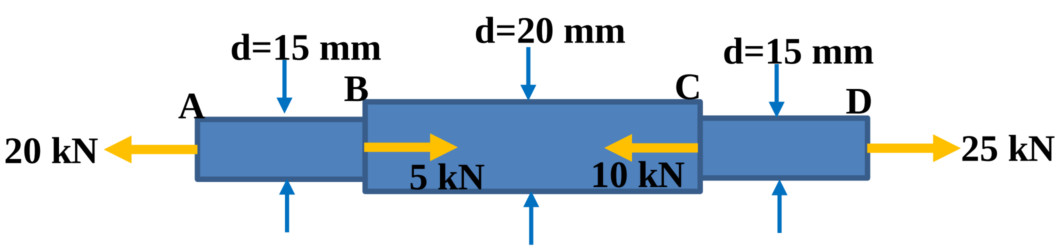


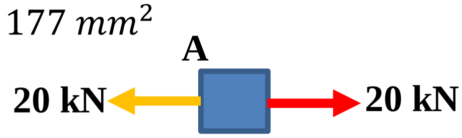
Figure (1.12)

Solution:

- The area of the first and third parts of the bar are:

$$A_1 = A_3 = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(15)^2 = 177\,mm^2$$

- The stress in the first part is:

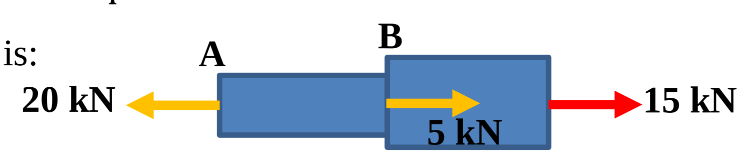


$$\sigma_t = \frac{P}{A} = \frac{20 \times 10^3}{177} = 113\,MPa$$

- And the area of the middle part of the bar is:

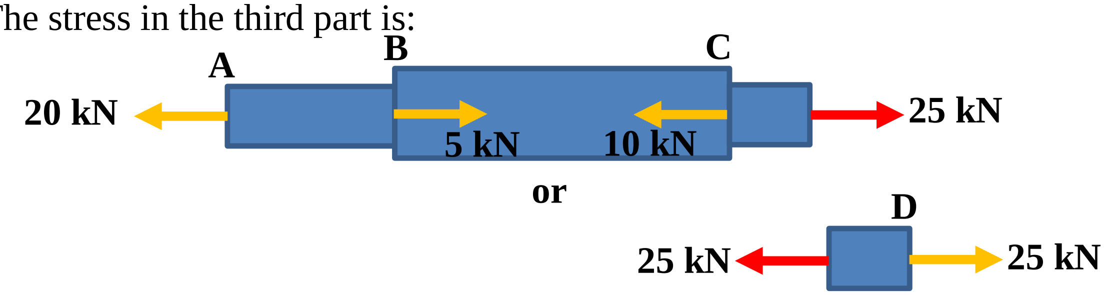
$$A_2 = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(20)^2 = 314\,mm^2$$

- The stress in the second part is:



$$\sigma_t = \frac{P}{A} = \frac{15 \times 10^3}{314} = 47.77\,MPa$$

- The stress in the third part is:



$$\sigma_t = \frac{P}{A} = \frac{25 \times 10^3}{177} = 141.24 \, MPa$$

Example (1-5): A round tapered alloy bar (4m) long is subjected to load as shown in Figure (1.13). Find the stresses in section at point (B) and section at point (C).

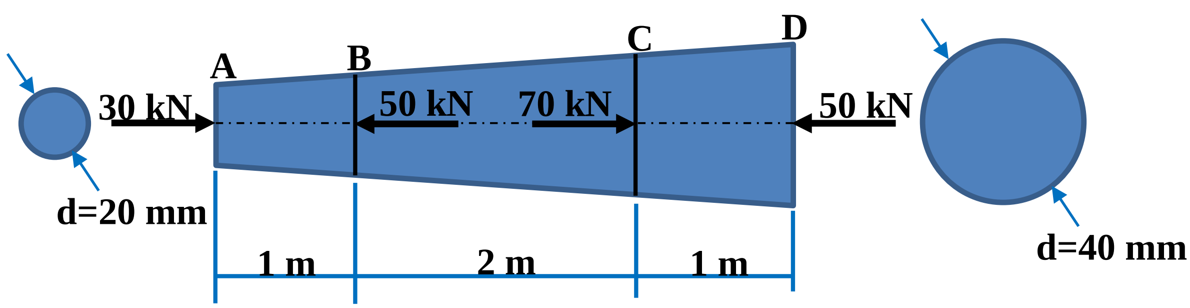


Figure (1.13)

Solution:

- Diameter of the bar at point (B) is:

$$d_B = 20 + (40 - 20)\left(\frac{1}{4}\right) = 25 \, mm$$

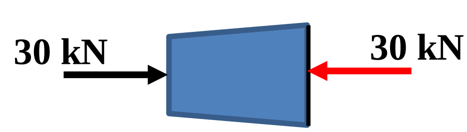
- Diameter of the bar at point (C) is:

$$d_C = 25 + (40 - 20)\left(\frac{2}{4}\right) = 35 \, mm$$

- The area at point (B) is:

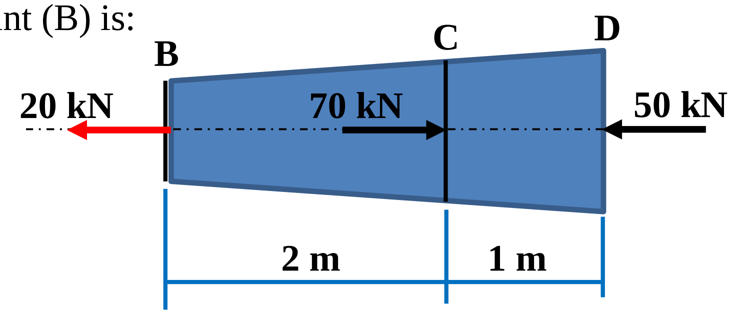
$$A_B = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(25)^2 = 490.87 \, mm^2$$

- Just to the left of point (B) is:



$$\sigma_B = \frac{P}{A} = \frac{30 \times 10^3}{490.87} = 61.12 \, MPa$$

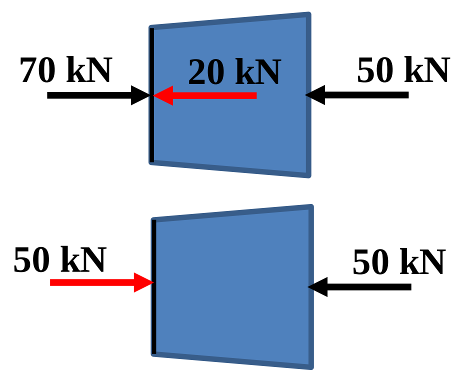
▶ Just to the right of point (B) is:



$$\sigma_B = \frac{P}{A} = \frac{20 \times 10^3}{490.87} = 40.74\ MPa$$

▶ The area at point (C) is:

$$A_C = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(35)^2 = 962.11\ mm^2$$



▶ Just to the left of point (C) is:

$$\sigma_C = \frac{P}{A} = \frac{20 \times 10^3}{962.11} = 20.79\ MPa$$

▶ Just to the right of point (C) is:

$$\sigma_C = \frac{P}{A} = \frac{50 \times 10^3}{962.11} = 51.97\ MPa$$

Example (1-6): The end chord of a timber truss is framed into the bottom chord as shown in Figure (1.14). Neglecting friction,

a) Compute dimension b, if the allowable shearing stress is (900 kPa) and

b) Determine dimension c so that the bearing stress does not exceed (7MPa).

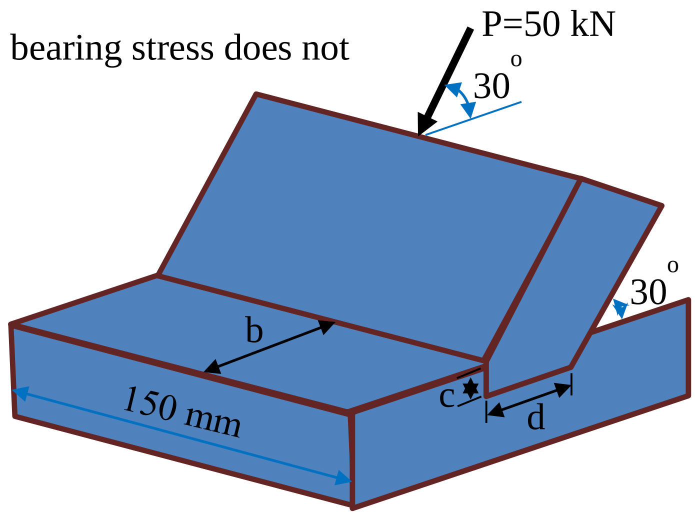


Figure (1.14)

Solution:

a)

$$\tau = \frac{V}{A} = \frac{50 \times 10^{6}. cos30}{b(150)} = 900 \times 10^{-3}$$

$$\rightarrow b = 320.7\ mm\ \cong 321\ mm$$

b)

$$\sigma_{bearing} = \frac{50 \times 10^{3}. cos30}{c(150)} = 7$$

$$\rightarrow c = 41.2\ mm$$

c)

$$\sigma_{bearing} = \frac{50 \times 10^{3}. sin30}{d(150)} = 7$$

$$\rightarrow d = 23.8\ mm$$

Example (1-7): The bell crank shown in Figure (1.15), is in equilibrium,

a) Determine the required diameter of the connecting rod (AB) if its axial stress is limited to (100 MPa).

b) Determine the shearing stress in the pin at point (D) if its radius the (20 mm).

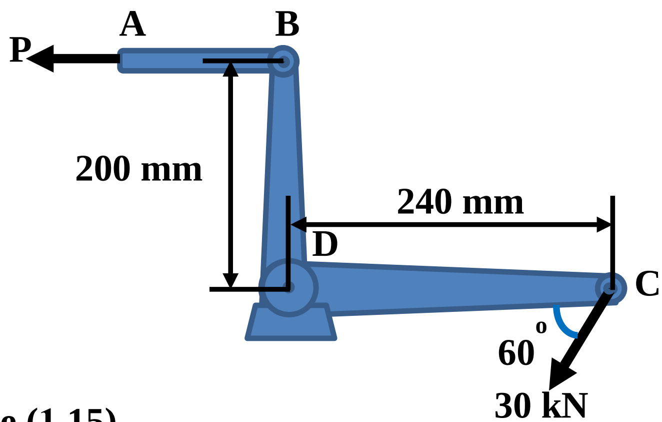


Figure (1.15)

Solution:

a) $$\sum M_D = 0 \rightarrow P(200) = 30sin60(240)$$

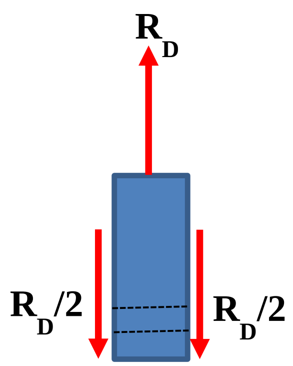
$$\rightarrow P = 31.176\ kN$$

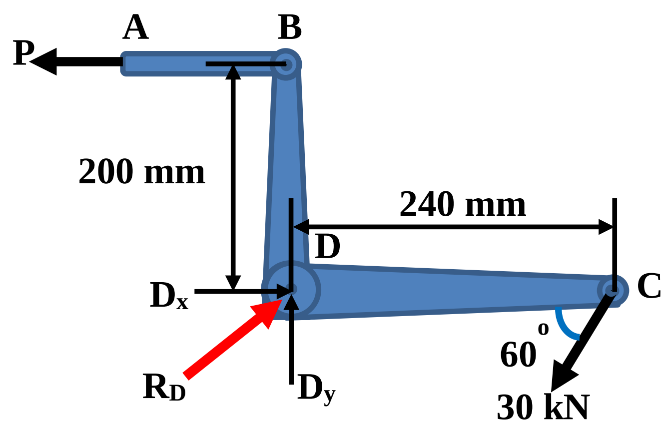
$$\sigma = \frac{P}{A} = \frac{31.176 \times 10^3}{A} = 100\ MPa$$

$$\rightarrow A = \frac{31.176 \times 10^3}{100} = 311.77\ mm^2$$

$$A = \pi(r^2) = 311.77\ \rightarrow r \cong 10\ mm$$

b) Frame as F.B.D:





$$\sum F_x = 0\ \rightarrow D_x = P + 30cos60\ \rightarrow D_x = 48.5\ kN$$

$$\sum F_y = 0\ \rightarrow D_y = 30sin60\ \rightarrow D_y = 25.98\ kN$$

$$R_D = \sqrt{(D_x)^2 + (D_y)^2} = \sqrt{(48.5)^2 + (25.98)^2} = 55\ kN$$

$$\tau = \frac{R_d/2}{\pi r^2} = \frac{55/2 \times 10^3}{\pi(20)^2} = 87.5\ MPa$$

Example (1-8): The lap joint shown in Figure (1.16) is fastened by three (20 mm) diameter rivets. Assuming that (P=50 kN), determine,

a) The shearing stress in each rivet.

b) The bearing stress in each plate, and the maximum average tensile stress in each plate. Assume that the applied load (P) is distributed equally among the three rivets.

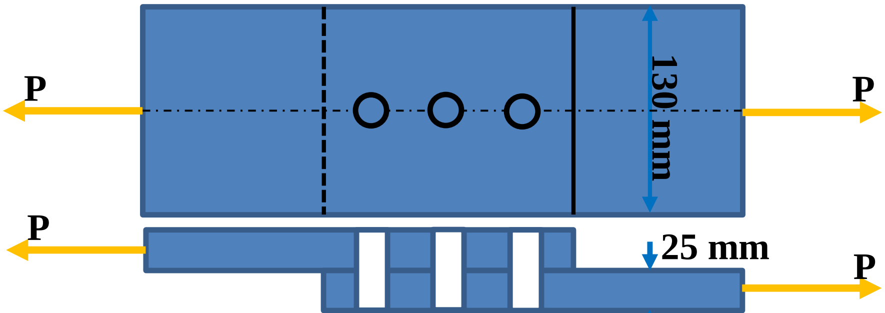
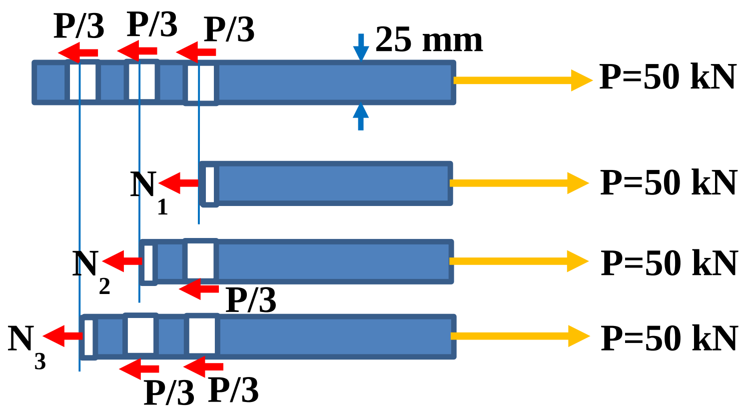


Figure (1.16)

Solution:



a)

$$\tau = \frac{P/3}{A_{rivet}} = \frac{50/3 \times 10^3}{\frac{\pi}{4}(20)^2} = 53\ MPa$$

b)

$$\sigma_b = \frac{P/3}{D(t)} = \frac{50/3 \times 10^3}{20(25)} = 33.33\ MPa$$

▶ For section (1-1):

$$\sigma_t = \frac{N_1}{(b-D)(t)} \quad , N_1 = P$$

$$\sigma_{t1} = \frac{50 \times 10^3}{(130-20)(25)} = 18.18\ MPa$$

- For section (2-2):

$$\sigma_t = \frac{N_2}{(b-D)(t)} \quad , N_2 = P - P/3$$

$$\sigma_{t2} = \frac{(50 - 50/3) \times 10^3}{(130-20)(25)} = 12.12 \; MPa$$

- For section (3-3):

$$\sigma_t = \frac{N_3}{(b-D)(t)} \quad , N_3 = P - 2P/3$$

$$\sigma_{t3} = \frac{(50 - 2(50)/3) \times 10^3}{(130-20)(25)} = 6.06 \; MPa$$

The maximum tensile stress is 18.18 MPa at section (1-1)

- In the same example, determine the maximum safe load (P) which may be applied if the shearing stress in the rivets is limited to (60 MPa), the bearing stress in the plates to (110 MPa), and the average tensile stress in the plate to (140 MPa).

Solution:

$$\tau = \frac{P/3}{A_{rivet}} \rightarrow 60 = \frac{P/3}{\frac{\pi}{4}(20^2)} \rightarrow P = 56.5 \; kN$$

$$\sigma_b = \frac{P/3}{D(t)} \rightarrow 110 = \frac{P/3}{20(25)} \rightarrow P = 165 \; kN$$

$$\sigma_t = \frac{P}{(b-D)(t)} \rightarrow 140 = \frac{P}{(130-20)(25)} \rightarrow P = 385 \; kN$$

- The maximum P is the smallest one P=56.5 kN