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Al-Mustaqbal University College
Air Conditioning and Refrigeration Techniques Engineering Department

Strength of Materials

Second Stage

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BEAMS

Shearing Force and Bending Moment Diagram

Introduction

- The transverse loading of a beam may consist of *concentrated loads* P_1, P_2, . . . , expressed in newton, pounds, or their multiples, kilo newton and kips (Figure 2.1a), of a *distributed load* w, expressed in N/m, kN/m, lb/ft, or kips/ft (Figure 2.1*b*), or of a combination of both. When the load w per unit length has a constant value over part of the beam (as between A and B in Figure 2.1*b*), the load is said to be *uniformly distributed* over that part of the beam.

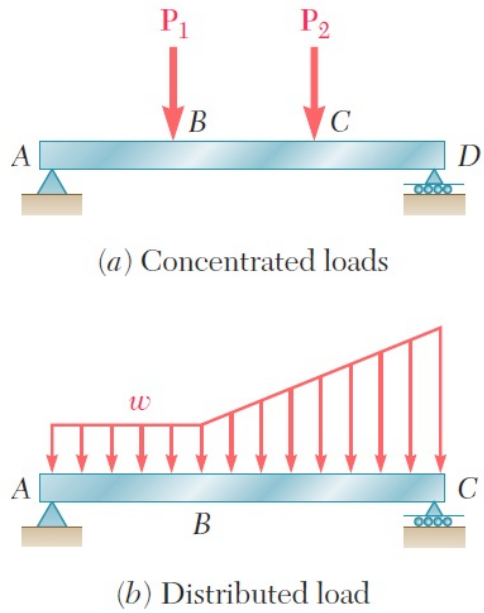
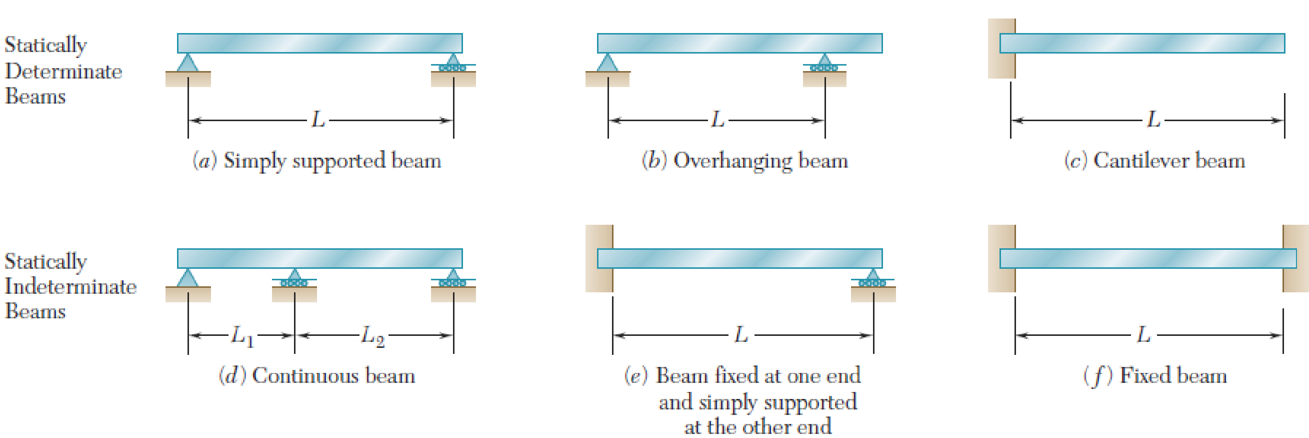


Figure 2.1: Transversely Loaded Beams

Classification of Beams

- Beams are classified according to the way in which they are supported. Several types of beams frequently used are shown in Figure 2.2. The distance L shown in the various parts of the figure is called the span.



- Sometimes two or more beams are connected by hinges to form a single continuous structure. Two examples of beams hinged at a point *H* are shown in Figure 2.3.

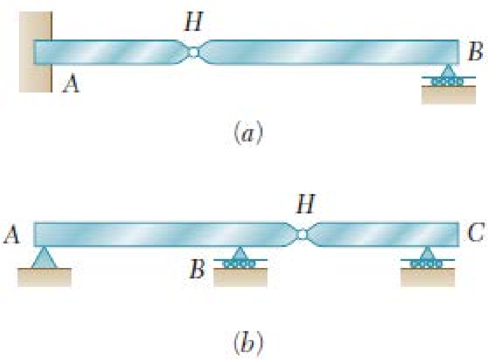


Figure 2.3: Beams Connected by Hinges

- When a beam is subjected to transverse loads, the internal forces in any section of the beam will generally consist of a shear force V and a bending couple M.
- Consider, for example, a simply supported beam *AB* carrying two concentrated loads and a uniformly distributed load (Figure 2.4*a*).
- To determine the internal forces in a section through point *C* we first draw the free-body diagram of the entire beam to obtain the reactions at the supports (Figure 2.4*b*).
- Passing a section through *C*, we then draw the free-body diagram of *AC* (Figure 2.4*c*), from which we determine the shear force V and the bending couple M.

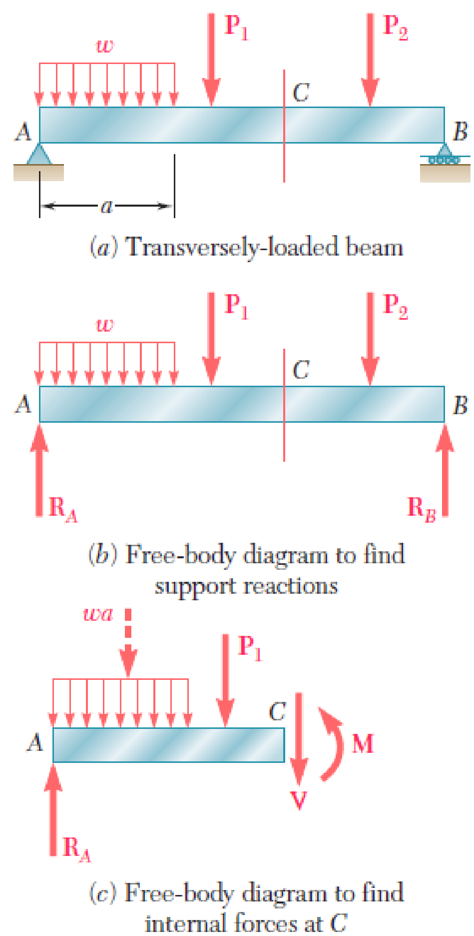
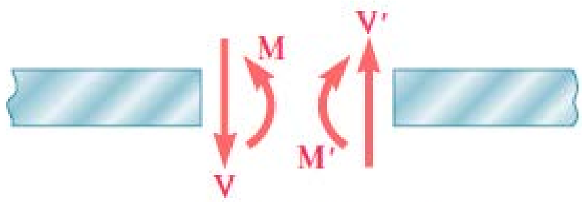


Figure 2.4

Sign Conventions

- The shear ***V*** and the bending moment ***M*** at a given point of a beam are said to be positive when the internal forces and couples acting on each portion of the beam are directed as shown in Figure 2.5.



(*a*) Internal forces
(positive shear and positive bending moment)

Figure 2.5

These conventions can be more easily remembered if we note that:

- The shear at any given point of a beam is positive when the external forces (loads and reactions) acting on the beam tend to shear off the beam at that point as indicated in Figure 2.6.

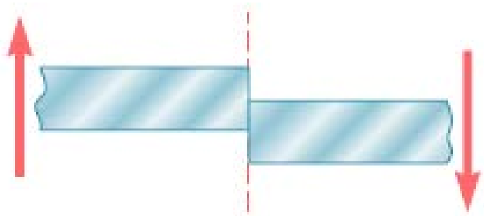

(*b*) Effect of external forces
(positive shear)

Figure 2.6

- The bending moment at any given point of a beam is positive when the external forces acting on the beam tend to bend the beam at that point as indicated in Figure 2.7.

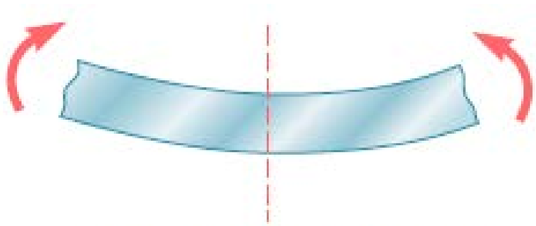

(*c*) Effect of external forces
(positive bending moment)

Figure 2.7

Basic Relationship between the Rate of Loading, Shear Force and Bending Moment

- A simple beam with a varying load indicated by *w(x)* is sketched in Figure 2.8. The coordinate system with origin at the left end A is established and distances to various sections in the beam are denoted by the variable x.

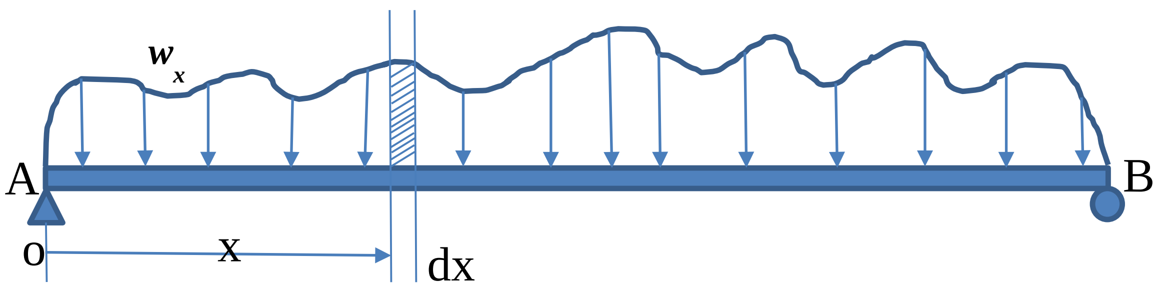


Figure 2.8

- Basic Relationship Between the Rate of Loading, Shear Force and Bending Moment
- For any value of ***x***, the relationship between the load ***w(x),*** the shearing force ***V*** and bending moment ***M*** is:

$$w = -\frac{dV}{dx} = -\frac{d^2M}{dx^2} \qquad (1)$$

- And for the relationship between shearing force ***V*** and bending moment ***M*** is:

$$V = \frac{dM}{dx} \qquad (2)$$

- Conclusions: From the above relations, the following important conclusions may be drawn:

- From Equ. (2), the area of the shear force diagram between any two points, from the basic calculus is the bending moment diagram.

$$M = \int V dx$$

- The slope of bending moment diagram is the shear force, thus:

$$V = \frac{dM}{dx}$$

- Thus, if ***V*=0**, the slope of the bending moment diagram is zero and the bending moment is therefore constant.

- The maximum or minimum bending moment occurs where

$$\frac{dM}{dx} = 0$$

- The slope of the shear force diagram is equal to the magnitude of the intensity of the distributed loading at any position along the beam. The (-*ve*) sign is as a consequence of our particular choice of sign conventions.

Examples

There are two methods to solve problems of shear and bending diagrams:

1) Method of Equations, and
2) Method of areas

Example (1): Draw the shear and bending moment diagrams for the beam shown in Figure (2.9).

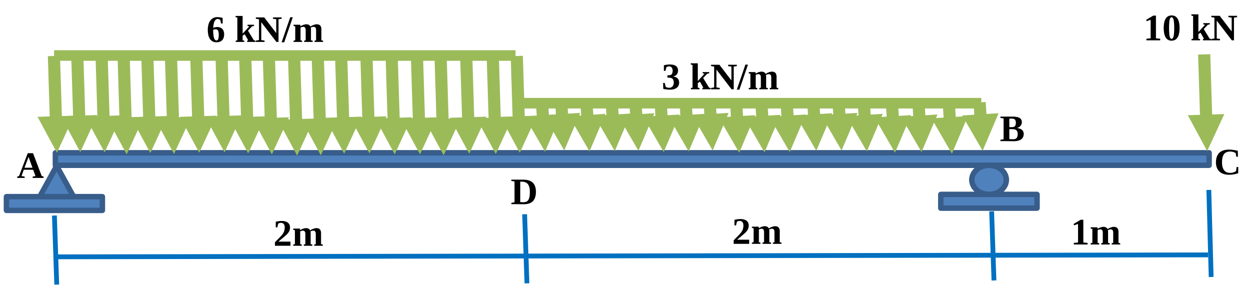


Figure (2.9)

Solution: Method of Equations

- First, find all reactions of the beam

$$\sum M_A = 0 \qquad \rightarrow 6(2)(1) + 3(2)(3) + 10(5) - 4(R_B) = 0$$

$$\rightarrow \quad R_B = 20\ kN\ \uparrow$$

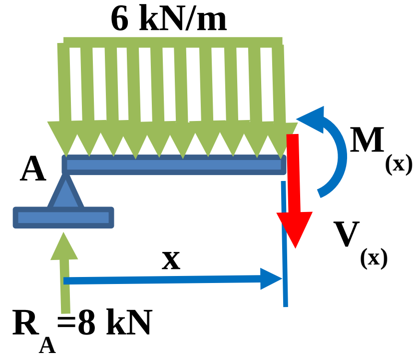
$$\sum F_y = 0 \qquad \rightarrow 6(2) + 3(2) + 10 - R_A - R_B = 0$$

$$\rightarrow \quad R_A = 8\ kN\ \uparrow$$

$$\sum F_x = 0 \qquad \rightarrow \quad A_x = 0$$

- Now divide the beam into three regions according to the change in loading. These regions are AD, DB, and BC.

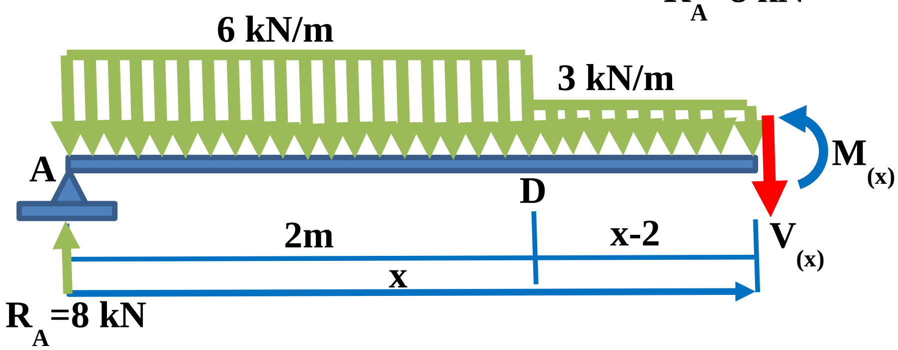
- Part AD: ($0 \leq x \leq 2$)



$$+\uparrow \sum F_y = 0 \quad \rightarrow 8 - 6x = V_{(x)}$$

$$\sum M_{sec.} = 0 \quad \rightarrow 8x - 6\frac{x^2}{2} = M_{(x)}$$

- Part DB: ($2 \leq x \leq 4$)



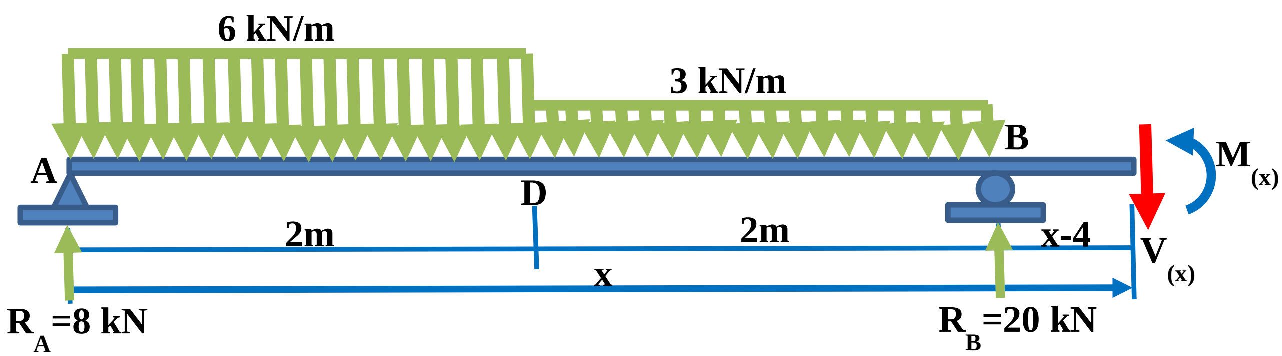
$$+\uparrow \sum F_y = 0 \rightarrow 8 - 6(2) - 3(x-2) = V_{(x)}$$

$$\rightarrow V_{(x)} = 2 - 3x$$

$$\sum M_{sec.} = 0 \rightarrow 8x - 6(2)(x-1) - 3\frac{(x-2)^2}{2} = M_{(x)}$$

$$\rightarrow M_{(x)} = 12 - 4x - 1.5(x-2)^2$$

- Part BC: ($4 \leq x \leq 5$)

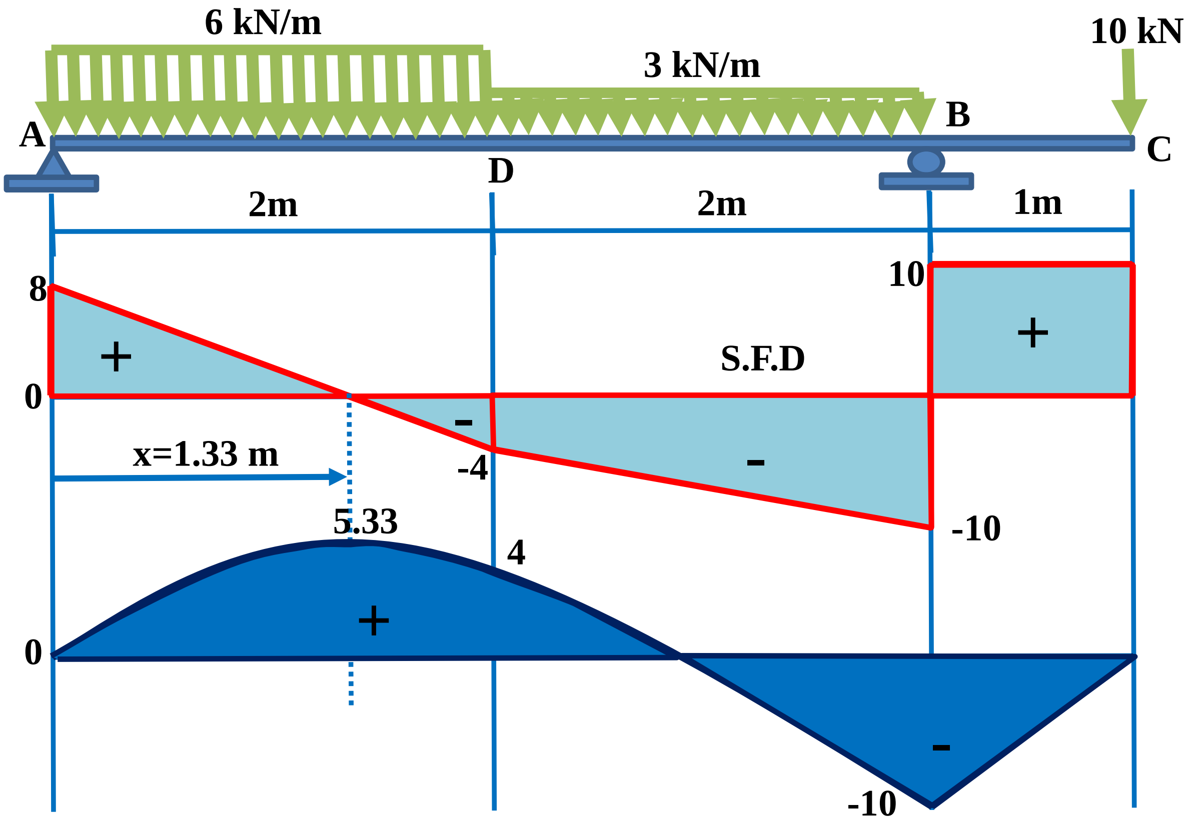


$$+\uparrow \sum F_y = 0 \quad \rightarrow 8 - 6(2) - 3(2) = V_{(x)}$$

$$\rightarrow V_{(x)} = 10$$

$$\sum M_{sec.} = 0 \qquad \rightarrow 8x - 6(2)(x-1) - 3(2)(x-3) + 20(x-4) = M_{(x)}$$

$$\rightarrow M_{(x)} = 10x - 50$$



Notes:

- ❑ If load to the n degree ,then:
- ❑ The shear is to the degree (n+1), and
- ❑ The moment is to the degree (n+2)

For this example,

- The load is to the zero degree (n=0),
- The shear is to the 1st degree (n=1), and
- The moment is to the 2nd degree (n=2)

Increasing and decreasing in slopes

Figure (2.10) shows how you can draw the increasing curve and the decreasing curve.

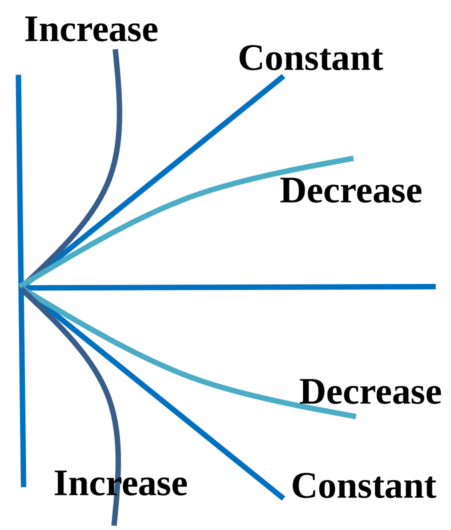


Figure (2.10)

Example (2): Draw the shear force and bending moment diagrams for the beam shown in Figure (2.11).

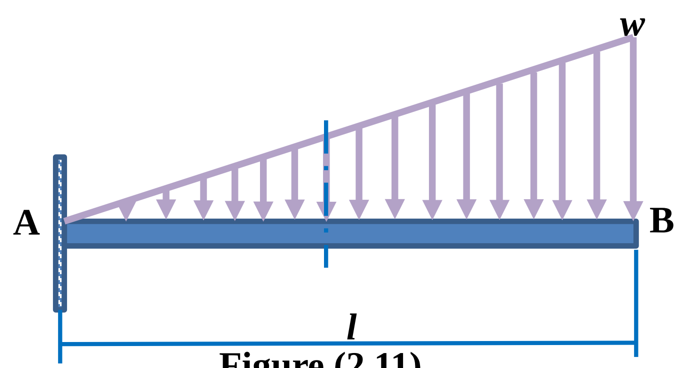


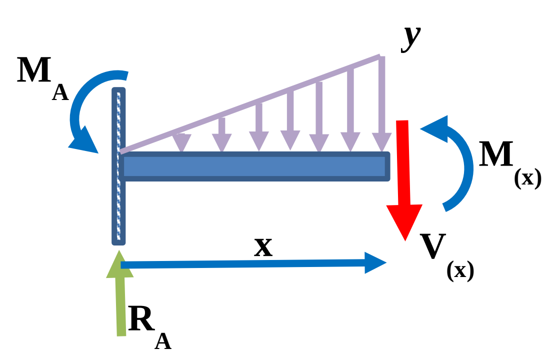
Figure (2.11)

Solution:

$$\sum M_A = 0 \qquad \rightarrow \frac{wl}{2}\left(\frac{2}{3}l\right) = M_A$$

$$\rightarrow M_A = \frac{wl^2}{3}$$

$$+\uparrow \sum F_y = 0 \qquad \rightarrow \quad R_A = \frac{wl}{2}$$



Now, for section:

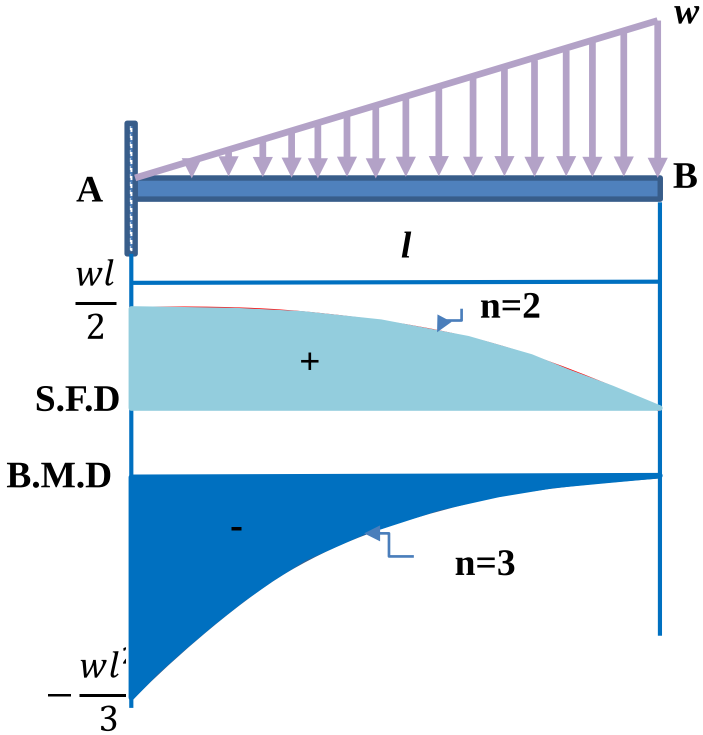
$$\frac{y}{x} = \frac{w}{l} \quad \rightarrow y = \frac{x}{l}w$$

$$+\uparrow \sum F_y = 0 \rightarrow V_{(x)} = \frac{wl}{2} - \frac{1}{2}xy$$

$$V_{(x)}=\frac{wl}{2}-\frac{1}{2}x\left(\frac{x}{l}w\right)=\frac{wl}{2}-\frac{1}{2}\frac{x^2}{l}w$$

$$\sum M_{sec}=0\rightarrow M_{(x)}=\frac{wl}{2}(x)-\frac{wl^2}{3}-\frac{1}{2}(xy)\left(\frac{x}{3}\right)$$

$$\rightarrow M_{(x)}=\frac{wl}{2}x-\frac{wl^2}{3}-\frac{wx^2}{6l}$$



2- Method of Areas

$$V_B=V_A+area\ of\ load\ between\ A\ and\ B$$

$$M_B=M_A+area\ of\ shear\ between\ A\ and\ B$$

Example (3): Resolve example (1) by method of areas.

Solution:

$$V_B=8+(-6\times 2)=-4\ kN$$

$$V_C=-4+(-3\times 2)=-10\ kN$$

$$V_D=-10+20+0=10\ kN$$

$$\frac{8+4}{2}=\frac{8}{x} \quad \rightarrow \quad x=1.33\ m$$

$$M_E=M_A+\frac{1}{2}(8)(1.33)=5.33\ kN.m$$

$$M_B=5.33-\frac{1}{2}(4)(2-1.33)=4\ kN.m$$

$$M_C=4-\left(\frac{4+10}{2}\right)(2)=-10\ kN.m$$

$$M_D=-10+10\times 1=0$$

Example (4): Draw the shear force and bending moment diagrams for the beam shown in Figure (2.12).

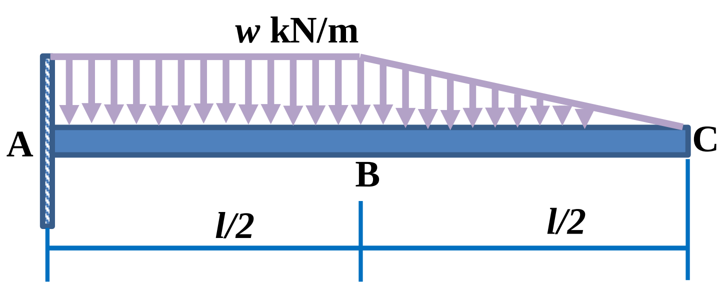


Figure (2.12)

Solution:

$$+\uparrow \sum F_y=0 \qquad \rightarrow \quad R_A=\frac{3wl}{4}$$

$$\sum M_A=0 \qquad \rightarrow M_A=w\frac{l}{2}\left(\frac{l}{4}\right)+\frac{1}{2}(w)\left(\frac{l}{2}\right)\left(\frac{l}{2}+\frac{1}{3}\frac{l}{2}\right)=\frac{7}{24}wl^2$$

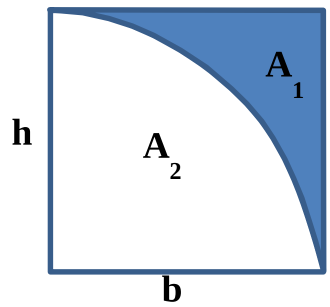
$$V_B=\frac{3wl}{4}-\frac{wl}{2}=\frac{wl}{4}$$

$$V_C=\frac{wl}{4}-\frac{1}{2}(w)\left(\frac{l}{2}\right)=0$$

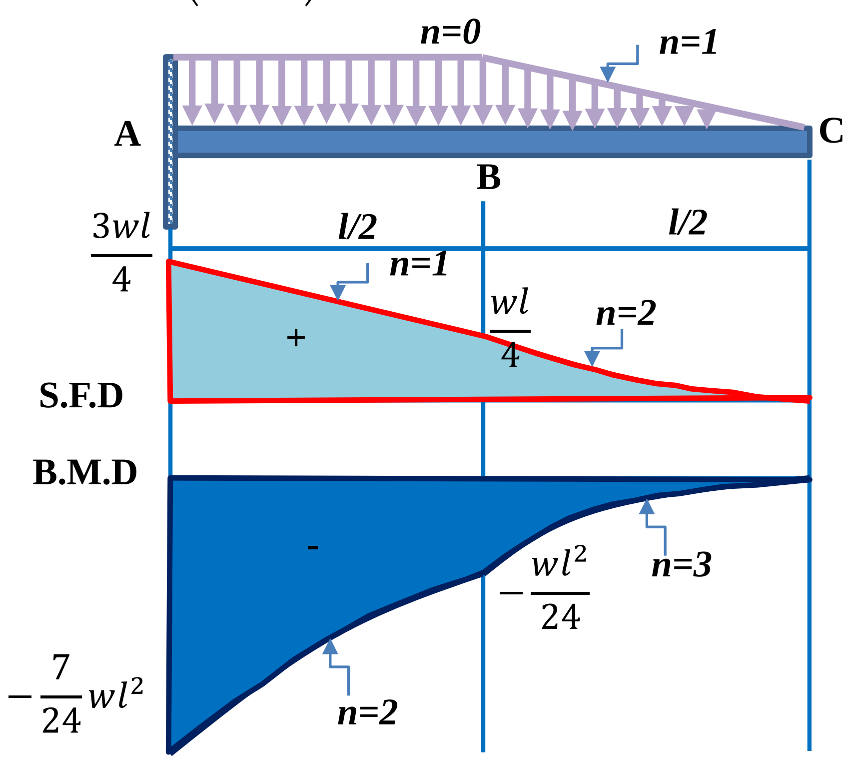
$$M_B=-\frac{7}{24}wl^2+\left(\frac{\frac{3wl}{4}+\frac{wl}{4}}{2}\right)\frac{l}{2}=-\frac{7}{24}wl^2+\frac{wl^2}{4}=-\frac{wl^2}{24}$$

$$A_1 = \frac{bh}{n+1}$$

$$A_2 = \frac{nbh}{n+1}$$



$$M_C = -\frac{wl^2}{24} + \left(\frac{\frac{l}{2} \times \frac{wl}{4}}{2+1} \right) = 0$$



Example (5): Draw the shear force and bending moment diagrams for the beam shown in Figure (2.13).

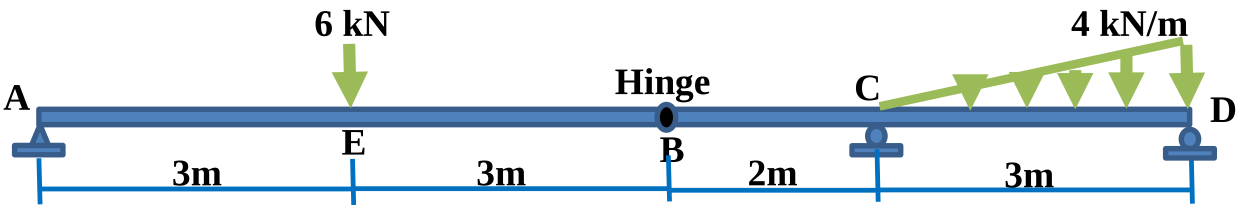
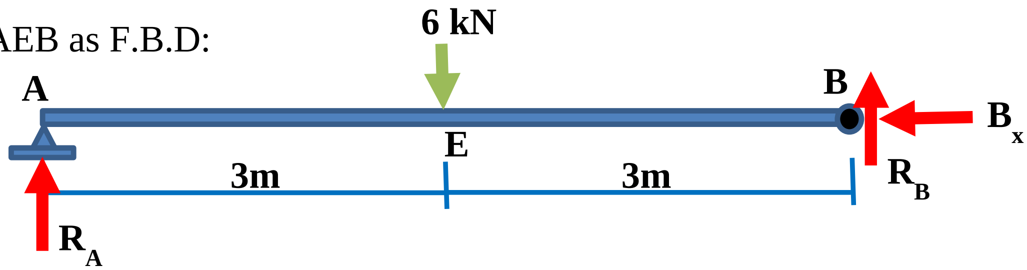


Figure (2.13)

Solution:

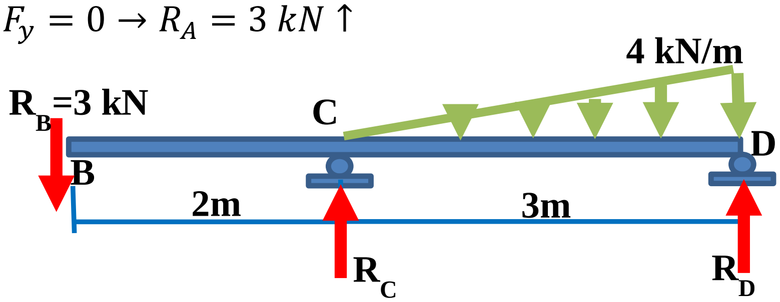
- Part AEB as F.B.D:



$$\sum M_A = 0 \rightarrow R_B = 3\ kN \uparrow$$

$$\sum F_y = 0 \rightarrow R_A = 3\ kN \uparrow$$

- Part BCD as F.B.D:



$$\sum M_A = 0 \rightarrow R_C = 7\ kN \uparrow$$

$$\sum F_y = 0 \rightarrow R_D = 2\ kN \uparrow$$

$$V_E = 3 + 0 = 3\ kN$$

$$V_B = -3 + 0 = -3\ kN$$

$$V_C = -3 + 0 = -3\ kN$$

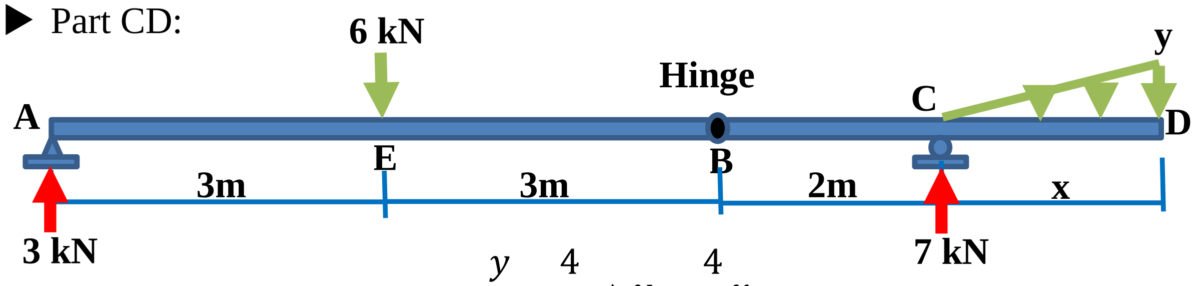
$$V_D = 4 - \frac{1}{2}(4)(3) = -2\ kN$$

$$M_E = 0 + 3(3) = 9\ kN.m$$

$$M_B = 9 - 3(3) = 0$$

$$M_C = 0 - 3(2) = -6\ kN.m$$

▶ Part CD:

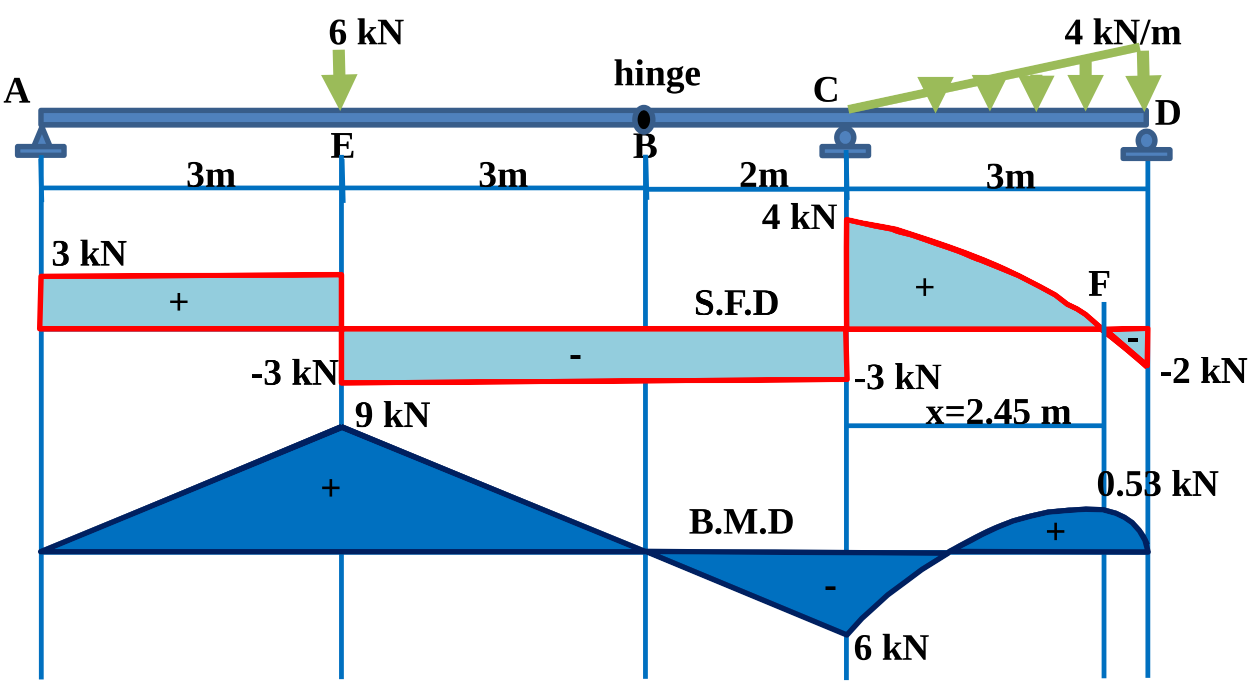


$$\frac{y}{x} = \frac{4}{3} \rightarrow y = \frac{4}{3}x$$

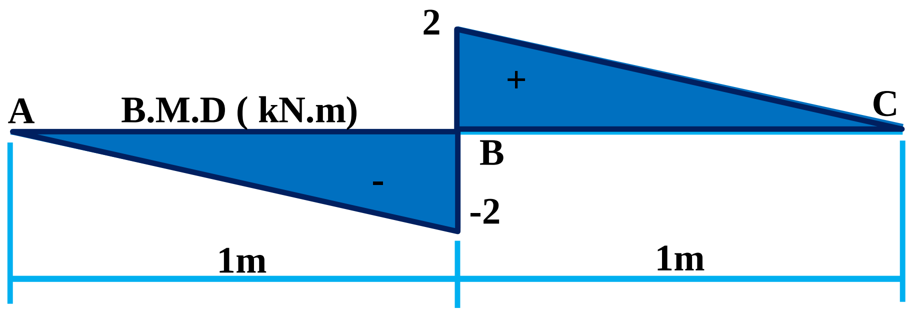
$$3 - 6 + 7 - \frac{1}{2}(xy)$$

$$V_{(x)} = 4 - \frac{2}{3}x^2 = 0 \rightarrow x = 2.45\ m$$

$$M_F = -6 + \frac{4(2.45)(2)}{2+1} = 0.53\ kN.m$$



Example (6): Find loading of the beam if B.M.D is as shown in Figure (2.14) simply support.



Solution:

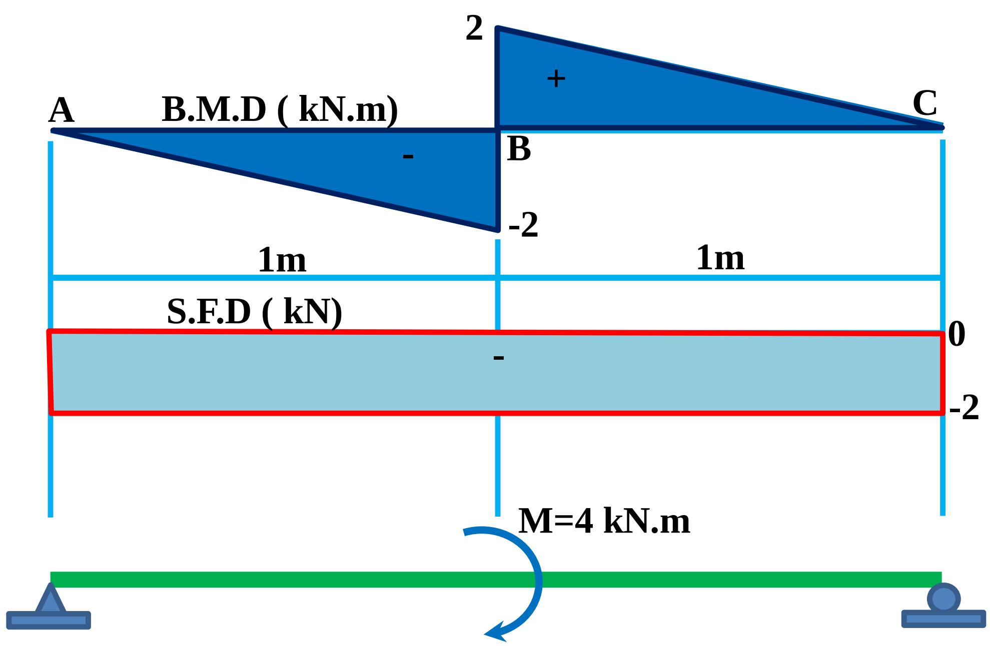
Figure (2.14)

$$M_B = M_A + area\ of\ shear\ AB$$

$$-2 = 0 + V(1) \ \rightarrow V = -2\ kN$$

$$M_{B,new} = M_{B,old} + M$$

$$2 = -2 + M \ \rightarrow M = 4$$



Example (7): A beam is supported at left end only and have S.F.D as shown in Figure (2.15). Draw loading and B.M.D for this beam.

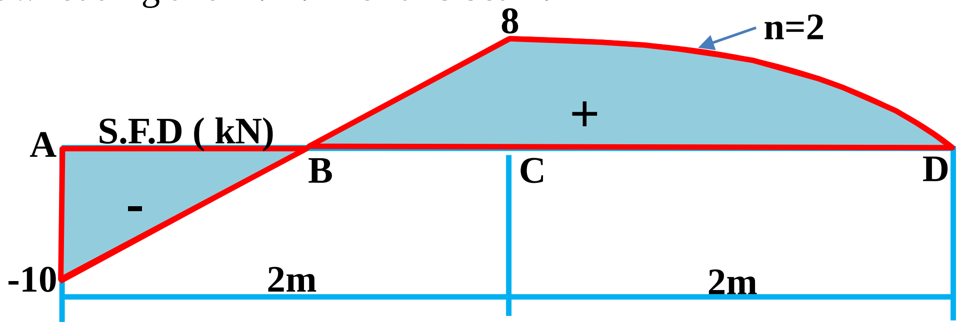


Figure (2.15)

Solution:

$$V_C = V_A + area\ of\ shear\ AC$$

$$8 = -10 + w(2) \quad \rightarrow w = 9\ kN/m$$

$$V_D = V_C + area\ of\ shear\ CD$$

$$0 = 8 + \frac{1}{2}(w_1)(2)$$

$$\rightarrow w_1 = -8\ kN/m$$

$$\sum M_A = 0 \rightarrow M_A = 8.67\ kN.m$$

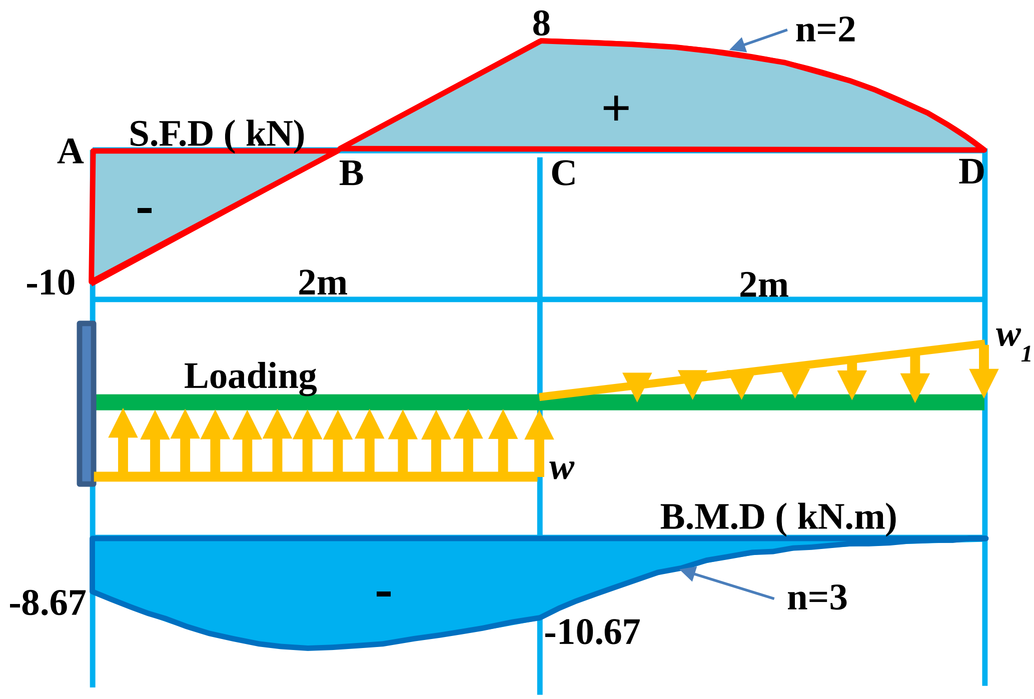
$$M_B = -8.67 + \frac{1}{2}(-10)(1.11)$$

$$\rightarrow M_B = -14.23\ kN.m$$

$$M_C = -14.23 + \frac{1}{2}(-8)(2 - 1.11)$$

$$\rightarrow M_C = -10.67\ kN.m$$

$$M_D = -10.67 + \frac{2(8)(2)}{2+1} = 0$$



Example (8): find the force (P) so that the B.M. becomes zero at point (C) for the beam shown in Figure (2.16).

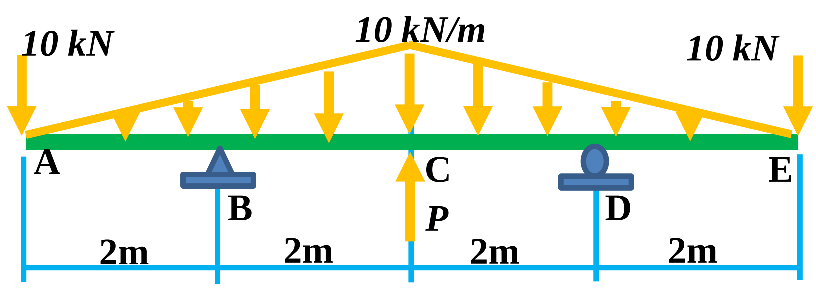


Figure (2.16)

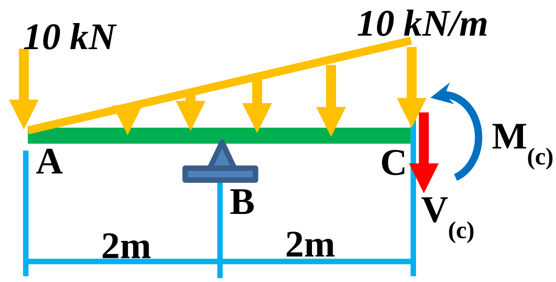
Solution:

$$\sum M_D = 0$$

$$\rightarrow 10(6) - R_B(4) - P(2) + \frac{1}{2}(10)(4)\left(\frac{4}{3} + 2\right) + \frac{1}{2}(10)(4)\left(2 - \frac{4}{3}\right) - 10(2) = 0$$

$$\rightarrow\ R_B = 30 - \frac{P}{2}$$

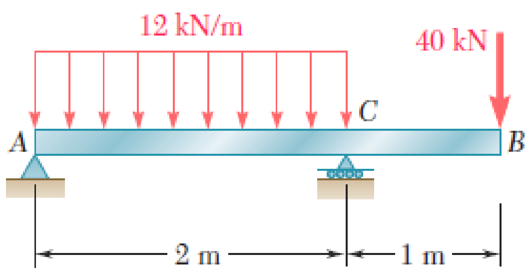
▶ Part ABC as F.B.D:



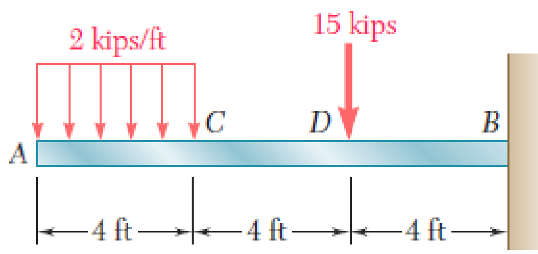
$$M_c = 0 = -10(4) + \left(30 - \frac{P}{2}\right)(2) - \frac{1}{2}(10)(4)\left(\frac{4}{3}\right)$$

$$\rightarrow P = -6.67\ kN$$

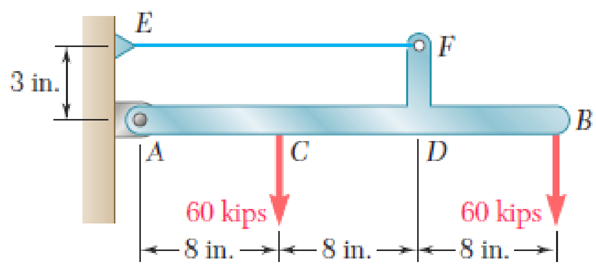
Problem 01: Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Problem 02: Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Problem 03: Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Problem 04: Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.