# CHAPTER 1

## Tension and Compression

### 1.1Internal Effects of Forces

The bodies themselves will no longer be considered to be perfectly rigid as was assumed in statics; instead, the calculation of the deformations of various bodies under a variety of loads will be one of our primary concerns in the study of strength of materials.

### Axially Loaded Bar

The simplest case to consider at the start is that of an initially straight metal bar of constant cross section, loaded at its ends by a pair of oppositely directed collinear forces coinciding with the longitudinal axis of the bar and acting through the centroid of each cross section. For static equilibrium the magnitudes of the forces must be equal. If the forces are directed away from the bar, the bar is said to be in tension; if they are directed toward the bar, a state of compression exists. These two conditions are illustrated in Fig. 1-1.

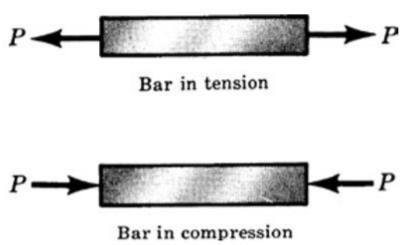


Fig. 1-1 Axially loaded bars.

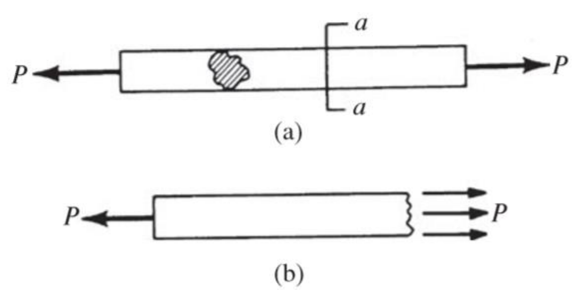


Fig. 1-2 Internal force.

Under the action of this pair of applied forces, internal resisting forces are set up within the bar and their characteristics may be studied by imagining a plane to be passed through the bar anywhere along its length and oriented perpendicular to the longitudinal axis of the bar. Such a plane is designated as a-ain Fig. 1-2(a). If for purposes of analysis the portion of the bar to the right of this plane is considered to be removed, as in Fig. 1-2(b), then it must be replaced by whatever effect it exerts upon the left portion. By this technique of introducing a cutting plane, the originally internal forces now become external with respect to the remaining portion of the body. For equilibrium of the portion to the left this ‘‘effect’’ must be a horizontal force of magnitude P. However, this force P acting normal to the cross section a-a is actually the resultant of distributed forces acting over this cross section in a direction normal to it.

At this point it is necessary to make some assumption regarding the manner of variation of these distributed forces, and since the applied force P acts through the centroid it is commonly assumed that they are uniform across the cross section.

**Normal Stress**

Instead of speaking of the internal force acting on some small element of area, it is better for comparative purposes to treat the normal force acting over a unit area of the cross section. The intensity of normal force per unit area is termed the normal stress and is expressed in units of force per unit area, N/m2. If the forces applied to the ends of the bar are such that the bar is in tension, then tensile stresses are set up in the bar; if the bar is in compression we have compressive stresses. The line of action of the applied end forces passes through the centroid of each cross section of the bar.

**Normal Strain**

Let us suppose that the bar of Fig. 1-1 has tensile forces gradually applied to the ends. The elongation per unit length, which is termed normal strain and denoted by Δ, may be found by dividing the total elongation Δ by the length L, i.e.,

$$\varepsilon = \frac{\Delta}{L}$$

The strain is usually expressed in units of meters per meter and consequently is **dimensionless**.

**Stress-Strain Curve**

As the axial load in Fig. 1-1 is gradually increased, the total elongation over the bar length is measured at each increment of load and this is continued until fracture of the specimen takes place. Knowing the original cross-sectional area of the test specimen, the normal stress, denoted by s, may be obtained for any value of the axial load by the use of the relation

$$\sigma = \frac{P}{A}$$

where (P) denotes the axial load in newtons and (A) The original cross-sectional area. Having obtained numerous pairs of values of normal stress s and normal strain

, experimental data may be plotted with these quantities considered as ordinate and abscissa, respectively. This is the stress-strain curve or diagram of the material for this type of loading. Stress-strain diagrams assume widely differing forms for various materials. Figure 1-3(a) is the stress-strain diagram for a medium-carbon structural steel, Fig. 1-3(b) is for an alloy steel, and Fig. 1-3(c) is for hard steels and certain nonferrous alloys. For nonferrous alloys and cast iron the diagram has the form indicated in Fig. 1-3(d).

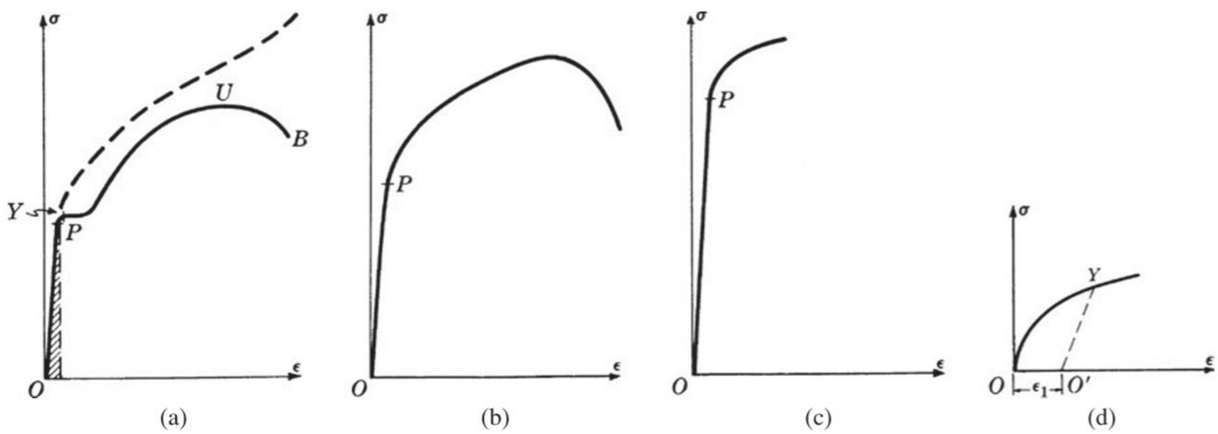
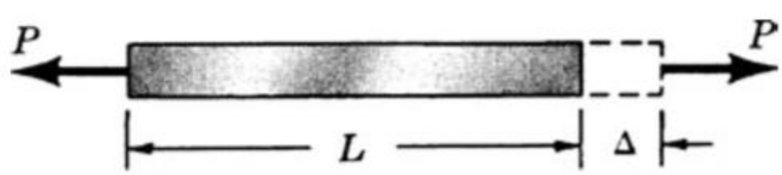



Fig. 1-3 Stress-strain diagrams.

Ex1/ In Fig. 1-8, determine an expression for the total elongation of an initially straight bar of length L, cross-sectional area A, and modulus of elasticity E if a tensile load P acts on the ends of the bar.

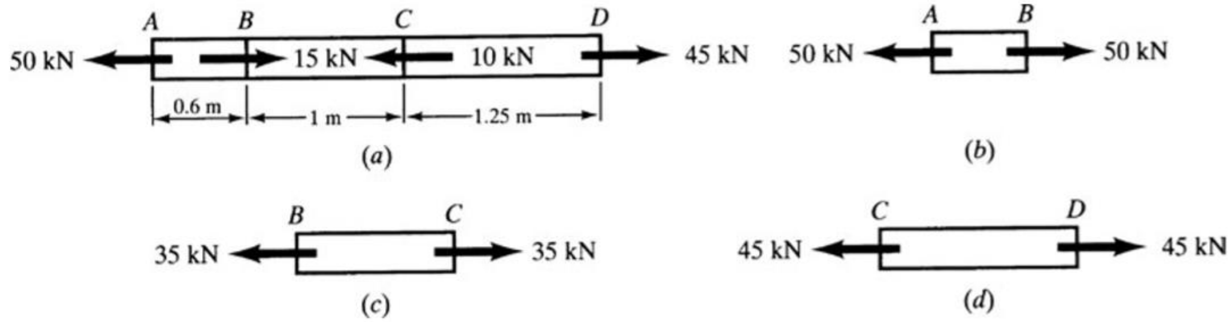



SOLUTION:

The unit stress in the direction of the force P is merely the load divided by the cross-sectional area, i.e.,($\sigma = P/A$). Also the unit strain $\varepsilon$ is given by the total elongation $\Delta$ divided by the original length, i.e., $\varepsilon = \Delta/L$. By definition the modulus of elasticity E is the ratio of $\sigma$ to $\varepsilon$, i.e.,

$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta/L} = \frac{P * L}{A * \Delta} \quad or \quad \frac{P * L}{A * E}$$

Ex2/ A steel bar of cross section 500 mm2 is acted upon by the forces shown in Fig. Determine the total elongation of the bar. For steel, consider E = 200 GPa.




SOLUTION: The entire bar is in equilibrium, and hence are all portions of it. The portion between A and B has a resultant force of 50 kN acting over every cross section and a free-body diagram of this 0.6-m length appears as in Fig.(b). The force at the right end of this segment must be 50 kN to maintain equilibrium with the applied load at A. The elongation of this portion is

$$\Delta_1 = \frac{P_1 L_1}{AE} = \frac{(50\,000\,\text{N})(0.6\,\text{m})}{(500\times10^{-6}\ \text{m}^2)(200\times10^{9}\,\text{N/m}^2)} = 0.0003\ \text{m}$$

The force acting in the segment between B and C is found by considering the algebraic sum of the forces to the left of any section between B and C, i.e., a resultant force of 35 kN acts to the left, so that a tensile force exists. The free-body diagram of the segment between B and C is shown in Fig.(c) and the elongation of it is

$$\Delta_2 = \frac{P_2 L_2}{AE} = \frac{(35000)(1)}{(500\times10^{-6})(200\times10^{9})} = 0.00035\ \text{m}$$

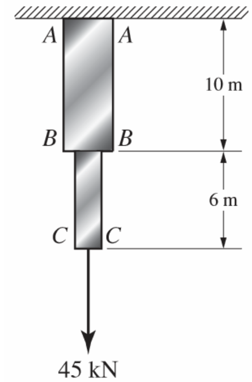
Similarly, the force acting over any cross section between C and D must be 45 kN to maintain equilibrium with the applied load at D. The elongation of CD is

$$\Delta_3 = \frac{P_3 L_3}{AE} = \frac{(45000)(1.25)}{(500\times10^{-6})(200\times10^{9})} = 0.00056\ \text{m}$$

The total elongation is

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 = 0.00121\ \text{m} \qquad \text{or} \qquad 1.21\ \text{mm}$$

Ex3/ Two prismatic bars are rigidly fastened together and support a vertical load of 45 kN, as shown in Fig. 1-12. The upper bar is steel having length 10 m and cross-sectional area 60 cm2. The lower bar is brass having length 6 m and cross-sectional area 50 cm2. For steel E = 200 GPa, for brass E = 100 GPa. Determine the maximum stress in each material.




SOLUTION: The maximum stress in the brass bar occurs just below the junction at section B-B. There, the vertical normal stress is caused by the combined effect of the load of 45 000 N together with the weight of the entire brass bar below B-B. Use specific weight in Table 1-3.

The weight of the brass bar is Wb = 6 × (50 × $10^{-4}$) × 84000 = 2520 N. The stress at this section is

$$\sigma = \frac{P}{A} = \frac{45000 + 2520}{50 \times 10^{-4}} = 9.5 \times 10^{6}\ \text{N/m}^2 \qquad \text{or} \qquad 9.5\ \text{MPa}$$

The maximum stress in the steel bar occurs at section A-A, the point of suspension, because there the entire weight of the steel and brass bars gives rise to normal stress. The weight of the steel bar is Ws = 10 × (60 × $10^{-4}$) × 77000 = 4620 N. The stress across section A-A is

$$\sigma = \frac{P}{A} = \frac{45000 + 2520 + 4620}{60 \times 10^{-4}} = 8.69 \times 10^{6}\ \text{N/m}^2 \qquad \text{or} \qquad 8.69\ \text{MPa}$$

## Ductile and Brittle Materials

Metallic engineering materials are commonly classified as either ductile or brittle materials. A ductile material is one having a relatively large tensile strain up to the point of rupture (for example, structural steel or aluminum) whereas a brittle material has a relatively small strain up to this same point. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes of materials. Cast iron and concrete are examples of brittle materials.

## Hooke's Law

For any material having a stress-strain curve of the form shown in Fig. 1-3(a), (b), or (c), it is evident that the relation between stress and strain is linear for comparatively small values of the strain. This linear relation between elongation and the axial force causing it is called Hooke's law. To describe this initial linear range of action of the material we may consequently write

$$\sigma = E * \varepsilon$$

where E denotes the slope of the straight-line portion OP of each of the curves in Figs.1-3(a), (b), and (c). The quantity E, i.e., the ratio of the unit stress to the unit strain, is the modulus of elasticity of the material in tension, or, as it is often called, Young's modulus. Values of E for various engineering materials are tabulated in handbooks. Table 1-3 for common materials appears at the end of this chapter. Since the unit strain  is a pure number (being a ratio of two lengths) it is evident that Ehas the same units as does the stress, N/m2. For many common engineering materials the modulus of elasticity in compression is very nearly equal to that found in tension. It is to be carefully noted that the behavior of materials under load as discussed in this book is restricted (unless otherwise stated) to the linear region of the stress-strain curve.

## 1.2Mechanical Properties of Materials

The stress-strain curve shown in Fig. 1-3(a) may be used to characterize several strength characteris tics of the material. They are:

## Proportional Limit

The ordinate of the point P is known as the proportional limit, i.e., the maximum stress that may be developed during a simple tension test such that the stress is a linear function of strain. For a material having the stress-strain curve shown in Fig. 1-3(d), there is no proportional limit.

## Elastic Limit

The ordinate of a point almost coincident with P is known as the elastic limit, i.e., the maximum stress that may be developed during a simple tension test such that there is no permanent or residual deformation when the load is entirely removed. For many materials the numerical values of the elastic limit and the proportional limit are almost identical and the terms are sometimes used synonymously. In those cases where the distinction between the two values is evident, the elastic limit is almost always greater than the proportional limit.

**Elastic and Plastic Ranges**

The region of the stress-strain curve extending from the origin to the proportional limit is called the elastic range. The region of the stress-strain curve extending from the proportional limit to the point of rupture is called the plastic range.

**Yield Point**

The ordinate of the point Y in Fig. 1-3(a), denoted by syp, at which there is an increase in strain with no increase in stress, is known as the yield point of the material. After loading has progressed to the point Y, yielding is said to take place. Some materials exhibit two points on the stress-strain curve at which there is an increase of strain without an increase of stress. These are called upper and lower yield points.

**Ultimate Strength or Tensile Strength**

The ordinate of the point Uin Fig. 1-3(a), the maximum ordinate to the curve, is known either as the ultimate strength or the tensile strength of the material.

**Breaking Strength**

The ordinate of the point B in Fig. 1-3(a) is called the breaking strength of the material.

**Modulus of Resilience**

The work done on a unit volume of material, as a simple tensile force is gradually increased from zero to such a value that the proportional limit of the material is reached, is defined as the modulus of resilience. This may be calculated as the area under the stress-strain curve from the origin up to the proportional limit and is represented as the shaded area in Fig. 1-3(a). The unit of this quantity is N · m/m3 in the SI system. Thus, resilience of a material is its ability to absorb energy in the elastic range.

**Modulus of Toughness**

The work done on a unit volume of material as a simple tensile force is gradually increased from zero to the value causing rupture is defined as the modulus of toughness. This may be calculated as the entire area under the stress-strain curve from the origin to rupture. Toughness of a material is its ability to absorb energy in the plastic range of the material.

### Percentage Reduction in Area

The decrease in cross-sectional area from the original area upon fracture divided by the original area and multiplied by 100 is termed percentage reduction in area. It is to be noted that when tensile forces act upon a bar, the cross-sectional area decreases, but calculations for the normal stress are usually made upon the basis of the original area. This is the case for the curve shown in Fig. 1-3(a). As the strains become increasingly larger it is more important to consider the instantaneous values of the cross-sectional area (which are decreasing), and if this is done the truestress-strain curve is obtained. Such a curve has the appearance shown by the dashed line in Fig. 1-3(a).

### Percentage Elongation

The increase in length of a bar after fracture divided by the initial length and multiplied by 100 is the percentage elongation. Both the percentage reduction in area and the percentage elongation are considered to be measures of the ductility of a material.

### Working Stress

The above-mentioned strength characteristics may be used to select a working stress. Frequently such a stress is determined merely by dividing either the stress at yield or the ultimate stress by a number termed the safety factor. Selection of the safety factor is based upon the designer’s judgment and experience. Specific safety factors are sometimes specified in design codes.

### Strain Hardening

If a ductile material can be stressed considerably beyond the yield point without failure, it is said to strainharden. This is true of many structural metals. The nonlinear stress-strain curve of a brittle material, shown in Fig. 1-3(d), characterizes several other strength measures that cannot be introduced if the stress-strain curve has a linear region. They are:

### Yield Strength

The ordinate to the stress-strain curve such that the material has a predetermined permanent deformation or ‘‘set’’ when the load is removed is called the yield strength of the material. The permanent set is often taken to be either 0.002 or 0.0035 mm per mm. These values are of course arbitrary. In Fig. 1-3(d) a set 1 is denoted on the strain axis and the line O′Y is drawn parallel to the initial tangent to the curve. The ordinate of Y represents the yield strength of the material, sometimes called the proof stress.

## Tangent Modulus

The rate of change of stress with respect to strain is known as the tangent modulus of the material. It is essentially an instantaneous modulus given by

$$E_t = \frac{d\sigma}{d\varepsilon}$$

## Coefficient of Linear Expansion

This is defined as the change of length per unit length of a straight bar subject to a temperature change of one degree and is usually denoted by a. The value of this coefficient is independent of the unit of length but does depend upon the temperature scale used. For example, from Table 1-3 at the end of this chapter the coefficient for steel is 12 × 10–6/°C. Temperature changes in a structure give rise to internal stresses, just as do applied loads. The thermal strain due to a temperature change ΔT is

$$\varepsilon t = \alpha * \Delta T$$

Ex4/ A solid brass of length L=100mm of diameter D= 15mm is used to fix two rigid surfaces. Find the stresses induced into it if the temperature of the bar is raised by 20c̊. Given α= 19x$10^{-6}$/c̊ and E=103GPa/m².

**Solution:**

$\sigma = -E\alpha\Delta T \ (or - E\alpha T)$

$\sigma x = -103 \times 10^9 \times 19 \times 10^{-6} \times 20$

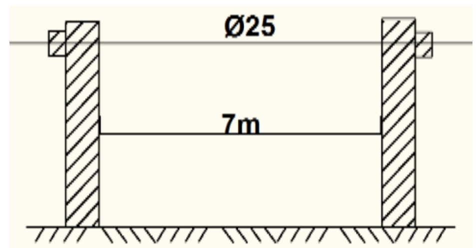
$\quad = -39.1 \ MPa$

It is interesting to note that this result does not depend on either the length or diameter of the cylinder

Ex5/ Two parallel walls 7m apart, are held by steel bar of 25mm diameter, the bar passes through a metal plate and nut at each end, the nuts are screwed up to the plates which the bar is at 150$\mathring{c}$. Find the pull exerted by the bar after cooling to 16$\mathring{c}$. If

**1-** the ends do not yield.
**2-** the total yield at the two ends is 6.25mm. Given Es = 220 GPa and α = 11x$10^{-6}$/$\mathring{c}$

solution




1- No yield

Total axial strain is:

$$\varepsilon = \varepsilon_T + \varepsilon_S$$

But:

$$\varepsilon = 0 \qquad (no\ change\ of\ length)$$

$$\varepsilon_T = \alpha \Delta T = 11 \times 10^{-6}(16 - 150)$$

$$= -1474 \times 10^{-6} \ \ (contraction\ to\ produce\ tension)$$

Substituting the strain due to constrain, in

$$\varepsilon_S = \varepsilon - \varepsilon_T$$

$$= 0 - (-1474 \times 10^{-6})$$

$$= 1474 \times 10^{-6}\ (tensile)$$

But:

$$\varepsilon_S = \frac{\sigma}{E}$$

Then:

$$\sigma = E \cdot \varepsilon_S = 220 \times 10^9 \times 1474 \times 10^{-6}$$

$$= 324.28\ MPa$$

The pull is

$$P = \sigma A$$

$$= 324.28 \times \frac{\pi d^2}{4}$$

$$= 324.28 \times \frac{\pi \times (0.025)^2}{4} = 159180.57\ N = 159.2\ kN$$

2- When yielding in ΔL=6.25 mm

The total axial strain in:

$\varepsilon = \varepsilon_T + \varepsilon_S$ ,

but:

$$\varepsilon = \frac{6.25}{7000} = -893 \times 10^{-6} \qquad (contraction)$$

Thermal strain is

$$\varepsilon_T = \alpha \Delta T = 11 \times 10^{-6}(16 - 150)$$

$$= -1474 \times 10^{-6} \ (contraction\ )$$

Therefore the induced strain is:

$$\varepsilon_S = \varepsilon - \varepsilon_T$$

$$= -893 \times 10^{-6} - (-1474 \times 10^{-6})$$

$$= 581 \times 10^{-6}\ (tensile)$$

The axial stress is:

$$\sigma = E \cdot \varepsilon_S = 220 \times 10^9 \times 581 \times 10^{-6}$$

$$= 127.82\ MPa$$

$$P = \sigma A$$

$$= 127.82 \times \frac{\pi \times (0.025)^2}{4} = 62743.5\ N = 62.7434\ kN$$