Integral التكامل

Some Rules of integral

1. $\int u^n du = \frac{u^{n+1}}{n+1} + c \;\; ; n \neq 1$

2. $\int \frac{du}{u} = \ln|u| + c$

3. $\int \sin u \, du = -\cos u + c$

4. $\int \cos u \, du = \sin u + c$

5. $\int \sec^2 u \, du = \tan u + c$

6. $\int \csc^2 u \, du = -\cot u + c$

7. $\int \sec u \tan u \, du = \sec u + c$

8. $\int \csc u \cot u \, du = -\csc u + c$

9. $\int a^u du = \frac{a^u}{\ln a} + c \;\; ; a > 0, a \neq 1$

10. $\int e^u \, du = e^u + c$

11. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + c_1 = -\cos^{-1}\frac{u}{a} + c_2$

12. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c_1 = -\frac{1}{a}\cot^{-1}\frac{u}{a} + c_2$

13. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + c_1 = -\frac{1}{a}\csc^{-1}\frac{u}{a} + c_2$

Example:: Find the integral I

1. $\int (x+2)\sqrt[3]{x^2+4x+25}\,dx = \frac{1}{2}\int 2(x+2)\sqrt[3]{x^2+4x+25}\,dx$
$= \frac{3}{8}(x^2+4x+25)^{4/3} + c$

2. $\int \frac{dx}{x(\ln x)^3} = -\frac{1}{2}(\ln x)^{-2} + c$

3. $\int x^2 e^{\sin x^3} \cos x^3 \, dx == \frac{1}{3}\int 3x^2 e^{\sin x^3}\cos x^3 \, dx = e^{\sin x^3} + c$

4. $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}} dx = 2\int \frac{\sec^2\sqrt{x}}{2\sqrt{x}} dx = 2\tan\sqrt{x} + c$

5. $\int \frac{e^x}{1+e^{2x}} dx = \int \frac{e^x}{1+(e^x)^2} dx = \tan^{-1} e^x + c$

6. $\int \frac{2dx}{x^2+2x+2} = \int \frac{2dx}{x^2+2x+1+1} = \int \frac{2dx}{(x+1)^2+1} = 2\tan^{-1}(x+1) + c$

7. $\int \frac{\sin 2x}{1+\cos 2x}dx = \frac{1}{2}\int \frac{2\sin 2x}{1+\cos 2x}dx = \frac{1}{2}\ln(\cos 2x) + c$

8. $\int \frac{2x+9}{x^2+8x+17}dx = \int \frac{2x+8+1}{x^2+8x+17}dx$

$$= \int \frac{2x+8}{x^2+8x+17}dx + \int \frac{1}{x^2+8x+16+1}dx$$

$$= \ln(x^2+8x+17) + \int \frac{1}{(x+4)^2+1}dx$$

$$= \ln(x^2+8x+17) + \tan^{-1}(x+4) + c$$

9. $\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{1-1+2x-x^2}} = \int \frac{dx}{\sqrt{1-(1-2x+x^2)}}$

$$= \int \frac{dx}{\sqrt{1-(x-1)^2}} = \sin^{-1}(x-1) + c$$

Exercises:. Find the integral H.W

1. $\int \frac{\sin(\ln x)}{x}dx$

2. $\int \frac{x\sin^{-1}x^2}{\sqrt{1-x^4}}\,dx$

3. $\int \frac{\tan^{-1}x}{1+x^2}dx$

4. $\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx$

5. $\int \frac{dx}{\sqrt{e^{2x}-1}}$

6. $\int \frac{(x+2)dx}{x^2+4x+5}$

7. $\int \frac{1-\cos 2x}{1+\cos 2x}\,dx$

8. $\int \frac{dx}{x\sqrt{4x^2-1}}$

9. $\int \frac{2dx}{x^2+4x+5}$

10. $\int \frac{2-\cos x}{\sin^2 x}\,dx$

11. $\int e^{\ln(\cot^2 x)}\,dx$

12. $\int \frac{(2x+1)dx}{x^2+4x+5}$

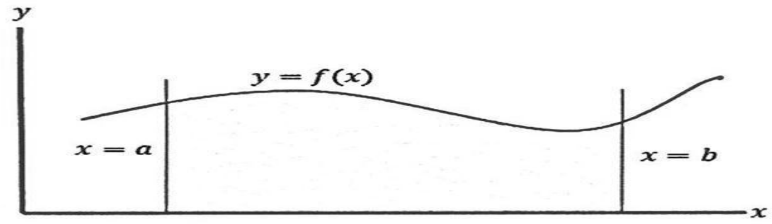
13. $\int \frac{\cos x}{1+\sin^2 x}\,dx$

14. $\int \frac{dx}{\sqrt{4-25x^2}}$

15. $\int \frac{dx}{\sqrt{-x^2+4x-1}}$

Definite integral التكامل المحدد

$\int_a^b f(x)\,dx$ is the Area between below $y=f(x)$ and above the x-axis for $x=a$ and $x=b$.



Properties of Definite integral

1. $\int_a^a f(x) = 0$
2. $\int_a^b f(x) = -\int_b^a f(x)$
3. $\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x) \qquad ; \quad a < c < b$

EX:. Find $I = \int_{-1}^{1} \frac{dx}{\sqrt{4-x^2}}$

$$\int_{-1}^{1} \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\frac{x}{2}\Big|_{-1}^{1} = \sin^{-1}\frac{1}{2} - \sin^{-1}\frac{-1}{2}$$

$$= \sin^{-1}\frac{1}{2} + \sin^{-1}\frac{1}{2}$$

$$= 2\sin^{-1}\frac{1}{2} = 2\times\frac{\pi}{6} = \frac{\pi}{3}$$

Ex: Find

$$I=\int_0^2 x\ln(x+2)\,dx$$

$$u=\ln(x+2) \quad , \quad dv = x\,dx$$

$$then \quad du=\frac{dx}{x+2} \quad and \quad v=\frac{x^2}{2}$$

$$\therefore \quad \int_0^2 x\ln(x+2)\,dx = \frac{x^2}{2}\ln(x+2)\Big|_0^2 - \int_0^2 \frac{x^2\,dx}{2(x+2)}$$

$$\begin{array}{r|l} & x-2 \\ \hline (x+2) & x^2 \\ & \mp x^2 \pm 2x \\ \hline & -2x \\ & \pm 2x \pm 4 \\ \hline & 4 \end{array}$$

$$=\frac{4}{2}\ln 4 - 0 - \frac{1}{2}\left[\int_0^2 (x-2)dx + \int_0^2 \frac{4dx}{(x+2)}\right]$$

$$=2\ln 4 - \frac{1}{2}\left[\frac{x^2}{2} - 2x + 4\ln(x+2)\right]_0^2$$

$$=4\ln 2 - \frac{1}{2}\left[\frac{4}{2} - 4 + 4\ln 4 - 0 + 0 - 4\ln 2\right]$$

$$=4\ln 2 - \frac{1}{2}[-2 + 8\ln 2 - 4\ln 2]$$

$$=4\ln 2 + 1 - 2\ln 2 = 2\ln 2 + 1$$

Exercises: H.W

1. $\int_0^{\pi} \sqrt{1-\cos 2x}\,dx$
2. $\int_0^{\pi/10} \sqrt{1+\cos 5x}\,dx$
3. $\int_1^3 \frac{dx}{x^2-2x+5}$
4. $\int_3^5 \frac{(x^3+1)dx}{x^3-x}$
5. $\int_1^2 x\sec^{-1}x\,dx$
6. $\int_1^4 \sec^{-1}\sqrt{x}\,dx$

Methods of integrantion طرق التكامل

Integrantion by parts : أولاً : التكامل بالتجزئة

x لـ funct. $v = v(x)$, $u = u(x)$ If

then $\boxed{\int u\,dv = u\,v - \int v\,du}$

Example: Find $I = \int \ln x\,dx$

$u = \ln x$ and $dv = dx$

$du = \dfrac{dx}{x}$ and $v = x$

$$\therefore \int \ln x\,dx = x\ln x - \int x.\frac{dx}{x}$$

$$= x\ln x - \int dx$$

$$= x\ln x - x + c$$

Example: Find $I = \int x^2 \cos 2x\,dx$

$u = x^2$ and $dv = \cos 2x\,dx$

$du = 2xdx$ and $v = \dfrac{1}{2}\sin 2x$

$$\therefore \int x^2 \cos 2x\,dx = x^2.\frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x\,(2xdx) = \frac{x^2}{2}\sin 2x - \int x\sin 2x\,dx$$

Example: Find $I = \int x\sin 2xdx$

$U = x$ and $dV = \sin 2xdx \rightarrow dU = dx$ and $V = -\dfrac{1}{2}\cos 2x$

$$\therefore \int x\sin 2xdx = x\left(-\frac{1}{2}\cos 2x\right) - \int -\frac{1}{2}\cos 2x\;dx = -\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x + c_1$$

$$\therefore \int x^2\cos 2x\,dx = \frac{x^2}{2}\sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x + c$$

We can SoL. for $\left(x^n e^{ax}\right)$ $\left(x^n \sin ax\right)$ by new method

u and it's D.		*v and it's I.*
x^2	+	$\cos 2x$
$2x$	-	$\frac{1}{2}\sin 2x$
2	+	$-\frac{1}{4}\cos 2x$
0		$-\frac{1}{8}\sin 2x$

$$\therefore \int x^2 \cos 2x\, dx = x^2 \times \frac{1}{2}\sin 2x + 2x \times \frac{1}{4}\cos 2x - 2 \times \frac{1}{8}\sin 2x + c$$

$$= \frac{x^2}{2}\sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x + c$$

Example:. Find $I = \int \sin^{-1} 2x\, dx$

$u = \sin^{-1} 2x \quad \textit{and} \quad dv = dx$

$du = \dfrac{2dx}{\sqrt{1-4x^2}} \quad \textit{and} \;\; v = x$

$$\therefore \int \sin^{-1} 2x\, dx = x \sin^{-1} 2x - \int \frac{2x dx}{\sqrt{1-4x^2}}$$

Example : Find the area of the region bounded by the curve $y = xe^{-x}$ and the x-axis from $x = 0$ to $x = 4$. اوجد المساحة

Sol: by using integration by part تكامل بالتجزئة

$$\int_0^4 xe^{-x}\,dx.$$

Let $u = x$, $dv = e^{-x}\,dx$, $v = -e^{-x}$, and $du = dx$. Then,

$$\begin{aligned}\int_0^4 xe^{-x}\,dx &= -xe^{-x}\Big]_0^4 - \int_0^4 (-e^{-x})\,dx \\ &= [-4e^{-4} - (0)] + \int_0^4 e^{-x}\,dx \\ &= -4e^{-4} - e^{-x}\Big]_0^4 \\ &= -4e^{-4} - e^{-4} - (-e^0) = 1 - 5e^{-4} \approx 0.91.\end{aligned}$$

Integration of Rational Functions by Partial Fractions

Method of Partial Fractions ($f(x)/g(x)$ Proper)

1. **Let $x - r$ be a linear factor of $g(x)$. Suppose that $(x - r)^m$ is the highest power of $x - r$ that divides $g(x)$. Then, to this factor, assign the sum of the m partial fractions:**

$$\frac{A_1}{x-r} + \frac{A_2}{(x-r)^2} + \cdots + \frac{A_m}{(x-r)^m}.$$

Do this for each distinct linear factor of $g(x)$.

2. **Let $x^2 + px + q$ be a quadratic factor of $g(x)$. Suppose that $(x^2 + px + q)^n$ is the highest power of this factor that divides $g(x)$. Then, to this factor, assign the sum of the n partial fractions:**

$$\frac{B_1x + C_1}{x^2+px+q} + \frac{B_2x + C_2}{(x^2+px+q)^2} + \cdots + \frac{B_nx + C_n}{(x^2+px+q)^n}.$$

Do this for each distinct quadratic factor of $g(x)$ that cannot be factored into linear factors with real coefficients.

3. Set the original fraction $f(x)/g(x)$ equal to the sum of all these partial fractions. Clear the resulting equation of fractions and arrange the terms in decreasing powers of x.

4. Equate the coefficients of corresponding powers of x and solve the resulting equations for the undetermined coefficients.

Example: Find the solution of the integral جد تكامل

$$\int \frac{6x + 7}{(x + 2)^2}\, dx$$

Sol: by using P . Frac. تجزئة الكسور

$$\frac{6x + 7}{(x + 2)^2} = \frac{A}{x + 2} + \frac{B}{(x + 2)^2}$$

$$6x + 7 = A(x + 2) + B \qquad \text{Multiply both sides by } (x + 2)^2.$$

$$= Ax + (2A + B)$$

Equating coefficients of corresponding powers of x gives

$A = 6$ and $2A + B = 12 + B = 7,$ or $A = 6$ and $B = -5.$

Therefore,

$$\int \frac{6x + 7}{(x + 2)^2}\, dx = \int \left(\frac{6}{x + 2} - \frac{5}{(x + 2)^2}\right) dx$$

$$= 6 \int \frac{dx}{x + 2} - 5 \int (x + 2)^{-2}\, dx$$

$$= 6 \ln |x + 2| + 5(x + 2)^{-1} + C$$

Exercises: Find the solution of the integral تمارين H.W

• $\int \frac{5x-13}{(x-3)(x-2)} \quad dx$

• $\int \frac{dx}{x(x+1)^2}$