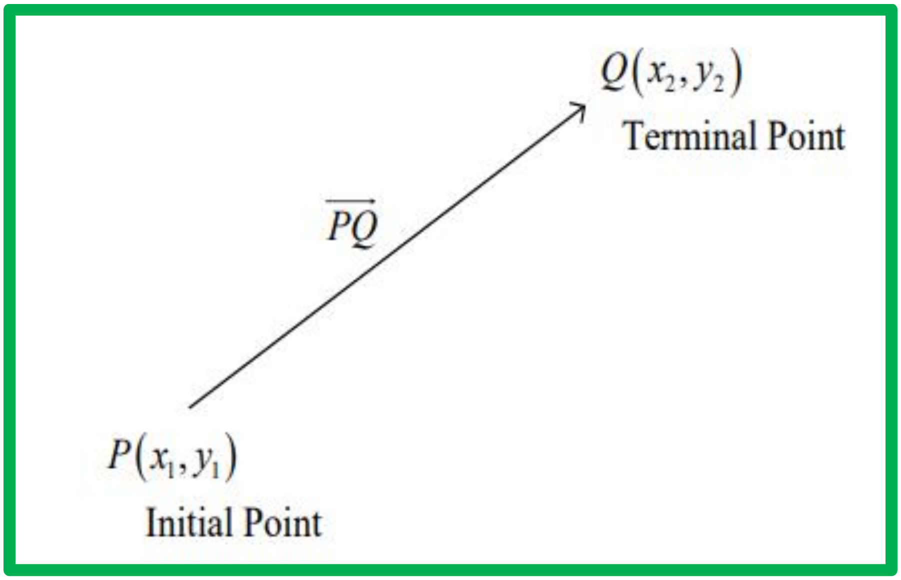
INTRODUCTION TO VECTORS

A vector can be written as PQ , or a. The order of the letters is important. PQ means the vector is from P to Q or the position vector Q relative to P, QP means vector is from Q to P or the position vector P relative to Q.



If $P\ (x_1, y_1)$ is the initial point and $Q(x_2, y_2)$ is the terminal point of a directed line segment, $\overrightarrow{PQ}$ then **component form** of vector **v** that represents $\overrightarrow{PQ}$ is

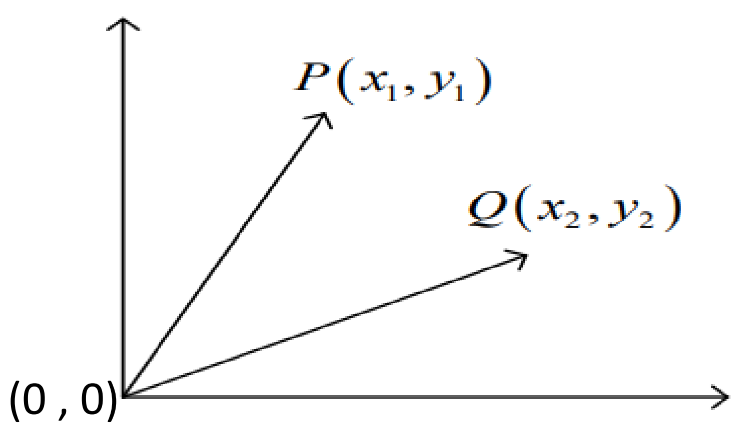
ولغرض معرفة القيمه للمتجهه بين النقطه P الى النقطه Q كما يلي:

لان المتجهه لها قيمه واتجاه ولنفرض القيمه تساوي "V "

the **magnitude** or the **length** of **v** is

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

وللتوضيح ادناه المثال التالي:



$$\overrightarrow{OP} = \langle x_1 - 0, y_1 - 0 \rangle = \langle x_1, y_1 \rangle$$

$$\overrightarrow{OQ} = \langle x_2 - 0, y_2 - 0 \rangle = \langle x_2, y_2 \rangle$$

والمتجه التي تظهر قيمتها واحد تسمى UNIT VECTOR

Any vector that has magnitude of 1 unit = unit vector.

EXAMPL NO. 1:

Find the component form and length of the vector **v** that has initial point $(3,-7)$ and terminal point $(-2,5)$.

Solution:

$$\bar{v} = \langle -2-3, 5+7 \rangle = \langle -5, 12 \rangle$$

$$|\bar{v}| = \sqrt{(-5)^2 + (12)^2} = \sqrt{25+144} = 13$$

An exercise تمرين يحل من قبلكم

Given $\mathbf{v} = \langle -2, 5 \rangle$ and $\mathbf{w} = \langle 3, 4 \rangle$, find each of the following vectors:

a) $\frac{1}{2}\mathbf{v}$ b) $\mathbf{w} - \mathbf{v}$ c) $\mathbf{v} + 2\mathbf{w}$

Answer:

a) $\langle -1, 5/2 \rangle$ b) $\langle 5, -1 \rangle$ c) $\langle 4, 13 \rangle$

a

Theorem:

If $\boldsymbol{a}$ is a non-null vector and if $\hat{\boldsymbol{a}}$ is a **unit vector** having the same direction as $\boldsymbol{a}$, then

$$\hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

في هذه الحاله اذا اعطيت متجهه "a" نستطيع ايجاد المتجهه التي قيمتها يساوي واحد بنفس اتجاه المتجهه "a " من خلال تقسيم المتجه على قيمتها الرقميه |a |

EXAMPLE NO.2:

Find a unit vector in the direction of $\mathbf{v} = \langle -2,5 \rangle$ and verify that it has length 1.

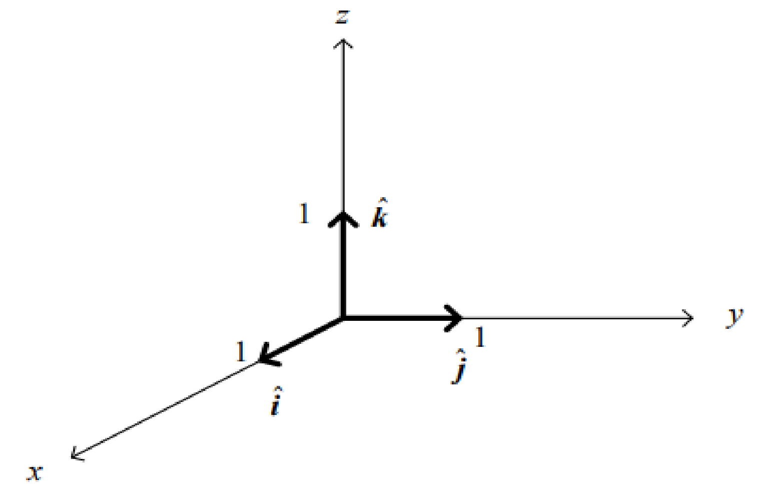
Solution:

$$\bar{v} = \frac{\langle -2,5 \rangle}{\sqrt{(-2)^2 + 5^2}} = \frac{1}{\sqrt{29}} \langle -2,5 \rangle$$

$$|\bar{v}| = \sqrt{\left(-2/\sqrt{29}\right)^2 + \left(5/\sqrt{29}\right)^2} = \sqrt{1} = 1$$

STANDARD VECTOR (i , j, k)

Three standard unit vectors are: i, j and k



<u>Vectors i, j and k can be written in components form:</u>

$\boldsymbol{i} = < 1, 0, 0 >,$

$\boldsymbol{j} = < 0, 1, 0 >$ and

$\boldsymbol{k} = < 0, 0, 1>$

and can interpreted as

$$\boldsymbol{a} = <x, y, z>$$

$$= x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

The vector $P\bar{Q}$ with initial point

$P(x_1, y_1, z_1)$and terminal point $Q(x_2, y_2, z_2)$

has the standard representation

$$P\bar{Q} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$

or

$$PQ = < x_2 - x_1, y_2 - y_1, z_2 - z_1 >$$

EXAMPLE NO. 3:

Let **u** be the vector with initial point (2,-5) and terminal point (-1,3), and let $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$.

Write each of the following vectors as a linear combination of **i** and **j.**

a) **u**

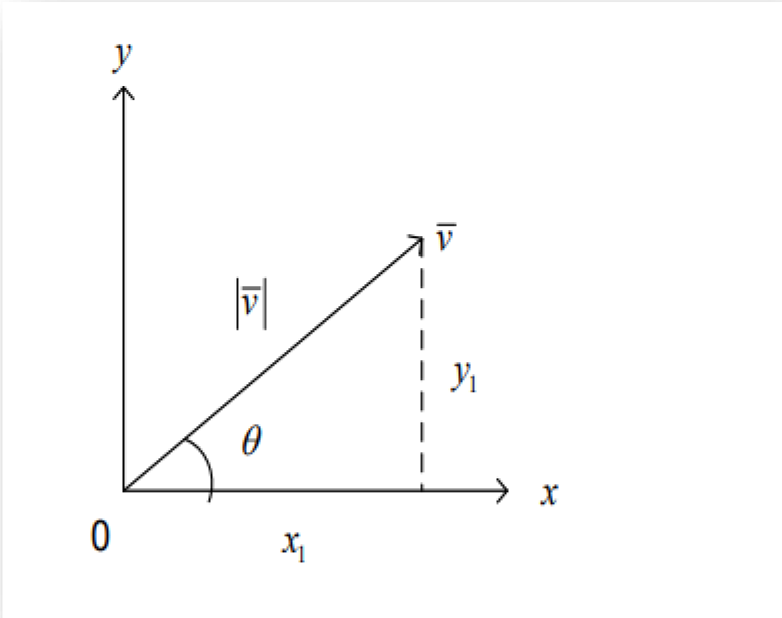
b) $\mathbf{w} = 2\mathbf{u} - 3\mathbf{v}$

SOLUTION:

u = (x1-x2)i + (y1-y2)j = 3i -8j

w= 2u – 3v = 6i -16j -6i + 3j = 0i -13j

اذا كانت قيمة المتجهه معلومه والزاويه بينها وبين محور السينات معلومه كما في الشكل التالي:



If θ is the angle between $\bar{v}$ and the positive x – axis then we can write

$$x = |\bar{v}|\cos\theta \text{ and } y = |\bar{v}|\sin\theta \,;\, |\bar{v}| = \sqrt{x_1^{\,2} + y_2^{\,2}} .$$

EXAMPLE NO.4

The vector **v** has a length of 3 and makes an angle of $30^o = \frac{\pi}{6}$ with the positive x-axis. Write **v** as a linear combination of the unit vectors **i** and **j**.

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solution

x = v cos 30 = 3 (0.866) = 2.6

y = v sin 30 = 3 (0.5) =1.5

v = 2.6 I + 1.5 j

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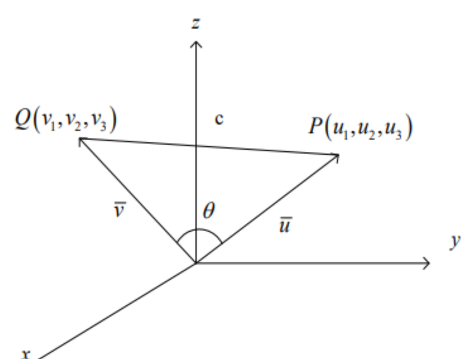
the angle between two vector:

Refer to the figure below, let

$$\bar{u} = \bar{u}(OP) = \langle u_1, u_2, u_3 \rangle,$$

$$\bar{v} = \bar{v}(OQ) = \langle v_1, v_2, v_3 \rangle$$

be two vectors and let θ be the angle between them, with $0 \le \theta \le \pi$.



Compute the distance, c between points P and Q in two ways.

1) *Using the Distance formula*

$$c^2 = (u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2$$

$$= u_1^2 + u_2^2 + u_3^2 + v_1^2 + v_2^2 + v_3^2 - 2(u_1v_1 + u_2v_2 + u_3v_3)$$

$$= |\overline{u}|^2 + |\overline{v}|^2 - 2(u_1v_1 + u_2v_2 + u_3v_3) \quad \text{---(1)}$$

2) *Using the Law of Cosines*

$$c^2 = |\overline{u}|^2 + |\overline{v}|^2 - 2|\overline{u}||\overline{v}|\cos\theta \quad \text{---(2)}$$

Equating equation (1) and (2), we get

$$\cos\theta = \frac{u_1v_1 + u_2v_2 + u_3v_3}{|\overline{u}||\overline{v}|} = \frac{\overline{u}\cdot\overline{v}}{|\overline{u}||\overline{v}|}$$

The dot product= u.v= u1v1+u2v2+u3v3 this called dot product.

EXAMPLE NO.5:

If **v** = 2**i**-**j**+**k**, **w** = **i**+**j**+2**k** and the angle between **v** and **w** is 60°, find $\mathbf{v}\cdot\mathbf{w}$.

SOLUTION

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$$\cos\theta = \frac{u_1v_1 + u_2v_2 + u_3v_3}{|\overline{u}||\overline{v}|} = \frac{\overline{u}\cdot\overline{v}}{|\overline{u}||\overline{v}|}$$

نحسب

$$|\bar{u}||\bar{v}|$$

ونضرب ب cosθ

$$\bar{v}\cdot\bar{w}=\sqrt{2^2+(-1)^2+1^2}\cdot\sqrt{1^2+1^2+2^2}\cos(\pi/3)$$

$$=\sqrt{6}\cdot\sqrt{6}\cos(\pi/3)=6(1/2)=3$$
