CHAPTER TWO

LATERAL EARTH PRESSURE

2.1 Introduction:

Vertical or near-vertical slopes of soil are supported by retaining walls, cantilever sheet-pile walls, sheet-pile bulkheads, braced cuts, and other, similar structures. The proper design of those structures requires an estimation of lateral earth pressure, which is a function of several factors, such as

(a) The type and amount of wall movement,

(b) The shear strength parameters of the soil,

(c) The unit weight of the soil, and

(d) The drainage conditions in the backfill.

Figure 2.1 shows a retaining wall of height *H*. For similar types of backfill,

a. The wall may be restrained from moving (Figure 2.1a). The lateral earth pressure on the wall at any depth is called the *at-rest earth pressure*.

b. The wall may tilt away from the soil that is retained (Figure 7.1b). With sufficient wall tilt, a triangular soil wedge behind the wall will fail. The lateral pressure for this condition is referred to as *active earth pressure*.

c. The wall may be pushed into the soil that is retained (Figure 7.1c). With sufficient wall movement, a soil wedge will fail. The lateral pressure for this condition is referred to as *passive earth pressure*.

Figure 2.2 shows the nature of variation of the lateral pressure, at a certain depth of the wall with the magnitude of wall movement.

In the sections that follow, we will discuss various relationships to determine the at-rest, active, and passive pressures on a retaining wall. It is assumed that the reader has studied lateral earth pressure in the past, so this chapter will serve as a review.

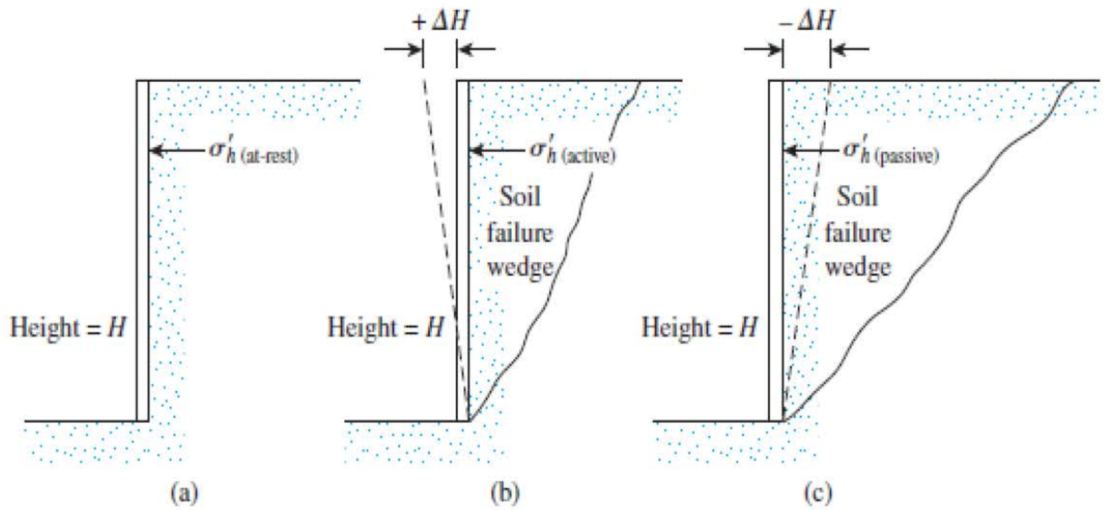


Figure 2.1: Nature of lateral earth pressure on a retaining wall.

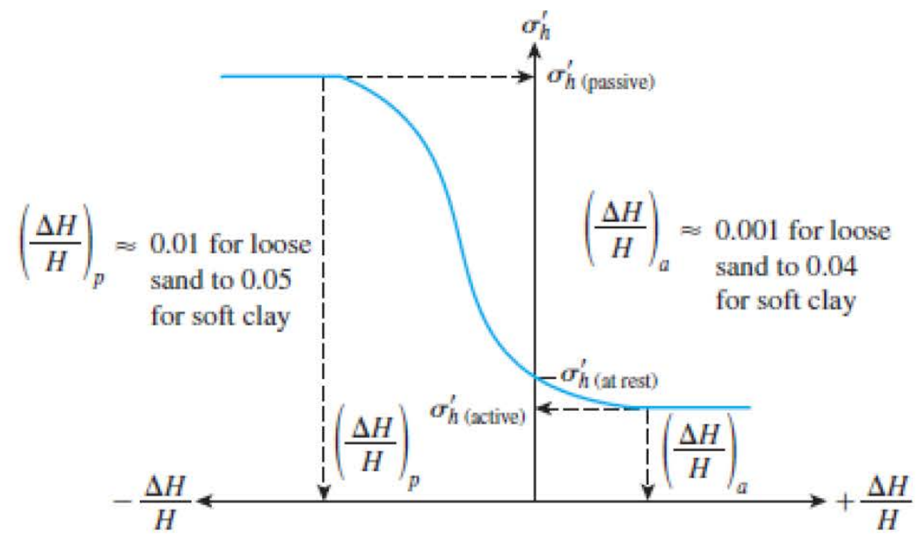


Figure 2.2: Nature of variation of lateral earth pressure at a certain depth.

2.2 Lateral Earth Pressure at Rest

Consider a vertical wall of height *H*, as shown in Figure 2.3, retaining a soil having a unit weight of γ. A uniformly distributed load, *q/unit area*, is also applied at the ground surface. The shear strength of the soil is

$$s = c' + \sigma' \tan \phi'$$

where

c' = cohesion

ϕ' = effective angle of friction

σ' = effective normal stress

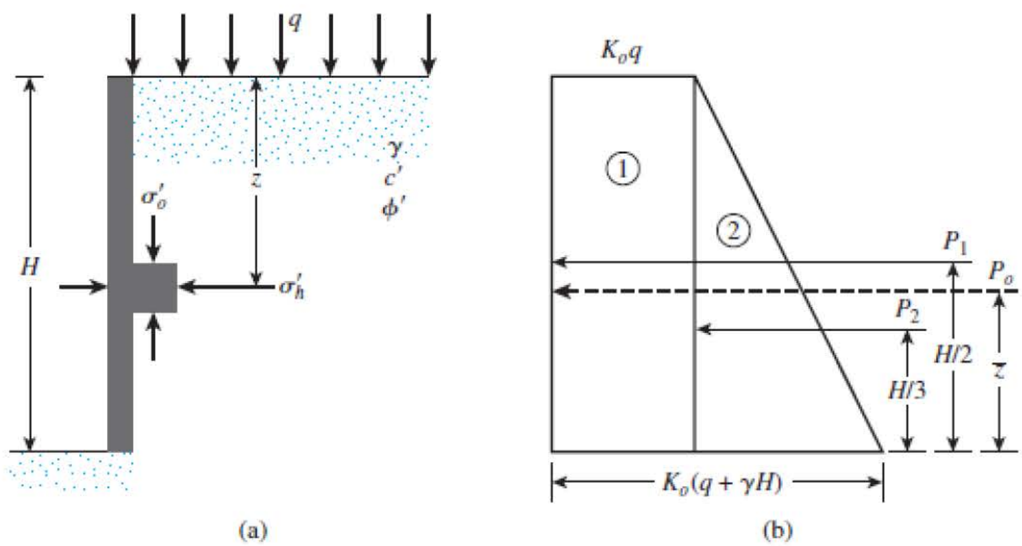


Figure 2.3: At-rest earth pressure.

At any depth *z* below the ground surface, the vertical subsurface stress is

$$\sigma_o' = q + \gamma z$$

If the wall is at rest and is not allowed to move at all, either away from the soil mass or into the soil mass (i.e., there is zero horizontal strain), the lateral pressure at a depth z is

$$\sigma_h = k_0 \sigma_o' + u$$

where

k_0 is coefficient of lateral earth pressure at rest.

u is pore water pressure.

For normally consolidated soil, the relation for k_0 (Jaky, 1944) is

$$k_0 = 1 - \sin\phi$$

For overconsolidated soil, the at-rest earth pressure coefficient may be expressed as (Mayne and Kulhawy, 1982):

$$k_{0_{O.C.S}} = (1 - \sin\phi).\sqrt{O.C.R}$$

1) With a properly selected value of the at-rest earth pressure coefficient, the variation of lateral earth pressure with depth z can be determined. Figure 2.3b shows the variation of σ_h with depth for the wall depicted in Figure 2.3a. Note that if the surcharge $q=0$ and the pore water pressure $u=0$, the pressure diagram will be a triangle. The total force Po, *per unit length* of the wall given in Figure 7.3a can now be obtained from the area of the pressure diagram given in Figure 7.3b and is:

$$P_o = P_1 + P_2 = qK_oH + \tfrac{1}{2}\gamma H^2 K_o$$

where

P_1 = area of rectangle 1
P_2 = area of triangle 2

The location of the line of action of the resultant force Po, can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o}$$

2) If the water table is located at a depth $z < H$, the pressure at-rest state diagram shown in Figure 2.3b will have to be somewhat modified, as shown in Figure 2.4. If the effective unit weight of soil below the water table equals γ' $(\gamma_{sat.} - \gamma_w)$ then

at $z = 0$, $\quad \sigma'_h = K_o\sigma'_o = K_oq$
at $z = H_1$, $\quad \sigma'_h = K_o\sigma'_o = K_o(q + \gamma H_1)$
and
at $z = H_2$, $\quad \sigma'_h = K_o\sigma'_o = K_o(q + \gamma H_1 + \gamma' H_2)$

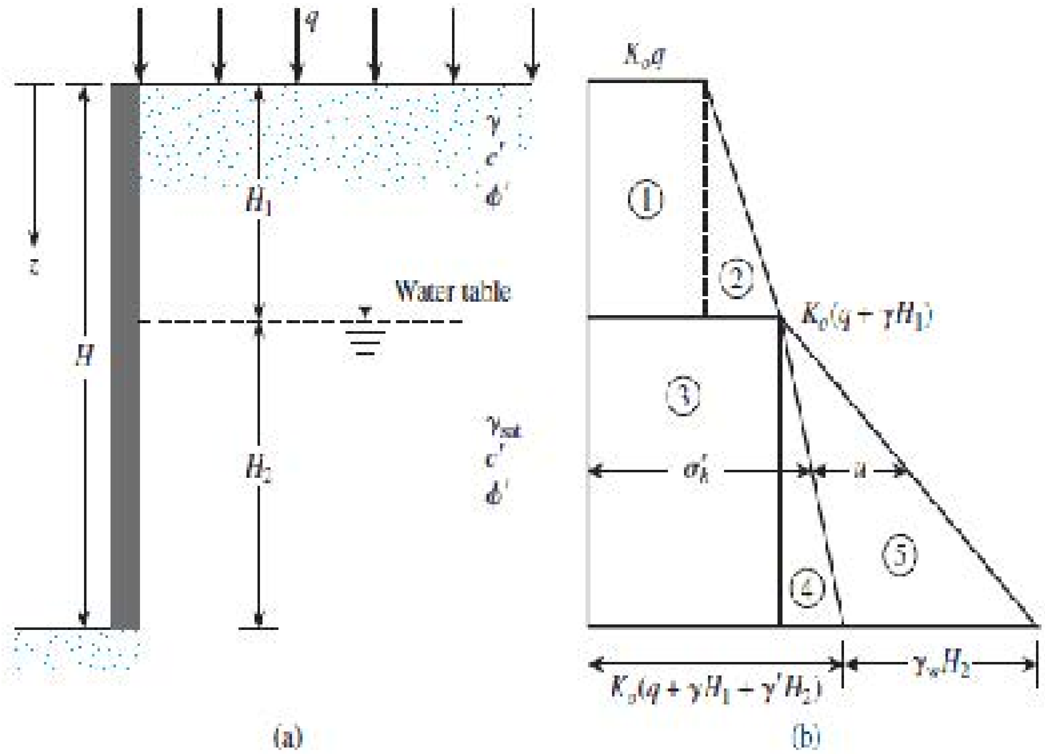


Figure 2.4: At-rest earth pressure with water table located at depth $z < H$.

Note that in the preceding equations, σ_o' and σ_h are effective vertical and horizontal pressures, respectively. Determining the total pressure distribution on the wall requires adding the hydrostatic pressure u, which is zero from z=o to z= H_1 and is $H_2\gamma_w$ at z= H_2. The variation of σ_h and u with depth is shown in Figure 2.4b. Hence, the total force per unit length of the wall can be determined from the area of the pressure diagram. Specifically,

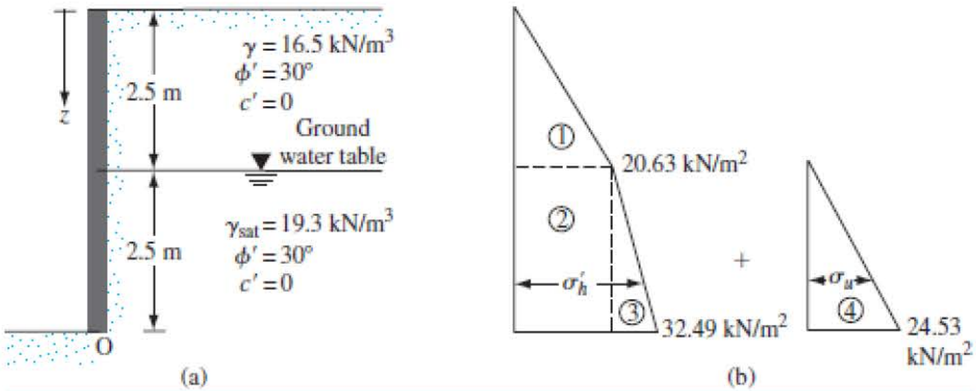
$$P_o = A_1 + A_2 + A_3 + A_4 + A_5$$

where A = area of the pressure diagram.

So,

$$P_o = K_o q H_1 + \tfrac{1}{2} K_o \gamma H_1^2 + K_o(q + \gamma H_1) H_2 + \tfrac{1}{2} K_o \gamma' H_2^2 + \tfrac{1}{2} \gamma_w H_2^2$$

Example 1: For the retaining wall shown in below Figure, determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force. Assume OCR =1.



Solution:

$$K_o = 1 - \sin\phi' = 1 - \sin 30° = 0.5$$

At $z = 0$, $\sigma'_o = 0$; $\sigma'_h = 0$

At $z = 2.5$ m, $\sigma'_o = (16.5)(2.5) = 41.25$ kN/m²;

$$\sigma'_h = K_o\sigma'_o = (0.5)(41.25) = 20.63 \text{ kN/m}^2$$

At $z = 5$ m, $\sigma'_o = (16.5)(2.5) + (19.3 - 9.81)2.5 = 64.98$ kN/m²;

$$\sigma'_h = K_o\sigma'_o = (0.5)(64.98) = 32.49 \text{ kN/m}^2$$

The hydrostatic pressure distribution is as follows:

From $z = 0$ to $z = 2.5$ m, $u = 0$. At $z = 5$ m, $u = \gamma_w(2.5) = (9.81)(2.5) = 24.53$ kN/m². The pressure distribution for the wall is shown in Figure 7.5b.

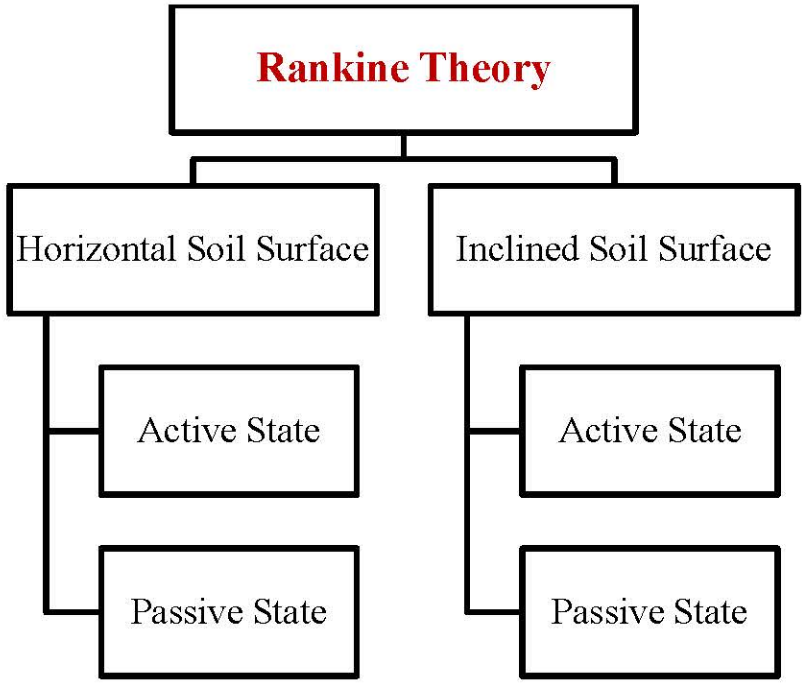
The total force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_o = \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4}$$
$$= \tfrac{1}{2}(2.5)(20.63) + (2.5)(20.63) + \tfrac{1}{2}(2.5)(32.49 - 20.63) + \tfrac{1}{2}(2.5)(24.53) = \mathbf{122.85\ kN/m}$$

The location of the center of pressure measured from the bottom of the wall (point O) =

$$\bar{z} = \frac{(\text{Area 1})\left(2.5+\frac{2.5}{3}\right)+(\text{Area 2})\left(\frac{2.5}{2}\right)+(\text{Area 3}+\text{Area 4})\left(\frac{2.5}{3}\right)}{P_o}$$
$$= \frac{(25.788)(3.33) + (51.575)(1.25) + (14.825 + 30.663)(0.833)}{122.85}$$
$$= \frac{85.87 + 64.47 + 37.89}{122.85} = \mathbf{1.53\ m}$$

2.3 Lateral Earth Pressure at Plastic Equilibrium:



2.3.1 Rankine Active Earth Pressure (Horizontal Surface)

Assuming that a gravity wall with no friction on its face is translated away from a soil mass that is initially at the at-rest condition, then the soil mass adjacent to the wall will pass into a failure state as shown in Figure 2.5. At this stage, the soil fails with the vertical stress unchanged from its original value, but with the lateral pressure decreased to a minimum value that can be defined using the Mohr-Coulomb failure criterion. **The minimum lateral pressure is known as the active pressure, and denoted by the symbol *Pa'* It is desirable to reach this condition if possible**, since it reduces the amount of load that the wall will have to carry while allowing the soil to share in the load-bearing process.

For the frictionless wall with a level backfill, the active pressure can be calculated from the geometry of the Mohr diagram in Figure 5.2 by the equation:

$$\sigma_a = k_a \gamma z - 2C\sqrt{k_a}$$

Where:

$$k_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \frac{1-\sin\phi}{1+\sin\phi}$$

k_a is coefficient of active earth pressure.
γ Unit weight of soil (kN/m^3)
ϕ angle of internal friction (degree).
C is the soil cohesion (kN/m^2).
z is the depth below the ground surface.

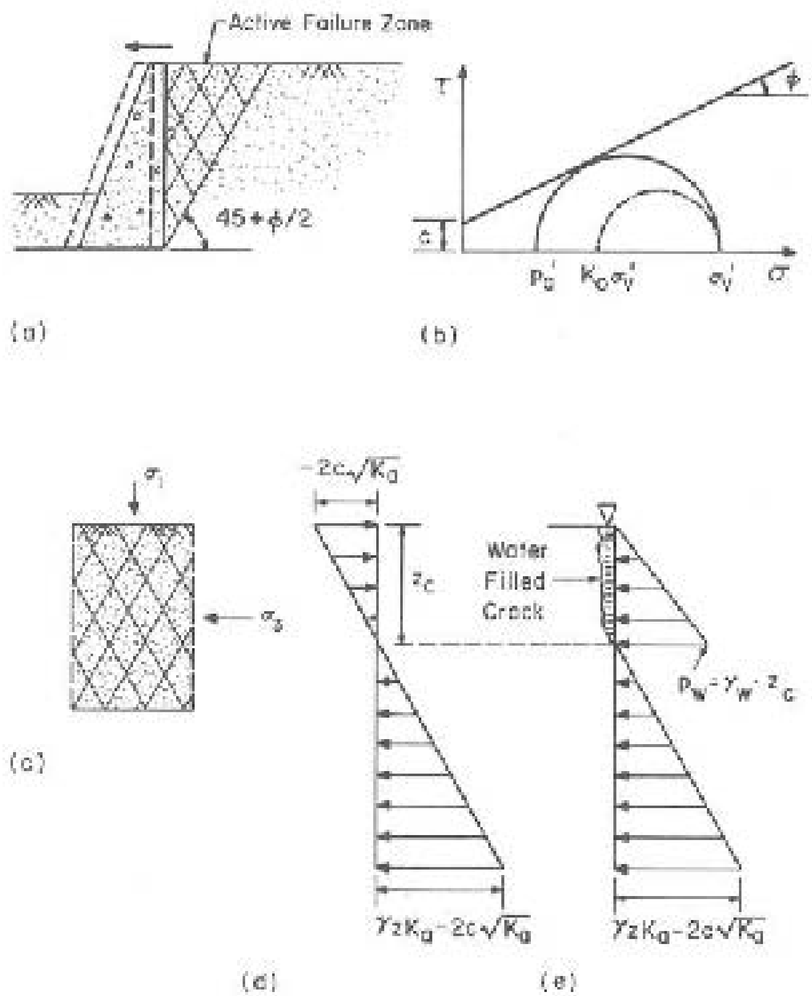


Figure 2.5: Active pressure-frictionless wall. (a) Frictionless wall moves away from backfill. (b) Stress state in active failure. (c) Active failure zone. (d) Theoretical active pressure distribution. (e) Water filled crack in tension zone.

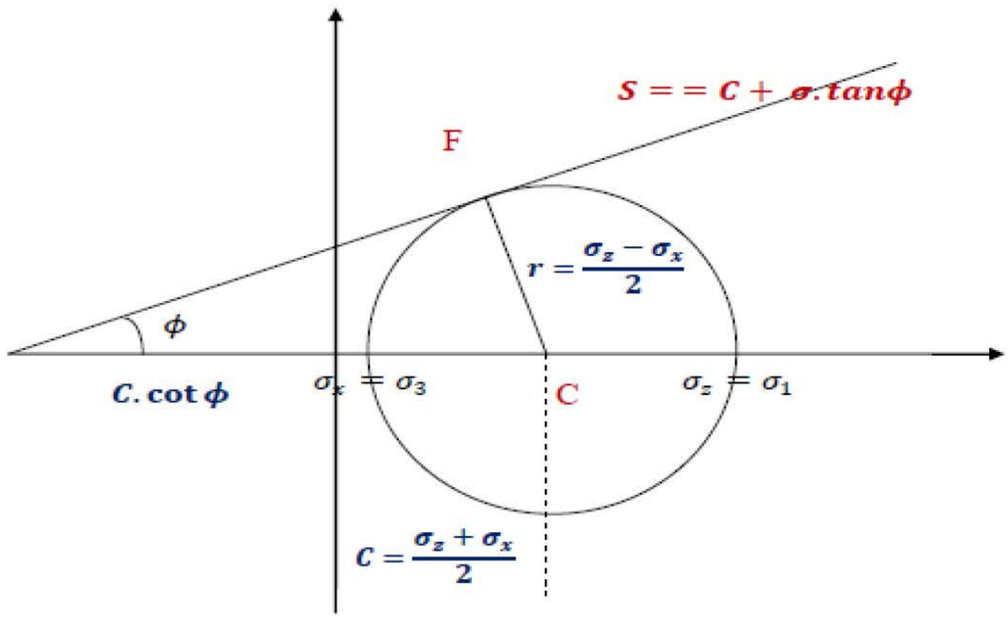


Figure 2.6: Mohr circle for active state at plastic equilibrium.

$$\sin\phi = \frac{r}{Center + C.\cot\phi}$$

$$\sin\phi = \frac{\frac{\sigma_z-\sigma_x}{2}}{\frac{\sigma_z+\sigma_x}{2}+C.\cot\phi}$$

$$\boldsymbol{\sigma_z - \sigma_x} = (\boldsymbol{\sigma_z - \sigma_x}) \times \sin\phi + 2C.\cot\phi.\sin\phi$$

$$\boldsymbol{\sigma_z - \sigma_x} = (\boldsymbol{\sigma_z - \sigma_x}) \times \sin\phi + 2C.\cos\phi$$

$$\boldsymbol{\sigma_x} + \boldsymbol{\sigma_x}\sin\phi = (\boldsymbol{\sigma_z} - \boldsymbol{\sigma_z}\sin\phi) - 2C.\cos\phi$$

$$\boldsymbol{\sigma_x}(\mathbf{1} + \sin\phi) = \boldsymbol{\sigma_z}(\mathbf{1} - \sin\phi) - 2C.\cos\phi$$

$$\boldsymbol{\sigma_x} = \boldsymbol{\sigma_z} \times \frac{(\mathbf{1}-\sin\phi)}{(\mathbf{1}+\sin\phi)} - 2C \times \frac{\cos\phi}{(\mathbf{1}+\sin\phi)}$$

$$\boldsymbol{\sigma_a} = \boldsymbol{\sigma_x} = \boldsymbol{\sigma_z} \times \frac{(\mathbf{1}-\sin\phi)}{(\mathbf{1}+\sin\phi)} - 2C \times \sqrt{\frac{(\mathbf{1}-\sin\phi)}{(\mathbf{1}+\sin\phi)}}$$

$$\boldsymbol{Since}\ \frac{(\mathbf{1}-\sin\phi)}{(\mathbf{1}+\sin\phi)} = \mathbf{tan}^2\left(\mathbf{45} - \frac{\phi}{2}\right) = \boldsymbol{K_a}$$

$$\boldsymbol{\sigma_a = k_a \gamma z - 2C\sqrt{k_a}}$$

1. Cohesive Soil:

If the soil has a cohesion component, the soil is in a state of tension of a depth of $\frac{2C}{\gamma\sqrt{k_a}}$. Ordinarily, it should not be assumed that this portion of the diagram will act on a wall, but rather that a tension crack will form to this depth, and fill with water, which then exerts a positive pressure on the wall as shown in Figures 2.7 and 2.8.

$$Z_o = \frac{2C}{\gamma\sqrt{k_a}} \qquad \textbf{(General State)}$$

$$Z_o = \frac{2C}{\gamma\sqrt{k_a}} - \frac{q}{\gamma} \qquad \textbf{(Surcharge Load)}$$

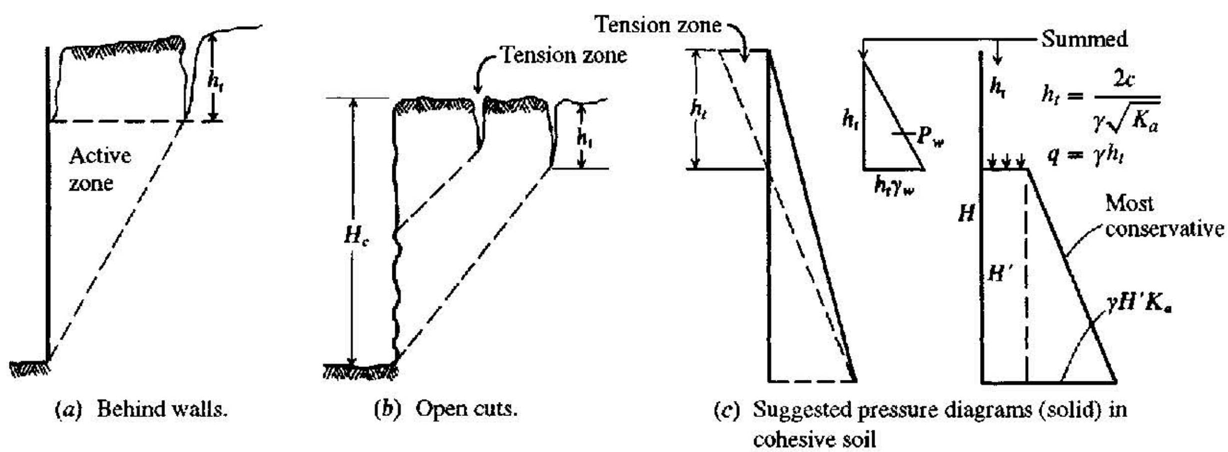


Figure 2.7: Tension crack and critical depth of an unbraced excavation. Tension cracks are often readily visible adjacent to excavations.

2. Surcharge and Non-homogeneous Conditions:

Design conditions often call for incorporation of a surcharge on the ground surface adjacent to the wall. In the case of a frictionless wall, the active pressure due to soil weight and surcharge, as shown in Figure 2.9, can be calculated using the equation:

$$\sigma_a = k_a(\gamma z + q) - 2C\sqrt{k_a}$$

Where:

q is surcharge pressure.

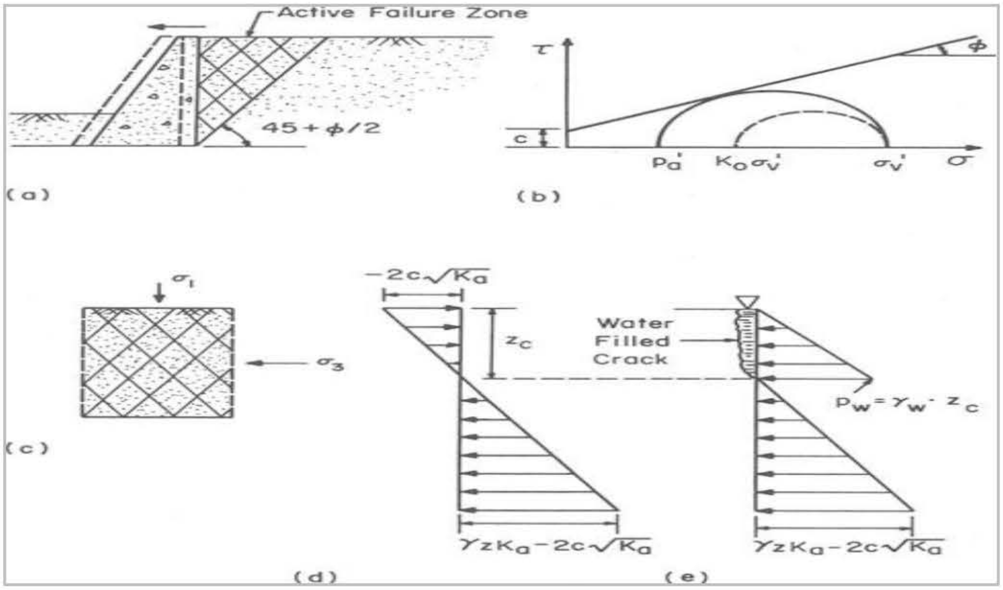


Figure 2.8: Active pressure-frictionless wall. (a) Frictionless wall moves away from backfill. (b) Stress state in active failure. (c) Active failure zone. (d) Theoretical active pressure distribution. (e) Water filled crack in tension zone.

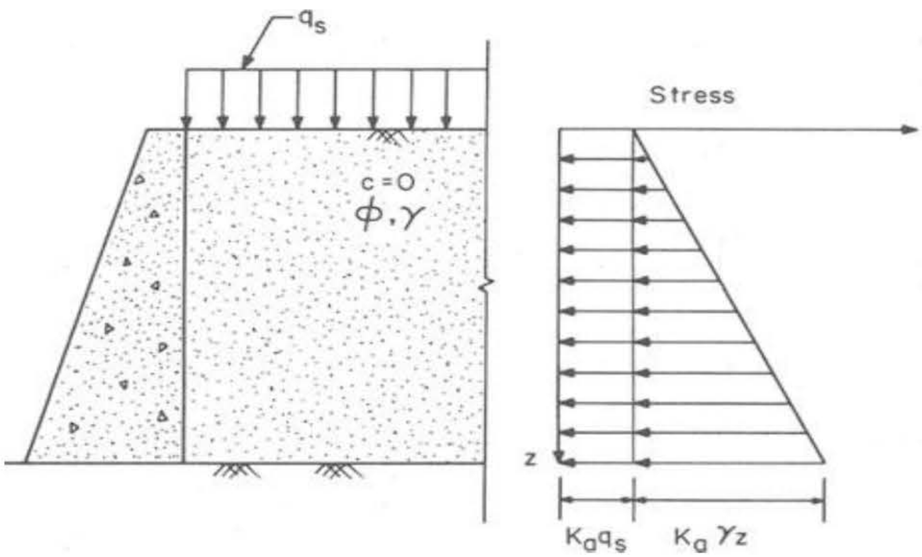


Figure 2.9: Frictionless wall with surcharge.

3. Water above the bottom of the wall:

Where a water table is situated above the bottom of the wall, or the soil involved is non-homogeneous, the above equations can be used if the proper allowance is made for the submergence effect and the changing properties for the soil layers. Figure 2.10 illustrates these considerations for active pressures of frictionless wall in **presence of groundwater table** and non-homogeneous cohesionless soil conditions.

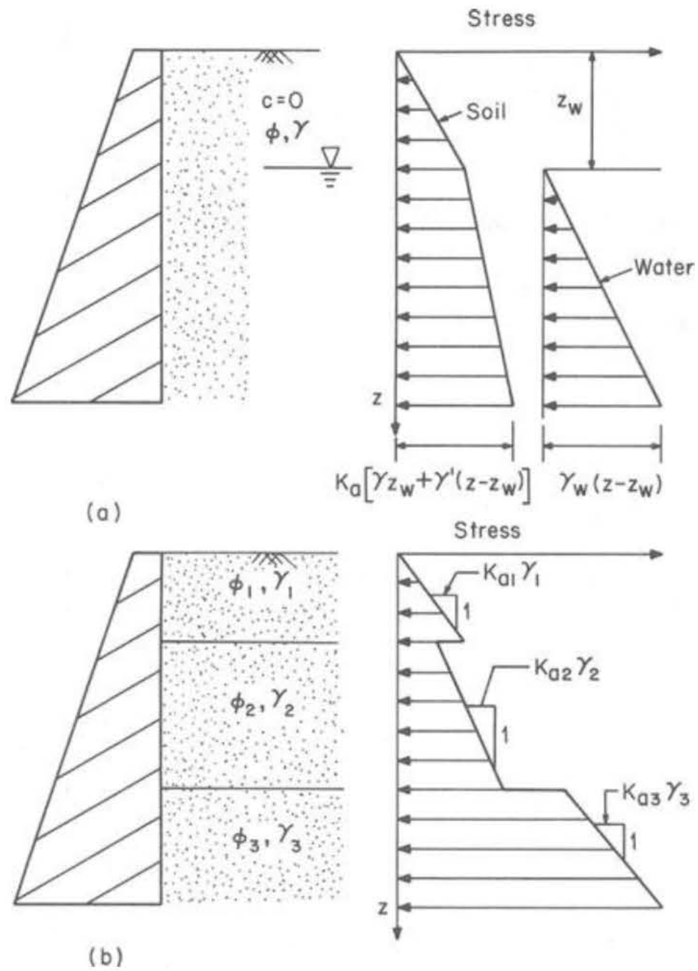


Figure 2.10 Active pressures for frictionless wall in presence of groundwater table and nonhomogeneous soil conditions. (a) Groundwater table. (b) Nonhomogeneous cohesionless soil.

2.3.2 Rankine Passive Earth Pressure (Horizontal Surface)

When the wall tilt toward the soil, soil will be compressed and contracted. The vertical stress σ_z remain constant, while the horizontal stress σ_x increases. The state of stress passes through the hydrostatic case ($\sigma_x = \sigma_y = \sigma_z$, then σ_x will be greater than σ_z and the failure condition is again fulfilled. This state of stresses can be represented by Mohr's circle as shown below Figure.

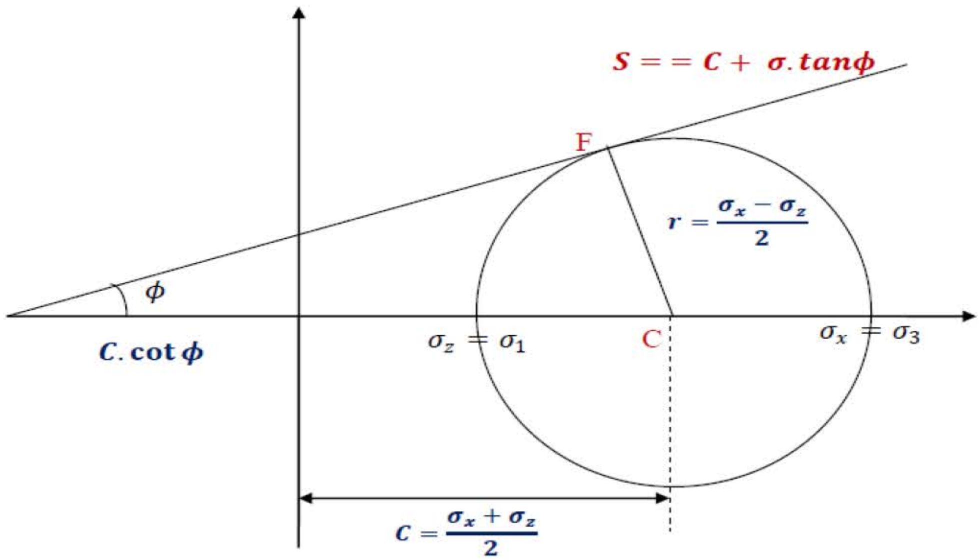


Figure 2.11: Mohr circle for passive state at plastic equilibrium.

$$\sin\phi = \frac{r}{Center + C.\cot\phi}$$

$$\sin\phi = \frac{\frac{\sigma_x - \sigma_z}{2}}{\frac{\sigma_x + \sigma_z}{2} + C.\cot\phi}$$

$$\boldsymbol{\sigma_x - \sigma_z} = (\boldsymbol{\sigma_x - \sigma_z}) \times \sin\phi + 2C.\cot\phi.\sin\phi$$

$$\boldsymbol{\sigma_x - \sigma_z} = (\boldsymbol{\sigma_x - \sigma_z}) \times \sin\phi + 2C.\cos\phi$$

$$\boldsymbol{\sigma_x - \sigma_x}\sin\phi = (\boldsymbol{\sigma_z + \sigma_z}\sin\phi) + 2C.\cos\phi$$

$$\boldsymbol{\sigma_x}(\mathbf{1} - \sin\phi) = \boldsymbol{\sigma_z}(\mathbf{1} + \sin\phi) + 2C.\cos\phi$$

$$\boldsymbol{\sigma_x} = \boldsymbol{\sigma_z} \times \frac{(1+\sin\phi)}{(1-\sin\phi)} + 2C \times \frac{\cos\phi}{(1+\sin\phi)}$$

$$\boldsymbol{\sigma_a} = \boldsymbol{\sigma_x} = \boldsymbol{\sigma_z} \times \frac{(1+\sin\phi)}{(1-\sin\phi)} + 2C \times \sqrt{\frac{(1+\sin\phi)}{(1-\sin\phi)}}$$

$$\boldsymbol{Since}\ \frac{(1+\sin\phi)}{(1-\sin\phi)} = \tan^2\left(45 + \frac{\phi}{2}\right) = \boldsymbol{K_p}$$

$$\boldsymbol{\sigma_p = k_p \gamma z - 2C\sqrt{k_p}}$$

2.3.3 Rankine Active Earth Pressure (Inclined Ground Surface)

Sometimes, the surface of the backfill will be inclined to the horizontal. This is considered to be a form of surcharge-inclined surcharge', and the angle of inclination of the backfill with the horizontal is called the 'angle of surcharge'. Rankine's theory for this case is based on the assumption that a 'conjugate' relationship exists between the vertical pressures and lateral pressures on vertical planes within the soil adjacent to a retaining wall. It may be shown that such a conjugate relationship would hold between vertical stresses and lateral stresses on vertical planes within an infinite slope. Thus, it would amount to assuming that the introduction of a retaining wall into the infinite slope does not result in any changes in shearing stresses at the surface of contact between the wall and the backfill. This inherent assumption in Rankine's theory means that the effect of 'wall friction' or friction between the wall and the backfill soil is neglected. Let us consider an element of soil of unit horizontal width at depth *z* below the surface of the backfill, the faces of which are parallel to the surface and to the vertical, as shown in Figure 2.12 (*a*).

The vertical stress and the lateral stress on the vertical plane are each parallel to the plane of the other and, therefore, are said to be conjugate stresses. Both have obliquities equal to the angle of inclination of the slope β.

The magnitude of the vertical stress acting on the face of the element parallel to the surface can be easily obtained as follows:

1. The weight of column of soil above the face = $\boldsymbol{\gamma}\ .\ \boldsymbol{z}$. Since the horizontal width is unity, the area of the parallelogram is $Z \times 1$, and the volume of the parallelogram is $Z \times 1 \times 1$ cubic units.
2. This force acts on an area $\frac{1}{\cos\beta} \times 1$
3. The vertical stress σv on the face of the element parallel to the slope is:

$$\sigma_v = \frac{\gamma Z}{(1/\cos\beta)} = \gamma . Z . \cos\beta$$

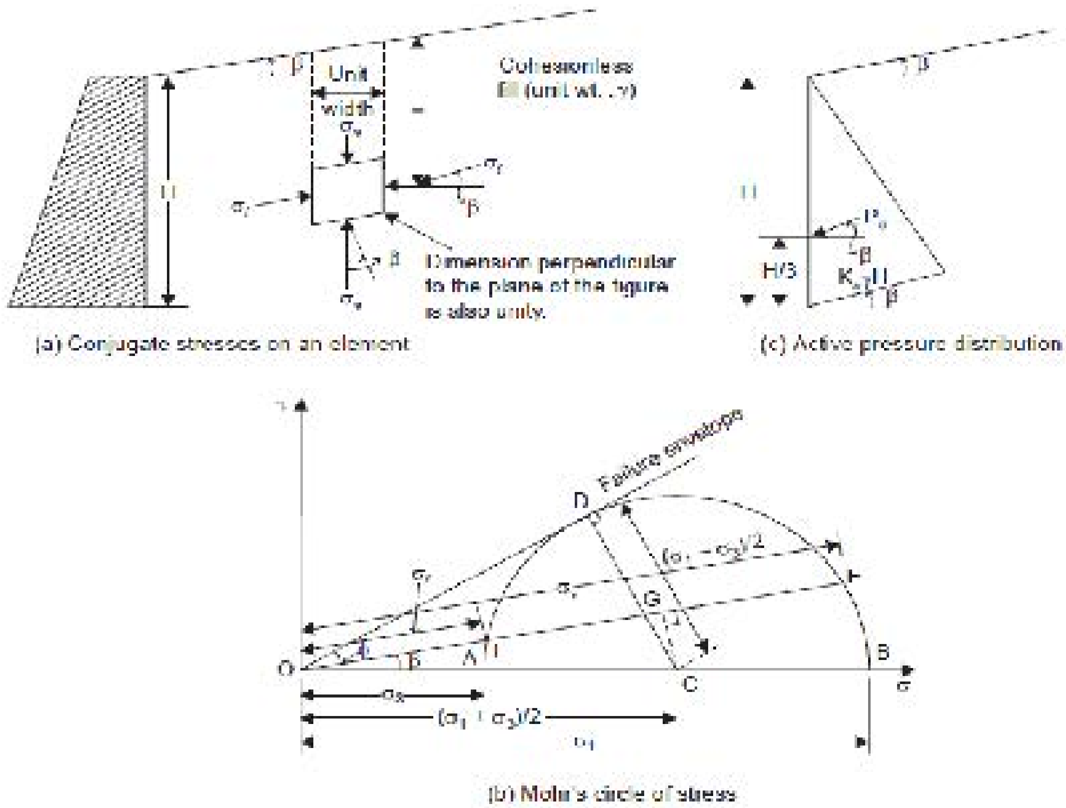


Figure 2.12: Inclined surcharge-Rankine's theory.

The conjugate nature of the lateral pressure on the vertical plane and the vertical pressure on a plane parallel to the inclined surface of the backfill may also be established from the Mohr's circle diagram of stresses, Fig. 13.12 (*b*). It is obvious that, from the very definition of conjugate relationship, the angle of obliquity of the resultant stress should be the same for both planes. Thus, in the diagram, if a line *OE* is drawn at an angle β, the angle of obliquity, with the σ-axis, to cut the Mohr's circle in *E* and *F*, *OE* represents σv and *OF* represents $\sigma 1$, for the active case (for the passive case, it is vice versa).

Now the relationship between σv and σ_1 may be derived from the geometry of the Mohr's circle, Fig. 2.12 (*b*), as follows.

$K_a = \frac{\sigma_l}{\sigma v} = \frac{OF}{OE}$

$K_a = \frac{OF}{OE} = \frac{OG-FG}{OG+EG}$

$FG = EG$

$K_a = \frac{OG-EG}{OG+EG}$ ………………. (1)

$OG = OC \times \cos\beta$ ……………..(2)

$CG = OC \times \sin\beta$

$CE^2 = CG^2 + EG^2$

$EG^2 = CE^2 - CG^2$

$EG = \sqrt{CE^2 - CG^2}$

$CE = CD$ (Radius of circle)

$CE = OC \times \sin\phi$

$EG = \sqrt{(OC \times \sin\phi)^2 - (OC \times \sin\beta)^2}$

$EG = OC\sqrt{(\sin\phi)^2 - (\sin\beta)^2}$ …………………….(3)

Sub. Equation (2) and (3) in equation (1).

$K_a = \frac{OG-EG}{OG+EG} = \frac{OC\times\cos\beta - OC\sqrt{(\sin\phi)^2-(\sin\beta)^2}}{OC\times\cos\beta + OC\sqrt{(\sin\phi)^2-(\sin\beta)^2}}$

$K_a = \frac{OC\{\cos\beta - \sqrt{(\sin\phi)^2-(\sin\beta)^2}\}}{OC\ \{\cos\beta + \sqrt{(\sin\phi)^2-(\sin\beta)^2}\}}$

$$\boldsymbol{K_a = \frac{\cos\beta - \sqrt{(\sin\phi)^2 - (\sin\beta)^2}}{\cos\beta + \sqrt{(\sin\phi)^2 - (\sin\beta)^2}}}$$

Therefore, the active stress acting parallel to the ground surface:

$$\boldsymbol{\sigma_a = k_a\gamma z.\cos\beta}$$

$$\boldsymbol{E_a = P_a = \frac{1}{2}\gamma.z^2.\cos\beta.K_a}$$

2.3.4 Rankine Passive Earth Pressure (Inclined Ground Surface)

For the passive state in inclined ground surface and for granular soil, the relationship between σv and σ_1 may be derived from the geometry of the Mohr's circle.

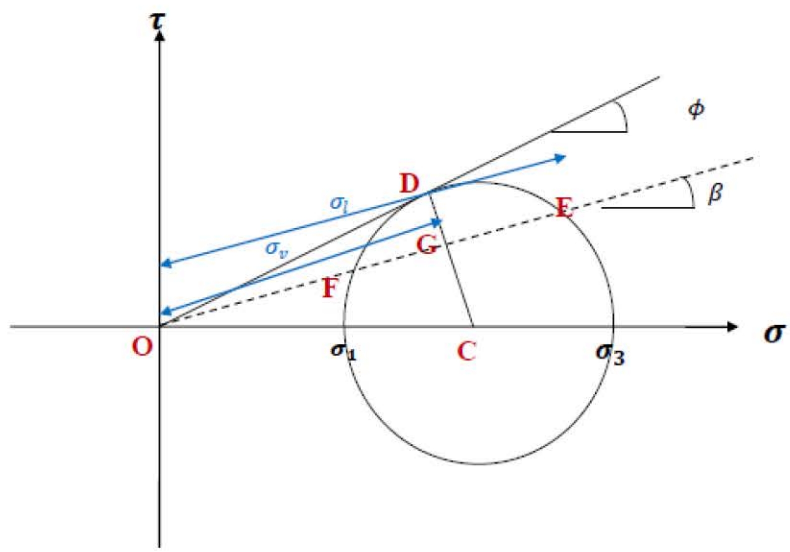


Figure 2.13: Inclined surcharge-Rankine's theory for passive state.

$K_p = \frac{\sigma_l}{\sigma_v} = \frac{OE}{OF}$

$K_p = \frac{OF}{OE} = \frac{OG+FG}{OG-EG}$

$FG = EG$

$K_p = \frac{OG+EG}{OG-EG}$ ………………… (1)

$OG = OC \times \cos\beta$ ………………(2)

$CG = OC \times \sin\beta$

$CE^2 = CG^2 + EG^2$

$EG^2 = CE^2 - CG^2$

$EG = \sqrt{CE^2 - CG^2}$

$CE = CD$ (Radius of circle)

$CE = OC \times \sin\phi$

$EG = \sqrt{(OC \times \sin\phi)^2 - (OC \times \sin\beta)^2}$

$EG = OC\sqrt{(\sin\phi)^2 - (\sin\beta)^2}$ ……………………….(3)

Sub. Equation (2) and (3) in equation (1).

$K_p = \frac{OG-EG}{OG+EG} = \frac{OC\times\cos\beta + OC\sqrt{(\sin\phi)^2-(\sin\beta)^2}}{OC\times\cos\beta - OC\sqrt{(\sin\phi)^2-(\sin\beta)^2}}$

$K_p = \frac{OC\{\cos\beta + \sqrt{(\sin\phi)^2-(\sin\beta)^2}\}}{OC\{\cos\beta - \sqrt{(\sin\phi)^2-(\sin\beta)^2}\}}$

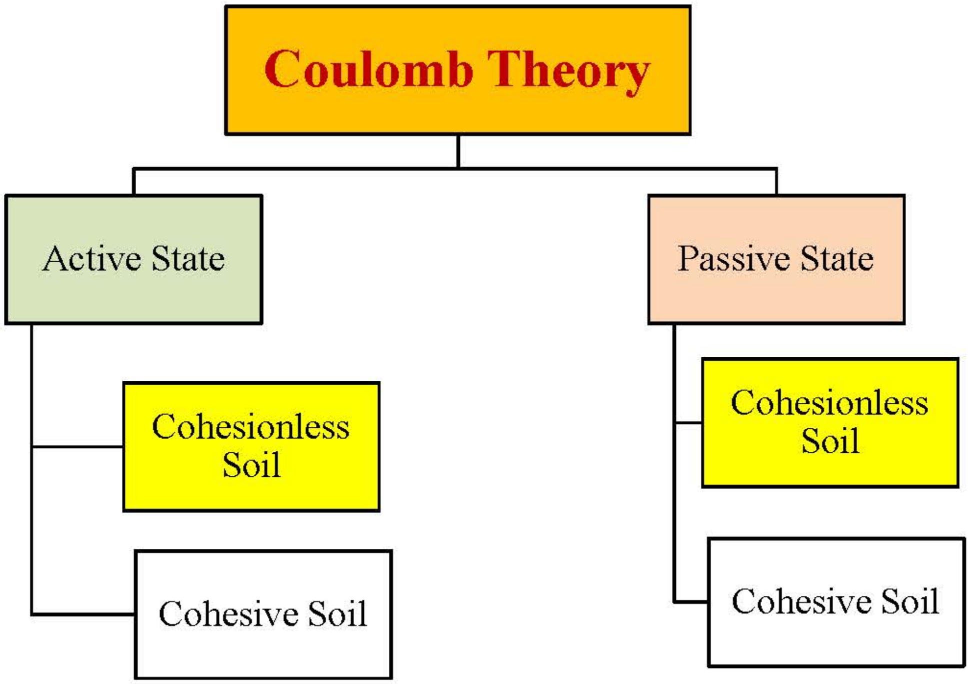
$$\boldsymbol{K_p = \frac{\cos\beta + \sqrt{(\sin\phi)^2 - (\sin\beta)^2}}{\cos\beta - \sqrt{(\sin\phi)^2 - (\sin\beta)^2}}}$$

Therefore, the passive stress acting parallel to the ground surface:

$$\boldsymbol{\sigma_p = k_p \gamma z . \cos\beta}$$

$$\boldsymbol{E_p = P_p = \frac{1}{2}\gamma . z^2 . \cos\beta . K_p}$$

2.4 Lateral Earth Pressure at Plastic Equilibrium (Coulomb Theory):



2.4.1 Coulomb Theory for Active Earth Pressure of Cohesionless Soil:

The Rankine active earth pressure calculations discussed in the preceding sections were based on the assumption that the wall is frictionless. In 1776, Coulomb proposed a theory for calculating the lateral earth pressure on a retaining wall with granular soil backfill. This theory takes wall friction into consideration. A simple case of active earth pressure on an inclined wall face with a uniformly sloping backfill may be considered first. The backfill consists of homogeneous, elastic and isotropic cohesionelss soil, and friction between soil and wall δ is taken into account. A unit length of the wall perpendicular to the plane of the paper is considered. The forces acting on the sliding wedge are:

(i) Weight of the soil contained in the sliding wedge W,

(ii) R, the soil reaction across the plane of sliding, and

(iii) The active thrust Pa against the wall,

In this case, the reaction from the wall on to the sliding wedge, as shown in Figure 2.14.

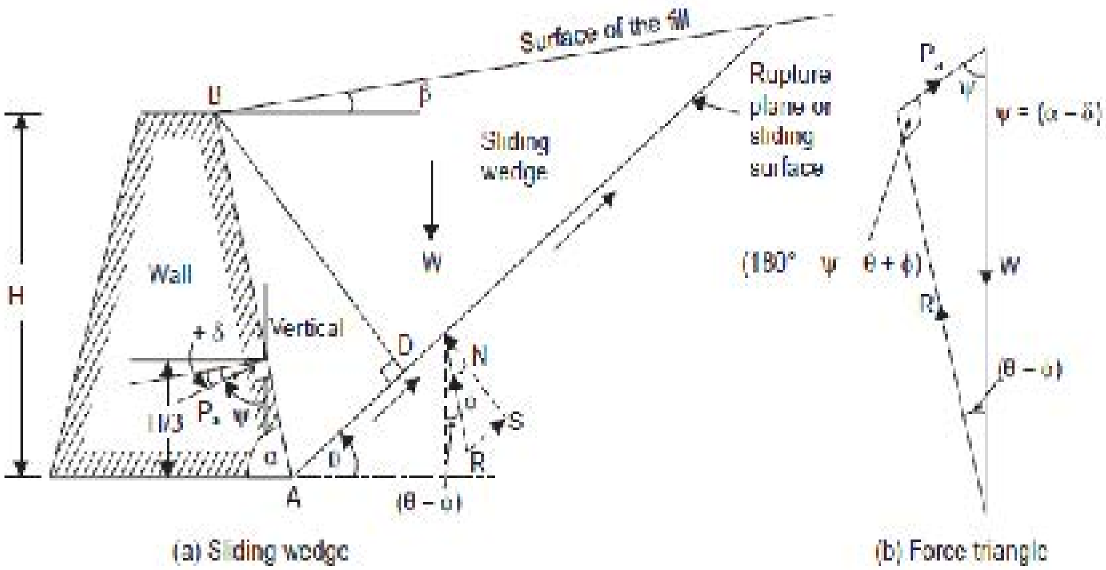


Figure 2.14: Active earth pressure of cohesionless soil-Coulomb's theory.

$$W = \frac{1}{2}\,\gamma\,(Area\ of\ the\ wedge)$$

$$W = \frac{1}{2}\,\gamma\,AC.BD.1$$

From sine law:

$$AC = AB.\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

$$BD = AB.\sin(\alpha+\theta)$$

$$W = \frac{1}{2}\,\gamma.AB.\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}.AB.\sin(\alpha+\theta)$$

Since $AB = \frac{H}{\sin\alpha}$

$$W = \frac{\gamma.H^2}{2\,\sin^2\alpha}.\sin(\alpha+\theta).\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From force triangle:

$$\frac{\boldsymbol{W}}{\sin(180-\psi-\theta+\phi)} = \frac{\boldsymbol{P_a}}{\sin(\theta-\phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$

Substitute equation (1) in (2) to obtain the lateral forced applied on the wall $\boldsymbol{P_a}$.

$$P_a = W.\frac{\sin(\theta-\phi)}{\sin(180-\psi-\theta+\phi)}$$

$$P_a = \frac{\gamma . H^2}{2\ \sin^2\alpha} . \frac{\sin(\theta-\phi)}{\sin(180-\psi-\theta+\phi)} . \frac{\sin(\alpha+\theta) . \sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

The maximum value of *Pa* is obtained by equating the first derivative of *Pa* with respect to θ to zero;or $\frac{dp}{d\theta} = 0$, and substituting the corresponding value of θ.

The value of *Pa* so obtained is written as:

$$P_a = \frac{1}{2}\gamma H^2 . \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha . \sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta).\sin(\alpha+\beta)}}\right]^2}$$

P_a is usually written as:

$$P_a = \frac{1}{2}\gamma H^2 . K_a$$

$$\therefore\ \boldsymbol{K_a} = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha . \sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta).\sin(\alpha+\beta)}}\right]^2}$$

Ka is the coefficient of active earth pressure.

In the case of $\delta = 0, \beta = 0, and\ \alpha = 90^0$

$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2(45-\frac{\phi}{2})$ ….. as Rankine theory.

2.4.2 Coulomb Theory for Passive Earth Pressure of Cohesionless Soil:

The passive case differs from the active case in that the obliquity angles at the wall and on the failure plane are of opposite sign. Plane failure surface is assumed for the passive case also in the Coulomb theory but the critical plane is that for which the passive thrust is minimum. The failure plane is at a much smaller angle to the horizontal than in the active case, as shown in Figure 2.15.

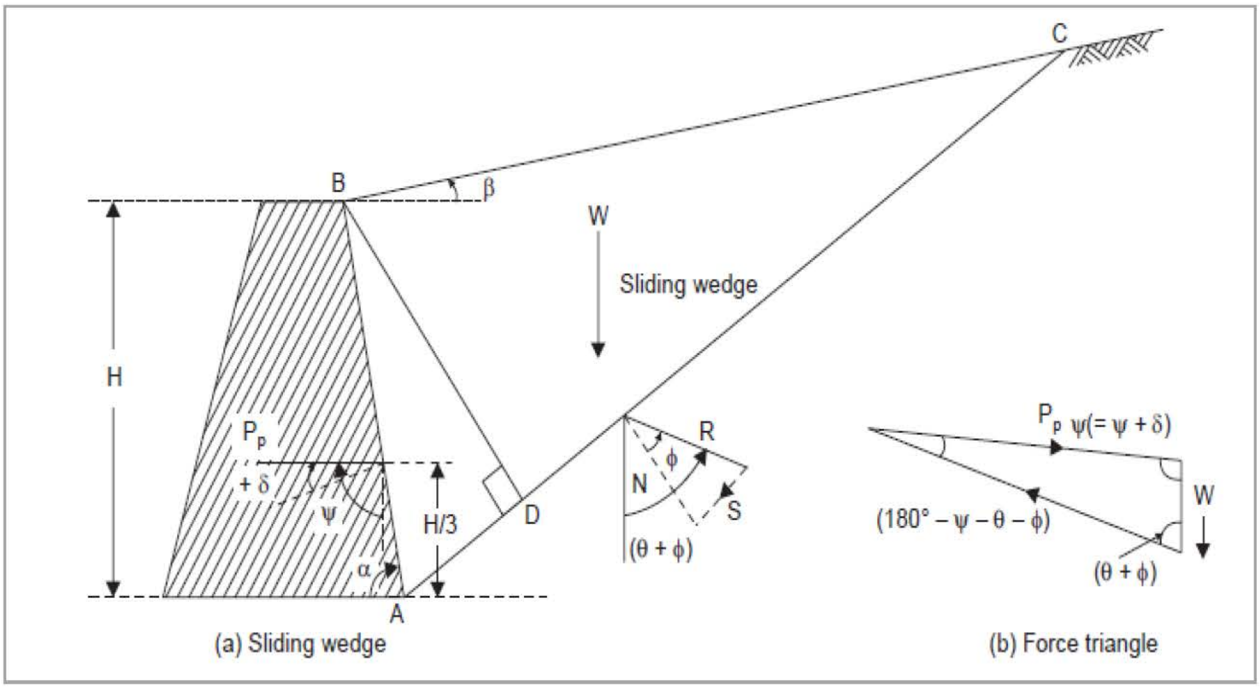


Figure 2.15: Passive earth pressure of cohesionless soil-Coulomb's theory.

$$W = \frac{1}{2}\,\gamma\,(Area\ of\ the\ wedge)$$

$$W = \frac{1}{2}\,\gamma\,AC.BD.1$$

From sine law:

$$AC = AB.\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

$$BD = AB.\sin(\alpha+\theta)$$

$$W = \frac{1}{2}\,\gamma.AB.\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}.AB.\sin(\alpha+\theta)$$

Since $AB = \frac{H}{\sin\alpha}$

$$W = \frac{\gamma.H^2}{2\,\sin^2\alpha}\,.\sin(\alpha+\theta).\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From force triangle:

$$\frac{\boldsymbol{W}}{\mathbf{sin(180}-\boldsymbol{\psi}-\boldsymbol{\theta}-\boldsymbol{\phi})} = \frac{\boldsymbol{P_p}}{\mathbf{sin}(\boldsymbol{\theta}+\boldsymbol{\phi})} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substitute equation (1) in (2) to obtain the lateral forced applied on the wall $\boldsymbol{P_p}$.

$$P_p = W.\frac{\sin(\theta+\phi)}{\sin(180-\psi-\theta-\phi)}$$

$$P_p = \frac{1}{2}.\frac{\gamma.H^2}{\sin^2\alpha}.\frac{\sin(\theta+\phi)}{\sin(180-\psi-\theta-\phi)}.\frac{\sin(\alpha+\theta).\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

The minimum value of *Pp* is obtained by equating the first derivative of *Pp* with respect to θ to zero value; or $\frac{dp}{d\theta} = \mathbf{0}$, and substituting the corresponding value of θ. The value of *Pp* so obtained is written as:

$$P_p = \frac{1}{2}\gamma H^2.\frac{\sin^2(\alpha-\phi)}{\sin^2\alpha.\sin(\alpha+\delta)\left[1-\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta).\sin(\alpha+\beta)}}\right]^2}$$

P_p is usually written as:

$$P_p = \frac{1}{2}\gamma H^2.K_p$$

$$\therefore\ \boldsymbol{K_p} = \frac{\mathbf{sin^2}(\boldsymbol{\alpha}-\boldsymbol{\phi})}{\mathbf{sin^2}\,\boldsymbol{\alpha}.\mathbf{sin}(\boldsymbol{\alpha}+\boldsymbol{\delta})\left[\mathbf{1}-\sqrt{\frac{\mathbf{sin}(\boldsymbol{\phi}+\boldsymbol{\delta}).\mathbf{sin}(\boldsymbol{\phi}-\boldsymbol{\beta})}{\mathbf{sin}(\boldsymbol{\alpha}-\boldsymbol{\delta}).\mathbf{sin}(\boldsymbol{\alpha}+\boldsymbol{\beta})}}\right]^{\mathbf{2}}}$$

Kp is the coefficient of passive earth pressure.

In the case of $\delta = 0, \beta = 0, and\ \alpha = 90^0$

$$K_a = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2(45+\frac{\phi}{2}) \quad \ldots\ldots \text{as Rankine theory.}$$

2.4.3 Coulomb Theory for Active Earth Pressure of Cohesive Soil:

The lateral earth pressure of cohesive soil may be obtained from the Coulomb's wedge theory; however, one should take cognizance of the tension zone near the surface of the cohesive backfill and consequent loss of contact and loss of adhesion and friction at the back of the wall and along the plane of rupture, so as to avoid getting erroneous results. The trial wedge method may be applied to this case a illustrated in Fig. 2.16. The following five forces act on a trial wedge:

1. Weight of the wedge including the tension zone, *W*.
2. Cohesion along the wall face or adhesion between the wall and the fill, *Ca*.
3. Cohesion along the rupture plane, *C*.
4. Reaction on the plane of failure, *R*, acting at φ to the normal to the plane of failure.
5. Active thrust, *Pa*, acting at δ to the normal to the face of the wall.
6. Cohesion force along the sliding surface ($C_a = C_a \times BF$

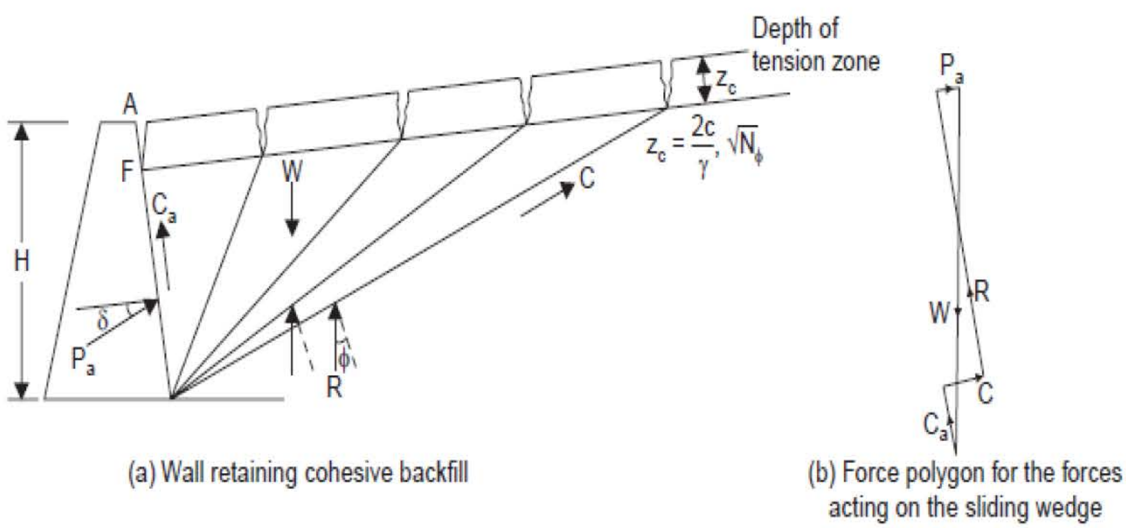


Figure 2.16: Active earth pressure of cohesive soil trial wedge method—Coulomb's theory.

The total direction of all forces is known and polygon of forces can be drawn to determine the value of (Pa). This method must be repeated for different trail wedges to find out maximum (Pa). Noting that, if the water is filled the crackes, the hydrostatic pressure must be taken into account.

2.4.4 Coulomb Theory for Passive Earth Pressure of Cohesive Soil:

The procedure adopted to determine the active earth pressure of cohesive soil from **Coulomb's theory** may also be used to determine the passive earth resistance of cohesive soil. The points of difference are that the signs of friction angles, φ and δ, will be reversed and the directions of *Ca* and *C* also get reversed.

Either the trial wedge approach or **Culmann's approach** may be used but one has also to consider the effect of the tensile zone in reducing *Ca* and *C*. However, it must be noted that the Coulomb theory with plane rupture surfaces is not applicable to the case of passive resistance. Analysis must be carried out, strictly speaking, using curved rupture surfaces such as logarithmic spirals (Terzaghi, 1943), so as to avoid overestimation of passive resistance.

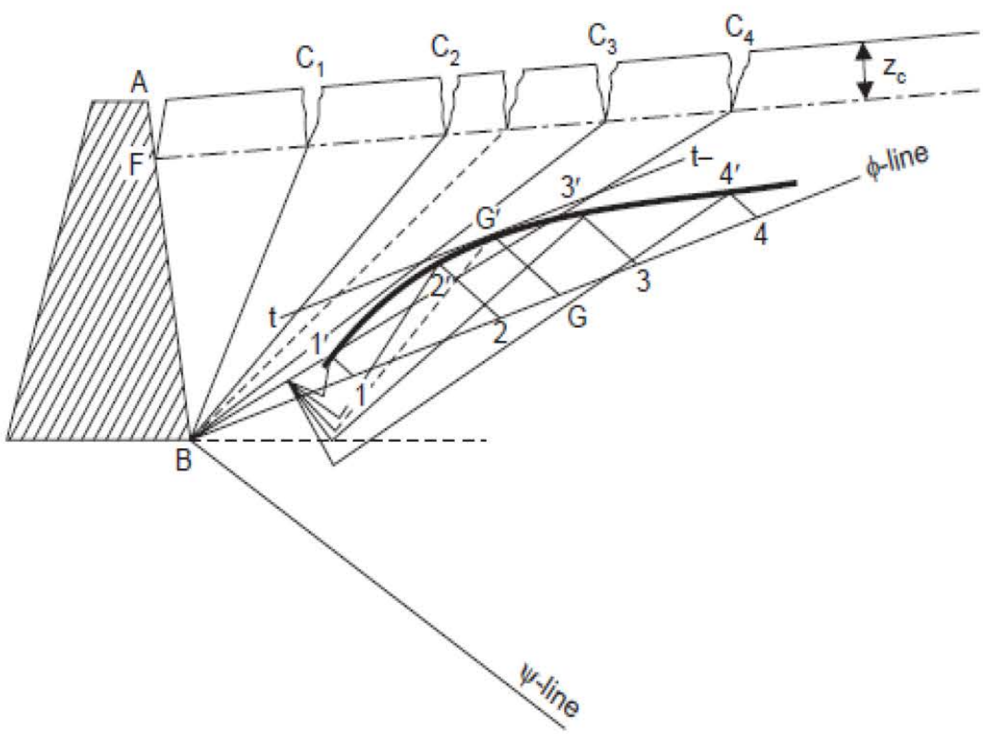


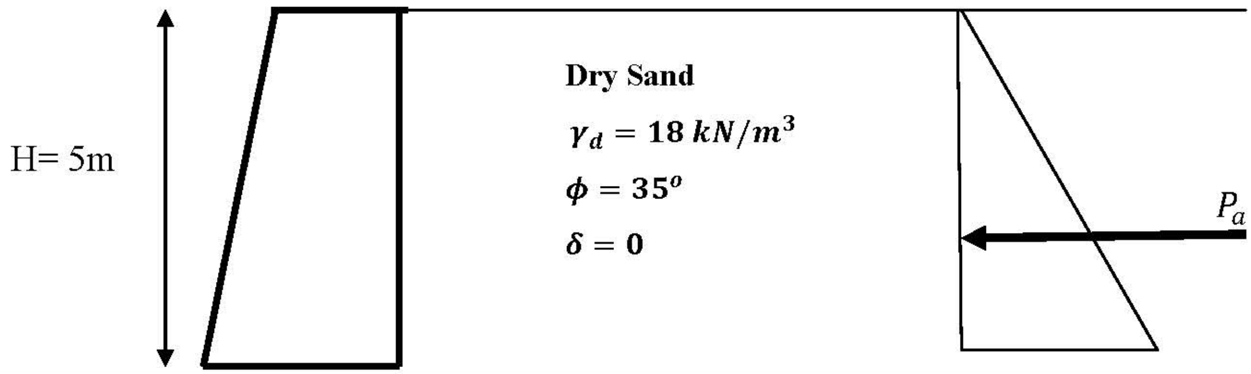
Figure 2.17: Culmann's method adapted to allow for passive earth pressure of cohesion

2.5: Comparison of Coulomb's Theory with Rankine's Theory:

The following are the important points of comparison:

1) Coulomb considers a retaining wall and the backfill as a system; he takes into account the friction between the wall and the backfill, while Rankine does not.
2) The backfill surface may be plane or curved in Coulomb's theory, but Rankine's allows only for a plane surface.
3) In Coulomb's theory, the total earth thrust is first obtained and its position and direction of the earth pressure are assumed to be known; linear variation of pressure with depth is tacitly assumed and the direction is automatically obtained from the concept of wall friction. In Rankine's theory, plastic equilibrium inside a semi-infinite soil mass is considered, pressures evaluated, a retaining wall is imagined to be interposed later, and the location and magnitude of the total earth thrust are established mathematically.
4) Coulomb's theory is more versatile than Rankine's in that it can take into account any shape of the backfill surface, break in the wall face or in the surface of the fill, effect of stratification of the backfill, effect of various kinds of surcharge on earth pressure, and the effects of cohesion, adhesion and wall friction. It lends itself to elegant graphical solutions and gives more reliable results, especially in the determination of the passive earth resistance; this is in spite of the fact that static equilibrium condition does not appear to be satisfied in the analysis.
5) Rankine's theory is relatively simple and hence is more commonly used, while Coulomb's theory is more rational and versatile although cumbersome at times; therefore, the use of the latter is called for in important situations or problems.

Example 1: Determine the active force for the retaining wall shown in below Figure:



Solution:

$\sigma_a = \gamma . H . K_a$

$K_a = \frac{1-\sin 35}{1+\sin 35} = \tan^2(45 + \frac{35}{2})$

$K_a = 0.27$

$\sigma_a = 5 \times 18 \times 0.27$

$\sigma_a = 24.3\ kN/m^2$

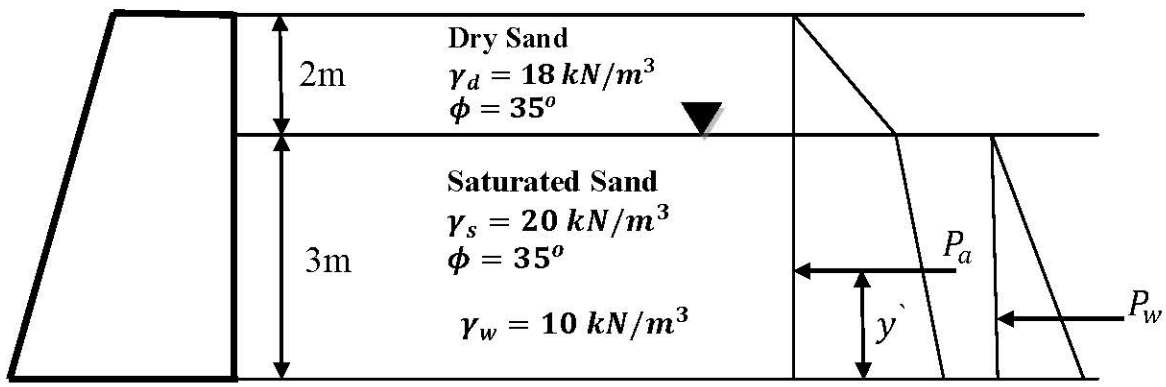
$P_a = \frac{1}{2} \times \gamma \times H^2 \times K_a$

$P_a = \frac{1}{2}\ (18)(5)^2(0.27)$

$P_a = 60.75\ kN/m$

$H` = \frac{H}{3} = \frac{5}{3} = 1.6667\ m$ (Location of the active lateral force applied on the wall).

Example 2: Determine the active force P_a for the retaining wall shown in below Figure:



Solution:

At (H=0 m):

$\sigma_a = \gamma.H.K_a \qquad K_a = \frac{1-\sin 35}{1+\sin 35} = \tan^2(45 + \frac{35}{2}) = 0.27$

$\sigma_{a(1)} = 18 \times 0 \times 0.27 = 0.0\ kN/m^2$

At (H=2 m):

$\sigma_{a(2)} = \gamma.H.K_a \qquad K_a = \frac{1-\sin 35}{1+\sin 35} = \tan^2(45 + \frac{35}{2}) = 0.27$

$\sigma_{a(2)} = 18 \times 2 \times 0.27 = 9.72\ kN/m^2$

At (H=5 m):

$\sigma_{a(3)} = [\gamma.H + (\gamma_{total} - \gamma_w) \times H_w].K_a \qquad K_a = \frac{1-\sin 35}{1+\sin 35} = \tan^2(45 + \frac{35}{2})$

$\sigma_{a(3)} = [18 \times 2 + (20 - 10) \times 3] \times 0.27$

$\sigma_{a(3)} = 17.82\ kN/m^2$

Due to water pressure:

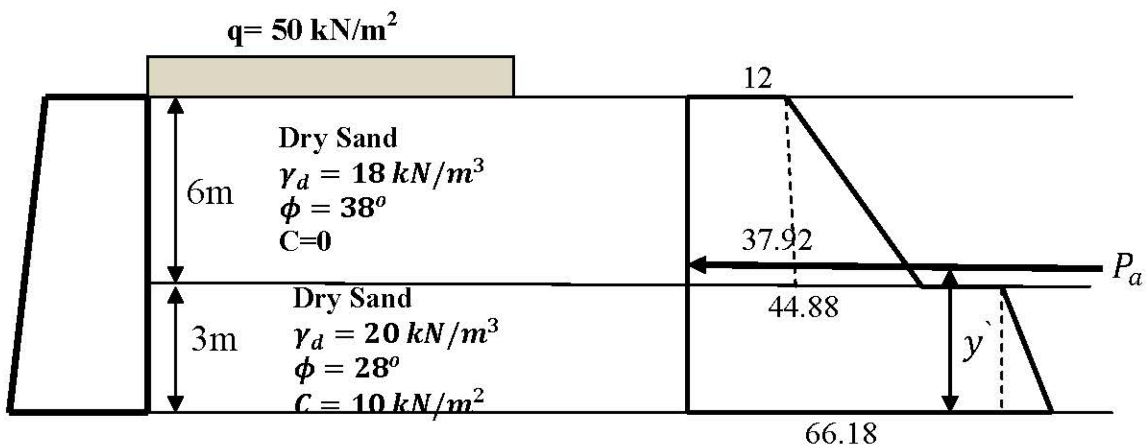
$\sigma_w = \gamma_w \times H_w = 10 \times (5 - 2) = 30\ kN/m^2$

$P_a = \left[\frac{1}{2} \times 9.72 \times 2\right] + [9.72 \times 3] + \left[\frac{(17.82-9.72)}{2} \times 3\right] + \left[\frac{1}{2} \times 30 \times 3\right]$

$\boldsymbol{P_a = 96.03\ kN/m}$

$\boldsymbol{y^`} = \frac{\left[\frac{1}{2}\times 9.72\times 2\times\left(\frac{2}{3}+3\right)\right]+\left[9.72\times 3\times\left(\frac{3}{2}\right)\right]+[(17.82-9.72)\times\frac{3}{2}\times 1]\times[\left[\frac{1}{2}\times 30\times 3\times\left(\frac{3}{3}\right)\right]}{96.03} = \boldsymbol{1.29\ m}$

Example 3: Determine the active force P_a for the retaining wall shown in below Figure:



Solution:

1. At (H=0 m):

$$\sigma_a = (\gamma.H + q)K_a \qquad K_{a(1)} = \frac{1-\sin 38}{1+\sin 38} = \tan^2(45 + \frac{38}{2}) = 0.24$$

$$\sigma_{a(1)} = (18 \times 0 + 50)\ 0.24 = 12\ kN/m^2$$

2. At (H=6 m):

$$\sigma_{a(2)} = \gamma.H.K_a \qquad K_{a(1)} = \frac{1-\sin 38}{1+\sin 38} = \tan^2(45 + \frac{38}{2}) = 0.24$$

$$\sigma_{a(2)} = (18 \times 6 + 50) \times 0.24 = 37.92\ kN/m^2$$

3. At (H=6 m):

$$\sigma_{a(3)} = (\gamma.H + q)K_{a(2)} - 2C\sqrt{K_{a(2)}}$$

$$K_{a(2)} = \frac{1-\sin 28}{1+\sin 28} = \tan^2(45 + \frac{28}{2}) = 0.36$$

$$\sigma_{a(3)} = (18 \times 6 + 50) \times 0.36 - 2 \times 10 \times \sqrt{0.36} = 44.88\ kN/m^2$$

4. At (H=9 m):

$$\sigma_{a(4)} = [\gamma_1.H_1 + \gamma_2.H_2 + q].K_{a(2)} - 2C\sqrt{K_{a(2)}}$$

$$K_{a(2)} = \frac{1-\sin 28}{1+\sin 28} = \tan^2(45 + \frac{28}{2}) = 0.36$$

$$\sigma_{a(4)} = [18 \times 6 + 20 \times 6 + 50] \times 0.36 - 2 \times 10 \times \sqrt{0.36}$$

$\sigma_{a(4)} = 66.48\ kN/m^2$

The active lateral force:

$$P_a = [12 \times 6] + \left[(37.92 - 12) \times \frac{6}{2}\right] + [44.88 \times 3] + \left[\frac{1}{2} \times (66.48 - 44.88) \times 3\right]$$

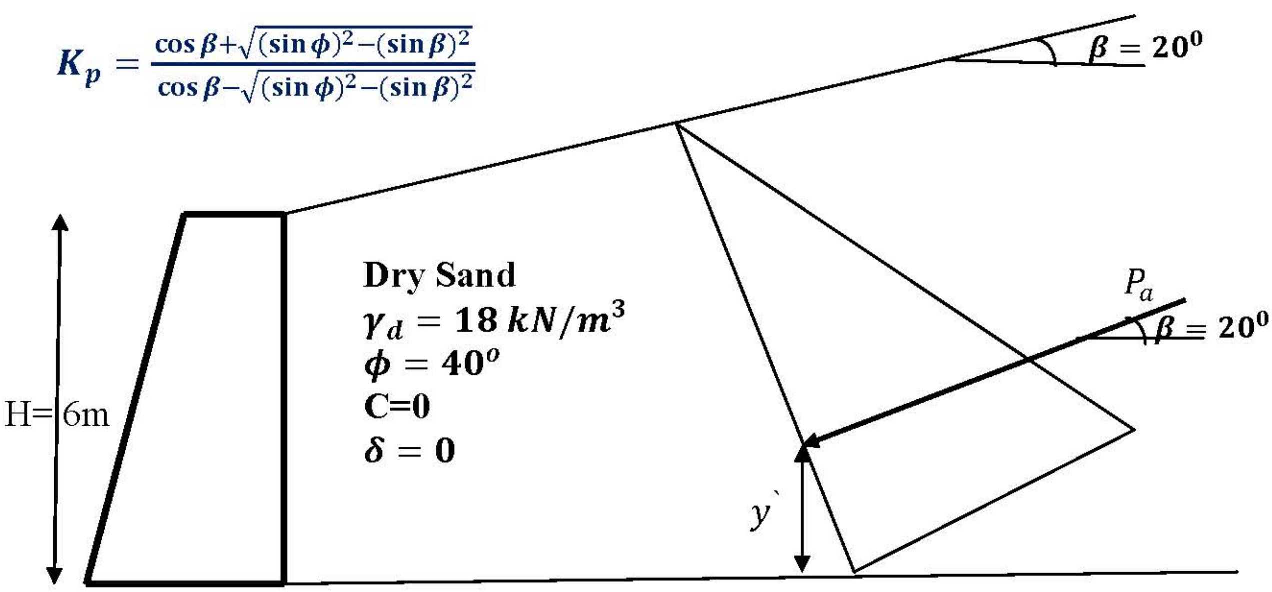
$$\boldsymbol{P_a = 72 + 77.76 + 134.64 + 32.4 = 316.8\ \ kN/m}$$

$$\boldsymbol{y^{`} = \frac{\left[72 \times \left(\frac{1}{2}(6) + 3\right)\right] + \left[77.76 \times \left(6 \times \left(\frac{1}{3}\right) + 3\right)\right] + \left[\left[134.64 \times 3 \times \left(\frac{1}{2}\right)\right] + \left[32.4 \times \frac{3}{3}\right]\right.}{280.8} = 3.3\ m}$$

$$\boldsymbol{y^{`} = \frac{[432] + [388.8] + [[201.96] + [32.4]}{316.8} = 3.3\ m}$$

Example 4: Determine the passive force, by using Rankine theory and draw the failure plane for the wall shown in below Figure.

$$\boldsymbol{K_p = \frac{\cos\beta + \sqrt{(\sin\phi)^2 - (\sin\beta)^2}}{\cos\beta - \sqrt{(\sin\phi)^2 - (\sin\beta)^2}}}$$



Solution:

$$\boldsymbol{K_p = \frac{\cos\beta + \sqrt{(\sin\phi)^2 - (\sin\beta)^2}}{\cos\beta - \sqrt{(\sin\phi)^2 - (\sin\beta)^2}}}$$

$$\boldsymbol{K_p = \frac{\cos 20 + \sqrt{(\sin 40)^2 - (\sin 20)^2}}{\cos 20 - \sqrt{(\sin 40)^2 - (\sin 20)^2}} = 3.75}$$

$$\sigma_z = \gamma . H . \cos\beta$$

$\sigma_z = 18 \times 6 \times \cos 20 = 101.49\ kN/m^2$

$\sigma_p = \gamma . H . \cos\beta . K_p$

$\sigma_p = 18 \times 6 \times \cos 20 \times 3.75 = 380.58\ kN/m^2$

$P_p = \frac{1}{2} \times \gamma \times H^2 \times \cos\beta \times K_p$

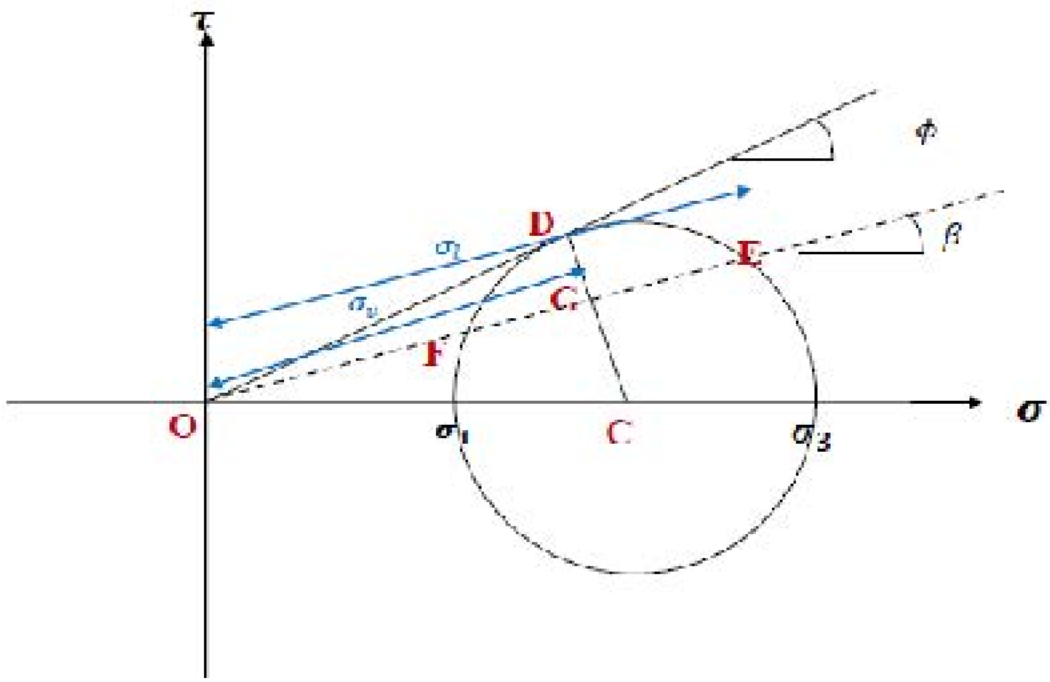
$$\boldsymbol{P_p = \frac{1}{2} \times 18 \times 6^2 \times \cos 20 \times 3.75 = 1141.73\ kN/m}$$

$\boldsymbol{y^` = 2m}$ **(From the bottom of the wall)**

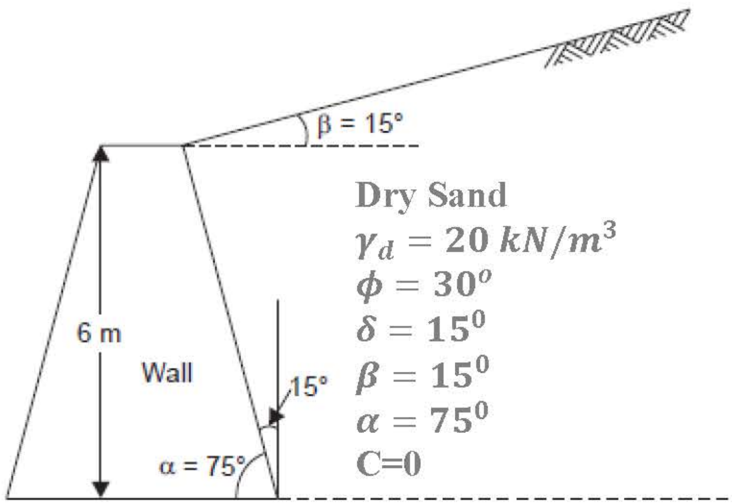
$OF = 101.49\ kN/m^2$

$OE = 380.58\ kN/m^2$

$ED\ is\ the\ failure\ plane$



Example 5: A retaining wall is battered away from the fill from bottom to top at an angle of 15° with the vertical. Height of the wall is 6 m. The fill slopes upwards at an angle 15° away from the rest of the wall. The friction angle is 30° and wall friction angle is 15°. Using Coulomb's wedge theory, determined the total active and passive thrusts on the wall, per meter assuming γ = 20 kN/m3.



$$\therefore\ K_a = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha\,.\sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta).\sin(\alpha+\beta)}}\right]^2}$$

$$\therefore\ K_p = \frac{\sin^2(\alpha-\phi)}{\sin^2\alpha\,.\sin(\alpha+\delta)\left[1-\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta).\sin(\alpha+\beta)}}\right]^2}$$

Solution:

$$K_a = \frac{\sin^2(75+30)}{\sin^2 75.\sin(75-15)\left[1+\sqrt{\frac{\sin(30+15).\sin(30-15)}{\sin(75-15).\sin(75+15)}}\right]^2}$$

$$K_a = \frac{\sin^2(105)}{\sin^2 75.\sin(60)\left[1+\sqrt{\frac{\sin(45).\sin(15)}{\sin(60).\sin(90)}}\right]^2} = 0.542$$

$$K_p = \frac{\sin^2(75-30)}{\sin^2 75.\sin(75+15)\left[1-\sqrt{\frac{\sin(30+15).\sin(30-15)}{\sin(75-\delta).\sin(75+15)}}\right]^2}$$

$$K_p = \frac{\sin^2(45)}{\sin^2 75.\sin(60)\left[1-\sqrt{\frac{\sin(45).\sin(15)}{\sin(60).\sin(90)}}\right]^2} = 6.25$$

Total active thrust, *Pa*, per meter of the wall:

$$P_a = \frac{1}{2} \times \gamma \times H^2 \times K_a$$

$$P_a = \frac{1}{2} \times 20 \times 6^2 \times 0.542$$

$$P_a = 195\ kN/m$$

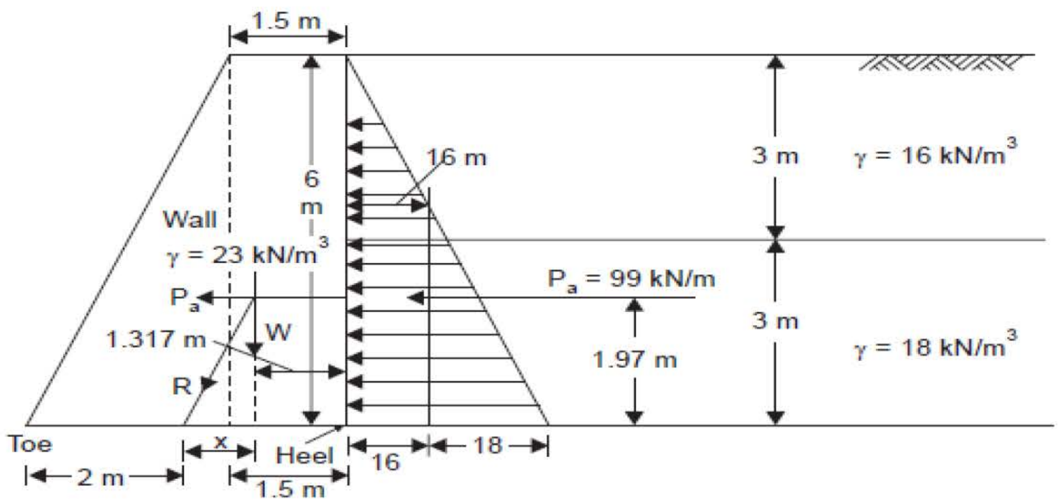
Total passive resistance, *Pp*, per meter of the wall:

$$P_p = \frac{1}{2} \times \gamma \times H^2 \times K_p$$

$$P_p = \frac{1}{2} \times 20 \times 6^2 \times 6.25$$

$$P_p = 2249\ kN/m$$

Example 6: A masonry retaining wall is 1.5 m wide at the top, 3.5 m wide at the base and 6 m high. It is trapezoidal in section and has a vertical face on the earth side. The backfill is level with top. The unit weight of the fill is 16 kN/m^3 for the top 3 m and 18 kN/m^3 for the rest of the depth. Determine the total lateral pressure on the wall per meter. Assume ϕ = 30° for both grades of soil.



Solution:

$$\phi = 30^\circ \; K_a = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = 1/3$$

Horizontal pressure of soil at 3 m depth $= K_a\gamma_1 H_1 = \frac{1}{3}\times 16\times 3 = 16 \text{ kN/m}^2$

Lateral earth pressure at 6 m depth $= K_a(\gamma_1 H_1 + \gamma_2 H_2)$

$$= \frac{1}{3}(16\times 3 + 18\times 3) = 34 \text{ kN/m}^2$$

Total active thrust per metre run of the wall,

$$P_a = \frac{1}{2}\times 3\times 16 + 3\times 16 + \frac{1}{2}\times 3\times 18 = 24 + 48 + 27 = 99 \text{ kN}$$

Let $\bar{z}$ metres be the height of the point of action above its base.

By taking moments about the base, $\bar{z} = \dfrac{(24\times 4 + 48\times 1.5 + 27\times 1)}{99} = 1.97$ m

Example 7: A 6-m-high retaining wall is to support a soil with unit weight $\gamma = 17.4$ kN/m^3, soil friction angle ϕ =26°, and cohesion $C`$ = 14.36 kN/m^2. Determine Rankine active force per unit length of the wall after the tensile crack occurs, and determine the line of action of the resultant.

Solution:

$$K_a = \frac{1-\sin 26}{1+\sin 26} = \tan^2(45 + \frac{26}{2}) = 0.39$$

$$\sigma`_a = \gamma \times H \times K_a - 2C`\sqrt{K_a}$$

1. At (H=0 m):

$$\sigma_a = \gamma.H.K_a - 2C`\sqrt{K_a}$$

$$\sigma_{a(1)} = 17.4 \times 0 \times 0.39 - 2 \times 14.3 \times \sqrt{0.39} = 17.95 \ kN/m^2$$

$$Z_c = \frac{2C`}{\gamma.\sqrt{K_a}} = \frac{2\times 14.36}{17.4\times\sqrt{0.39}} = 2.64 \ m$$

2. At (H=6 m):

$$\sigma_{a(2)} = \gamma.H.K_a - 2C`\sqrt{K_a}$$

$$\sigma_{a(2)} = 17.4 \times 6 \times 0.39 - 2 \times 14.3 \times \sqrt{0.39} = 22.85 \ kN/m^2$$

$$P_a = \left[\frac{1}{2} \times (H - Z_c)\right] \times \left[\gamma.H.K_a - 2C`\sqrt{K_a}\right] = \left[\frac{1}{2} \times (6 - 2.64) \times (22.85)\right]$$

$$\boldsymbol{P_a = 38.25 \ kN/m}$$

$$\boldsymbol{y` = \frac{[6-2.64]}{3} = 1.12 \ m}$$

Example 8: A wall, 5.4 m high, retains sand. In the loose state the sand has void ratio of 0.63 and φ = 27°, while in the dense state, the corresponding values of void ratio and φ are 0.36 and 45° respectively. Compare the ratio of active and passive earth pressure in the two cases, assuming $G_s = 2.64$.

Solution:

(*a*) Loose State:

$$G = 2.64 \quad e = 0.63$$

$$\gamma_d = \frac{G.\gamma_w}{(1+e)} = \frac{2.64 \times 1}{(1+0.63)} = 16.2 \text{ kN/m}^3$$

$$\phi = 27°$$

$$K_a = \frac{1-\sin 27°}{1+\sin 27°} 0.376; \; K_p = \frac{1+\sin 27°}{1-\sin 27°} = 2.663$$

Active pressure at depth H m = $K_a.\gamma.H$ = 0.376 × 16.2 H = 6.09. H kN/m^2

Passive pressure at depth H m = $K_p.\ \gamma H$ = 2.663 × 16.2 H = 43.14 H kN/m^2

(*b*) Dense State:

$$G = 2.64 \qquad e = 0.36$$

$$\gamma_d = \frac{2.64 \times 10}{(1+0.36)} = 19.4 \text{ kN / m}^3$$

$$\phi = 45°$$

$$K_a = \frac{1-\sin 45°}{1+\sin 45°} = 0.172; \; K_p = \frac{1+\sin 45°}{1-\sin 45°} = 5.828$$

Active pressure at depth H m = 0.172 × 19.4H = 3.34 H kN/m^2

Passive pressure at depth H m = 5.828 × 19.4 H = 113.06 H kN/m^2

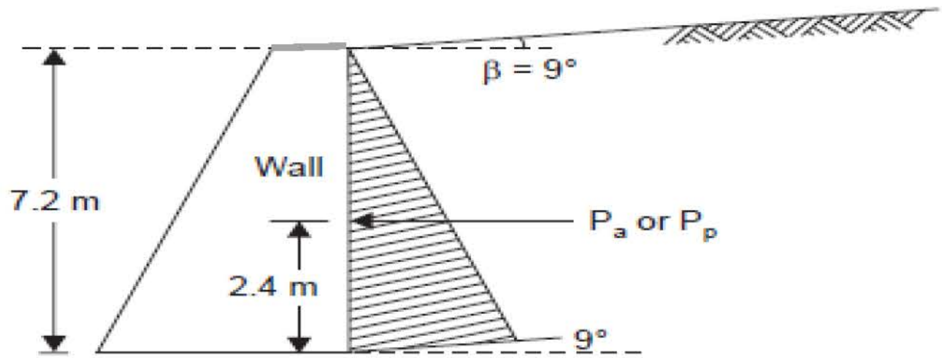
Ratio of active pressure in the dense state of that in the loose state = $\frac{0.334}{0.609}$ = **0.55**

Ratio of passive resistance in the dense state to that in the loose state = $\frac{11.306}{4.314}$ = **2.62**

Example 9: A vertical wall with a smooth face is 7.2 m high and retains soil with a uniform surcharge angle of 9°. If the angle of internal friction of soil is 27°, compute the active earth pressure and passive earth resistance assuming $\gamma = 20\ kN/m^3$.

$H = 7.2$ m $\beta = 9°$

$\phi = 27°$ $\gamma = 20$ kN/m³



According to Rankine's theory,

$$K_a = \cos\beta\left(\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}\right)$$

$$= \cos 9°\left(\frac{\cos 9° - \sqrt{\cos^2 9° - \cos^2 27°}}{\cos 9° + \sqrt{\cos^2 9° - \cos^2 27°}}\right)$$

$$= 0.988 \times 0.397 = 0.392$$

$$K_p = \cos\beta\left(\frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}\right) = 0.988 \times \frac{1}{0.397} = 2.488$$

Total active thrust per metre run of the wall

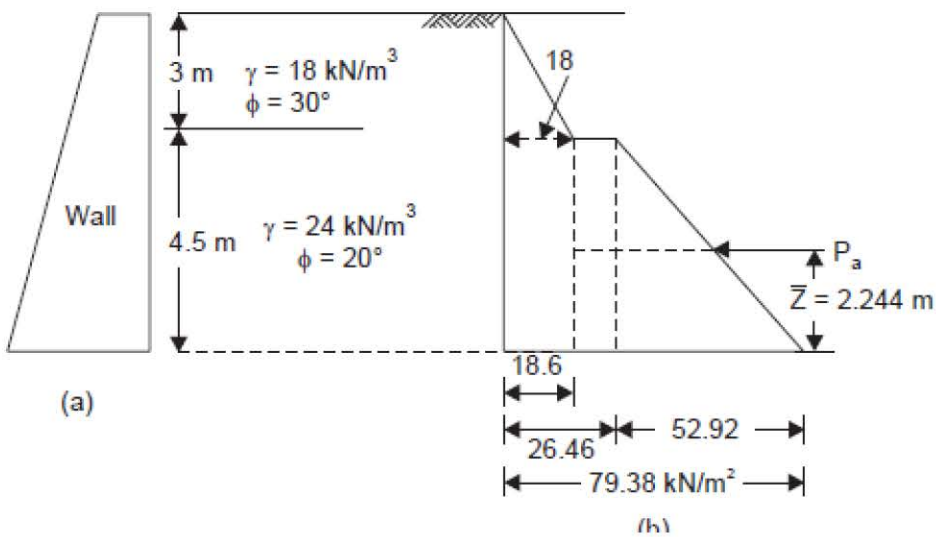
$$P_a = \frac{1}{2}\gamma H^2.K_a = \frac{1}{2} \times 20 \times (7.2)^2 \times 0.392 = \mathbf{203.2\ kN}$$

Total passive resistance per metre run of the wall

$$P_p = \frac{1}{2}\gamma H^2.K_p = \frac{1}{2} \times 20 \times (7.2)^2 \times 2.488 = \mathbf{1289.8\ kN}$$

Example 10: A retaining wall, 7.5 m high, retains a cohsionless backfill. The top 3 m of the fill has a unit weight of 18 kN/m^3 and φ = 30° and the rest has unit weight of 24 kN/m^3 and φ = 20°. Determine the pressure distribution on the wall.

Solution:



$$K_a \text{ for top layer} = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3}$$

$$K_a \text{ for bottom layer} = \frac{1-\sin 20^\circ}{1+\sin 20^\circ} = 0.49$$

Active pressure at 3 m depth – considering first layer

$$K_{a_1}.\sigma_v = \frac{1}{3} \times 3 \times 18 = 18 \text{ kN/m}^2$$

Active pressure at 3 m depth – considering second layer

$$K_{a_2}.\sigma_v = 0.49 \times 3 \times 18 = 26.46 \text{ kN/m}^2$$

Active pressure at the base of the wall :

$$K_{a_2} \times 3 \times 18 + K_{a_2} \times 4.5 \times 24 = 26.46 + 0.49 \times 4.5 \times 24 = 79.38 \text{ kN/m}^2$$

The pressure distribution with depth is shown in Fig. 13.57 (*b*).

Total active thrust, P_a, per metre run of the wall

= Area of the pressure distribution diagram

$$= \frac{1}{2} \times 3 \times 18 + 4.5 \times 26.46 + \frac{1}{2} \times 4.5 \times 52.92$$

= 27 + 119.07 + 119.07 = **265.14 kN**

The height of the point of application of this thrust above the base of the wall is obtained by taking moments, as usual.

$$\bar{z} = \frac{(27 \times 5.5 + 119.07 \times 2.25 + 11907 \times 1.5)}{265.14} \text{m} = \mathbf{2.244\ m}$$

Example 11: A retaining wall 9 m high retains granular fill weighing $\gamma = 18\ kN/m^3$ with level surface. The active thrust on the wall is 180 kN/m length of the wall. The height of the wall is to be increased and to keep the force on the wall within allowable limits; the backfill in the top-half of the depth is removed and replaced by cinders. If cinders are used as backfill even in the additional height, what additional height may be allowed if the thrust on the wall is to be limited to its initial value? The unit weight of the cinders is 9 kN/m^3. Assume the friction angle for cinders the same as that for the soil.

Solution:

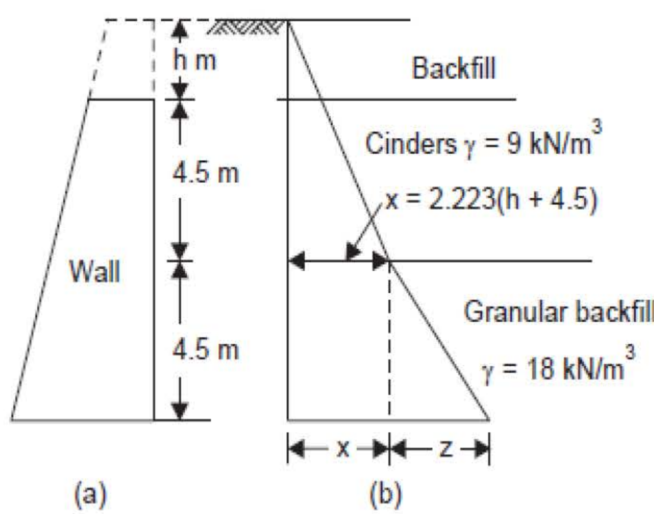
$$H = 9\ \text{m}$$
$$\gamma = 18\ \text{kN/m}^3$$
$$P_a = 180\ \text{kN/m. run}$$

Initially,

$$P_a = \frac{1}{2}\gamma H^2 . K_a$$

$$\therefore \quad 180 = \frac{1}{2} \times 18 \times 9^2 \times K_a$$

$$K_a = \frac{2 \times 180}{18 \times 9^2} = 0.247$$



Let the increase in the height of wall be h m.

The depth of cinders backfill will be $(h + 4.5)$ m and bottom 4.5 m is granular backfill with $K_a = 0.247$. Since the friction angles for cinders is taken to be the same as that for the granular soil, K_a for cinders is also 0.247, but γ for cinders is 9 kN/m^3.

The intensity of pressure at $(h + 4.5)$m depth $= 0.247 \times 9\,(h + 4.5)$ kN/m^2

$= 2.223\,(h + 4.5)$ kN/m^2

Intensity of pressure at the base $= 0.247\,[9\,(h + 4.5) + 18 \times 4.5]$ kN/m^2

$= 2.223\,(h + 4.5) + 20$ kN/m^2

$$\text{Total thrust } P_a' = 1.112\,(h + 4.5)^2 + 2.223 \times 4.5\,(h + 4.5) + \frac{1}{2} \times 4.5 \times 20$$

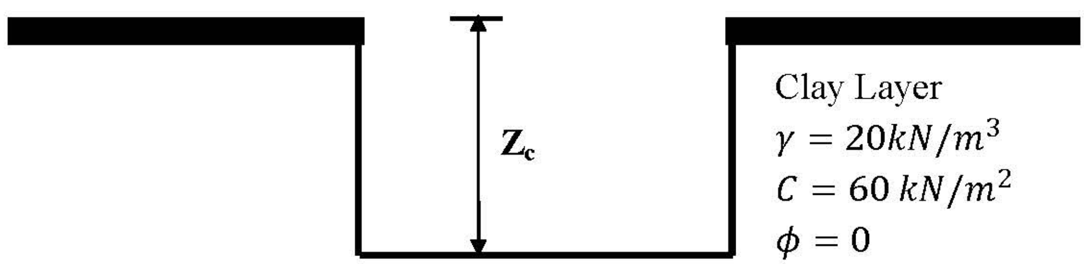
Equating this to the initial value P_a, or 180 kN, the following equation is obtained:

$$1.112\,h^2 + 20h - 67.5 = 0$$

Solving, $h = 2.90$ m

Thus, the height of the wall may be increased by **2.90 m** without increasing the thrust.

Example 12: Calculate the maximum depth of excavation for clay soil without need bracing supports.



Solution:

$$\sigma_a = \gamma . Z_c . K_a - 2C . \sqrt{K_a}$$

$$\sigma_a = 0.0$$

$$Z_c = \frac{2C}{\gamma \times \sqrt{K_a}}$$

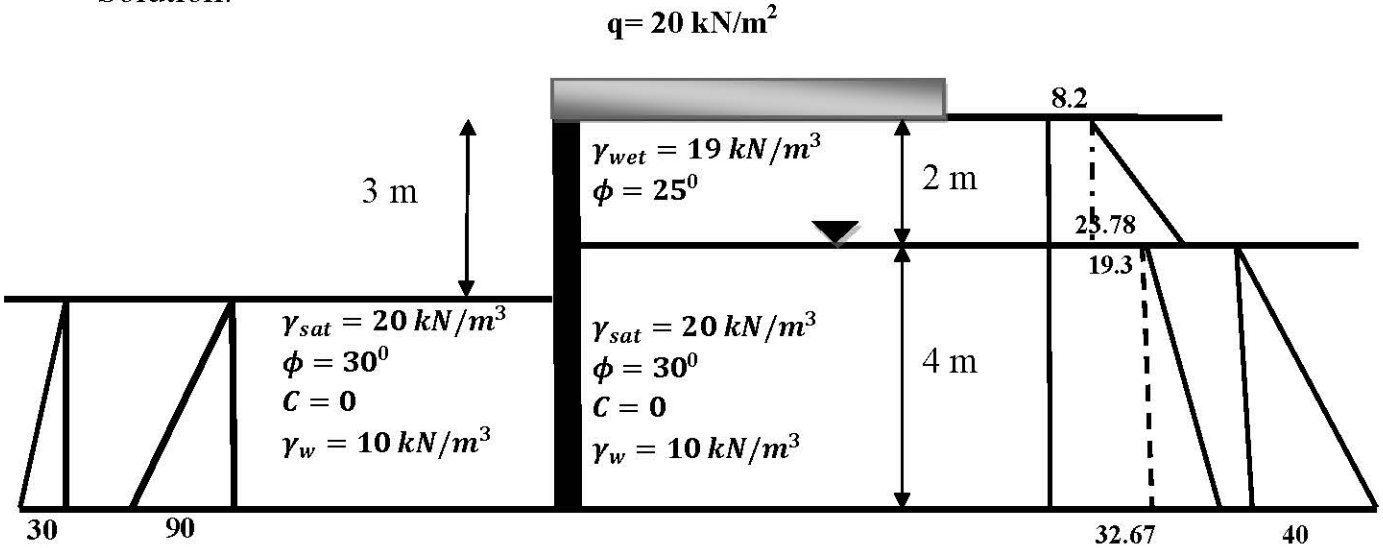
$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = 1$$

$Z_c = \frac{2 \times 60}{20 \times 1} = 6\ m$ (maximum depth without bracing)

Example 13: For the frictionless sheet pile shown in below Figure, determine the following:

1. The active lateral earth pressure distribution with depth.
2. The passive lateral earth pressure distribution with depth.
3. The magnitudes and location of active and passive forces.

Solution:



1) Active Side:

$$\sigma_a = (q + \gamma . Z). K_a - 2C.\sqrt{K_a} \qquad C = 0$$

$$\sigma_a = (q + \gamma . Z). K_a$$

a) Layer (1)

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 25}{1+\sin 25} = 0.41$$

$$\sigma_{a(1)} = (0 + 20) \times 0.41 = 8.2\ kN/m^2$$

$$\sigma_{a(2)} = (20 + 2 \times 19) \times 0.41 = 23.78\ kN/m^2$$

b) Layer (2)

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30}{1+\sin 30} = 0.334$$

$$\sigma_{a(2)} = (20 + 2 \times 19) \times 0.334 = 19.33\ kN/m^2$$

$$\sigma_{a(3)} = (20 + 2 \times 19 + (20 - 10) \times 4) \times 0.334 = 32.67\ kN/m^2$$

$$\sigma_{water} = \gamma_w \times H_w = 10 \times 4 = 40\ kN/m^2$$

The active lateral force:

$$P_a = [8.2 \times 2] + \left[\frac{1}{2} \times (37.78) \times 2\right] + [19.33 \times 4] + \left[\frac{1}{2} \times (32.67 - 19.33) \times 4\right] + [\left[\frac{1}{2} \times (40) \times 4\right]$$

$$\boldsymbol{P_a = 16.4 + 15.58 + 77.32 + 26.68 + 80 = 215.98\ kN/m}$$

$$y^{`} = \frac{\left[16.4 \times \left(\frac{1}{2}(2) + 4\right)\right] + \left[15.58 \times (2 \times \left(\frac{1}{3}\right) + 4)\right] + [\left[77 \times 4 \times \left(\frac{1}{2}\right)\right] + \left[26.68 \times \frac{4}{3}\right] + [80 \times \frac{4}{3}]}{215.98}$$

$\boldsymbol{y^{`} = 2.53\ m}$ **(Location of active force, from the base of the wall)**

2) Passive Side:

$$\sigma_p = (q + \gamma . Z) . K_p + 2C . \sqrt{K_a} \qquad \boldsymbol{C = 0}$$

$$\sigma_p = (\gamma . Z) . K_p$$

$$K_p = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1+\sin 25}{1-\sin 25} = 3$$

$$\sigma_{p(1)} = (0 \times 20) \times 3 = 0\ kN/m^2$$

$$\sigma_{p(2)} = (3 \times (20 - 10)) \times 3 = 90\ kN/m^2$$

$$\sigma_{water} = \gamma_w \times H_w = 10 \times 3 = 30\ kN/m^2$$

$$\boldsymbol{P_p = \left[\frac{1}{2} \times 90 \times 3\right] + \left[\frac{1}{2} \times (30) \times 3\right] = 135 + 45 = 180\ kN/m}$$

$\boldsymbol{y^{`} = \frac{\left[135 \times \left(\frac{1}{3}\right) \times 3\right] + \left[45 \times (3 \times \left(\frac{1}{3}\right))\right]}{180} = 1m}$ **(Location of passive force, from the base of the wall)**