CHAPTER THREE

RETAINING WALLS

3.1 Introduction:

In the previous chapter, the various theories of lateral earth pressure are discussed. Those theories will be used in this chapter to design various types of retaining walls. In general, retaining walls can be divided into two major categories: (a) conventional retaining walls and (b) mechanically stabilized earth walls.

Conventional retaining walls can generally be classified into four varieties:

- 1. Gravity retaining walls
- 2. Semi-gravity retaining walls
- 3. Cantilever retaining walls
- 4. Counter-fort retaining walls
- 1) *Gravity retaining walls*: as shown in Figure 3.1a are constructed with plain concrete or stone masonry. They depend for stability on their own weight and any soil resting on the masonry. This type of construction is not economical for high walls. In many cases, a small amount of steel may be used for the construction of gravity walls, thereby minimizing the size of wall sections. Such walls are generally referred to as
- 2) Semi-gravity walls: as shown in Figure 3.1b
- 3) Cantilever retaining walls: as shown in Figure 8.1c are made of reinforced concrete that consists of a thin stem and a base slab. This type of wall is economical to a height of about 8 m.
- **4)** *Counter-fort retaining walls:* as shown in Figure 3.1d are similar to cantilever walls. At regular intervals, however, they have thin vertical concrete slabs known as *counterforts* that tie the wall and the base slab together. The purpose of the counterforts is to reduce the shear and the bending moments.

To design retaining walls properly, an engineer must know the basic parameters the *unit weight* γ , *angle of friction* ϕ , and *cohesion* C of the soil retained behind

the wall and the soil below the base slab. Knowing the properties of the soil behind the wall enables the engineer to determine the lateral pressure distribution that has to be designed for.

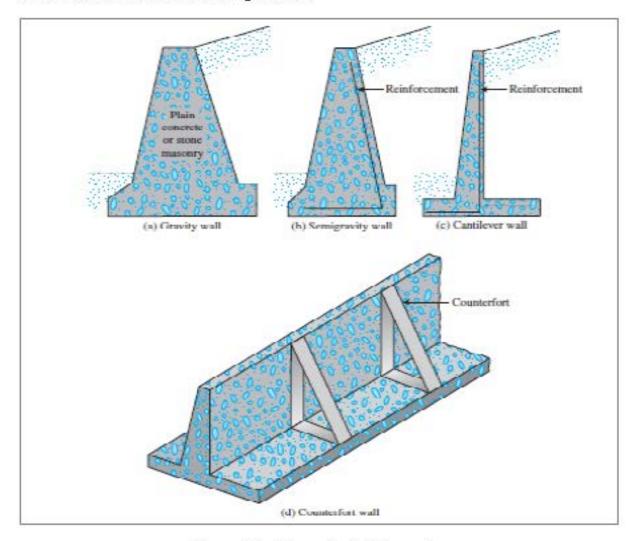


Figure 3.1: Types of retaining walls.

There are two phases in the design of a conventional retaining wall. First, with the lateral earth pressure known, the structure as a whole is checked for stability. The structure is examined for possible overturning, sliding, and bearing capacity failures. Second, each component of the structure is checked for strength, and the steel reinforcement of each component is determined.

This chapter presents the procedures for determining the stability of the retaining wall. Checks for strength can be found in any textbook on reinforced concrete. Some retaining walls have their backfills stabilized mechanically by

including reinforcing elements such as metal strips, bars, welded wire mats, geotextiles, and geogrids. These walls are relatively flexible and can sustain large horizontal and vertical displacements without much damage.

3.2 Proportioning Retaining Walls

In designing retaining walls, an engineer must assume some of their dimensions. Called *proportioning*, such assumptions allow the engineer to check trial sections of the walls for stability. If the stability checks yield undesirable results, the sections can be changed and rechecked. Figure 3.2 shows the general proportions of various retaining-wall components that can be used for initial checks. Note that the top of the stem of any retaining wall should not be less than about 0.3 m for proper placement of concrete. The depth, D, to the bottom of the base slab should be a minimum of 0.6 m. However; the bottom of the base slab should be positioned below the seasonal frost line. For counter-fort retaining walls, the general proportion of the stem and the base slab is the same as for cantilever walls. However, the counterfort slabs may be about 0.3 m thick and spaced at center-to-center distances of 0.3H to 0.7H.

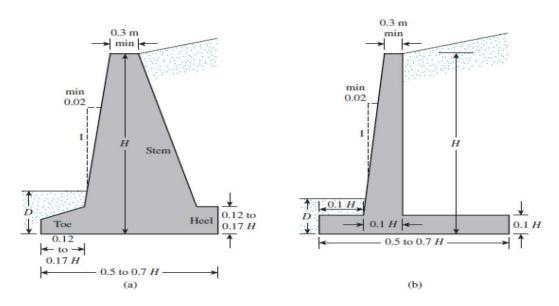


Figure 3.2: Approximate dimensions for various components of retaining wall for initial stability checks: (a) gravity wall; (b) cantilever wall.

3.3 Stability Requirements

The retaining wall must be satisfying the following stability requirements:

- 1. Safety against overturning (Rotation).
- 2. Safety against sliding.
- 3. Safety against bearing capacity failure.
- 4. Safety against overall stability.

3.3.1 Check for overturning (Rotation):

Consider the retaining wall shown in below Figure (3.3) the failure may occur with retaining wall rotation about point (C).

$$\sum M_c = 0$$

$$P_a \times l_a = P_p \times l_p + \sum W_R \times l_w$$

The usual minimum desirable value of the factor of safety with respect to overturning is 2 to 3.

$$F.S_{overturning} = \frac{\sum M_R}{\sum M_o} = \frac{P_p \times l_p + \sum W \times l_w}{P_a \times l_a}$$

$$F.S_{overturning} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + P_a \times y}{P_a \times \cos \alpha \times y}$$

Where:

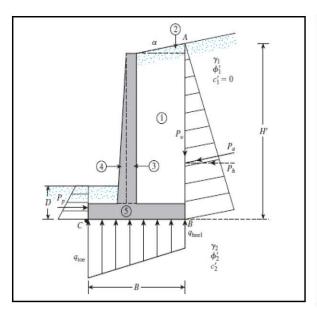
 $\sum M_o = \text{sum of the moments of forces tending to overturn about point } C$

 $\sum M_R$ = sum of the moments of forces tending to resist about point C

 P_a = Active force.

 P_p = Passive force.

 l_a and l_a = Moment arm measured from point C.



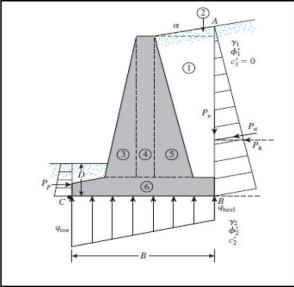


Figure 3.3: Check for overturning, assuming that the Rankine pressure is valid

Procedure for Calculating $(\sum M_R)$:

Section (1)	Area (2)	Weight/unit length of wall (3)	Moment arm measured from C (4)	Moment about <i>C</i> (5)
1	A_1	$W_1 = \gamma_1 \times A_1$	X_1	M_1
2	A_2	$W_2 = \gamma_1 \times A_2$	X_2	M_2
3	A_3	$W_3 = \gamma_c \times A_3$	X_3	M_3
4	A_4	$W_4 = \gamma_c \times A_4$	X_4	M_4
5	A_5	$W_5 = \gamma_c \times A_5$	X_5	M_5
6	A_6	$W_6 = \gamma_c \times A_6$	X_6	M_6
		P_v	B	M_v
		$\sum V$		$\sum M_R$

Note: γ_1 = unit weight of backfill, γ_c = unit weight of concrete)

3.3.2 Check for Sliding along the Base:

A slide results if the shear stress along some potential slip surface becomes equal to the shear strength. One possible slip surface is shown in below Figure (3.4).

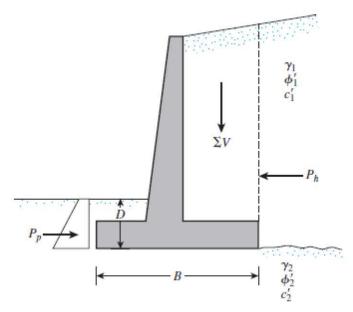


Figure 3.4: Check for sliding along the base.

The factor of safety against sliding may be expressed by the equation:

$$F.S_{Sliding} = \frac{\sum F_R}{\sum F_d}$$

Where:

 F_R = Sum of the horizontal resisting forces.

 F_d = Sum of the horizontal driving forces.

$$\sum F_R = (\sum V) \times tan \ \delta^{\hat{}} + B \ C_a + P_p$$
$$\sum F_d = P_a \times cos \ \beta$$

Where:

 P_p = The passive horizontal force.

 $\sum V = \text{Sum of the vertical forces.}$

 δ = The angle of friction between the soil and the base slab and may be = $\frac{2}{3} \phi$.

 C_a = Adhesion between the soil and the base slab.

 P_a = The active horizontal force.

 β = The angle of backfill slop.

$$F.S_{Sliding} = \frac{(\sum V) \times tan \,\delta^{\hat{}} + B \,C_{a} + P_{p}}{P_{a} \times cos \,\beta}$$

If the desired value of F. $S_{Sliding}$ is not achieved, several alternatives may be investigated as illustrated in Figure 3.5.

- 1. Increase the width of the base slab (i.e., the heel of the footing).
- 2. Use a key to the base slab.
- 3. Use a dead-man anchor at the stem of the retaining wall.

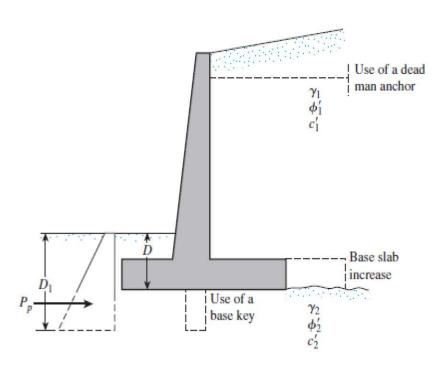


Figure 3.5: Alternatives for increasing the factor of safety with respect to sliding.

3.3.3 Check for Bearing Capacity Failure:

The vertical pressure transmitted to the soil by the base slab of the retaining wall should be checked against the ultimate bearing capacity of the soil. The nature of variation of the vertical pressure transmitted by the base slab into the soil is shown in Figure 3.6. Note that q_{toe} and q_{heel} are the *maximum* and the *minimum* pressures occurring at the ends of the toe and heel sections, respectively. The magnitudes of and can be determined in the following manner:

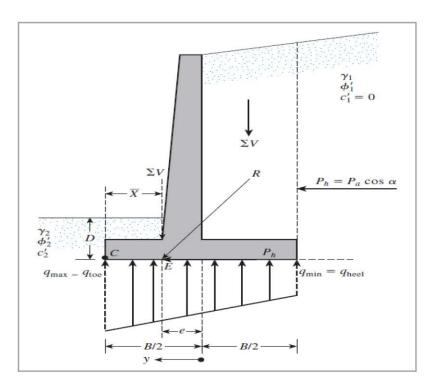


Figure 3.6: Check for bearing capacity failure.

$$q_{max.} = rac{\sum V}{Area} \left(1 + rac{6e}{B} \right)$$
 and $q_{min.} = rac{\sum V}{Area} \left(1 - rac{6e}{B} \right)$
 $Area = B. (1)$
 $q_{max.} = rac{\sum V}{B} \left(1 + rac{6e}{B} \right)$
 $q_{min.} = rac{\sum V}{B} \left(1 - rac{6e}{B} \right)$

Where:

 $\sum V$ = Algebraic sum of all the vertical forces includes the weight of the soil. B= width of the wall base.

e= the eccentricity of the resultant force on the base

$$\begin{split} \boldsymbol{e} &= \frac{\boldsymbol{B}}{2} - \frac{\sum \boldsymbol{M_R} - \sum \boldsymbol{M_o}}{\sum \boldsymbol{V}} \\ \sum \boldsymbol{M_R} &= P_p \times l_p + \sum \boldsymbol{W} \times l_w \\ \sum \boldsymbol{M_o} &= P_a \times l_a \end{split}$$

$$F.S_{Bearing\ Capacity} = \frac{q_u}{q_{max.}} \geq 3$$

- \triangleright Three different cases arise depending upon the value of e as shown in Figure 3.7.
- 1) $e < \frac{B}{6}$
- 2) $e = \frac{B}{6}$ in this case the maximum pressure can be computed as

$$q_{max.} = \frac{2\sum V}{B}$$
 and $q_{min.} = 0$

 $e > \frac{B}{6}$ in this case tension is supposed to have developed as shown. Since soil is considered incapable of resisting any tension, the pressure is taken to be redistributed along the intact base of width 3b', where b' is the distance of the line of action of $\sum V$ from the toe. q_{max} is then given by:

$$q_{max.} = \frac{3\sum V}{B'}$$
 and $q_{min.} = 0$
 $B' = \frac{B}{2} - e$

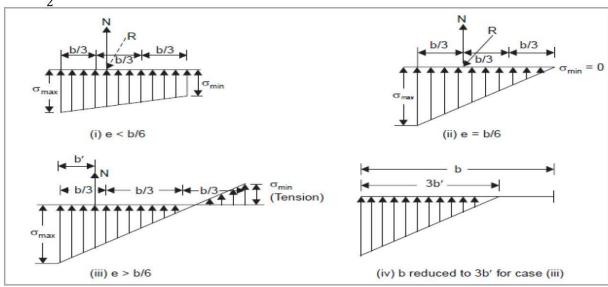
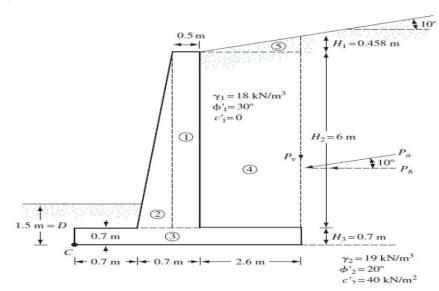


Figure 3.7: Distributions of base pressure for different values of eccentricity of the resultant force on the base.

Example (1): The cross section of a cantilever retaining wall is shown in below Figure. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity. Where the ultimate bearing capacity of the soil is 560 kN/m².



Solution: Calculation of stability of a retaining wall.

$$H = H_1 + H_2 + H_3$$

$$H = 2.6 \times \tan 10 + 6 + 0.7 = 0.458 + 6 + 0.7$$

$$H = 7.158 m$$

The Rankine active force per unit length of wall = P_a

$$P_a = \frac{1}{2} \times \gamma \times H^2 \times K_a$$

$$K_a=\frac{\cos\beta-\sqrt{(\sin\phi)^2-(\sin\beta)^2}}{\cos\beta+\sqrt{(\sin\phi)^2-(\sin\beta)^2}}$$
 , For $\beta=10$ and $\phi=30^o$

$$K_a = 0.3532$$

$$P_a = \frac{1}{2} \times 18 \times (7.158)^2 \times 0.3532$$

$$P_a = 162.9 \, kN/m$$

$$P_V = P_a \times \sin \beta = 28.29 \ kN/m$$

$$P_h = P_a \times \cos \beta$$

$$P_h = 160.43 \, kN/m$$

1) Factor of safety against overturning (Rotation):

$$\sum M_o = P_h \times \left(\frac{H}{3}\right) = 160.43 \times \frac{7.158}{3} = 382.79 \text{ kN. m/m}$$

The following table can now be prepared for determining the resisting moment M_R :

Section no.a	Area (m²)	Weight/unit length (kN/m)	Moment arm from point <i>C</i> (m)	Moment (kN-m/m)
1	$6 \times 0.5 = 3$	70.74	1.15	81.35
2	$\frac{1}{2}(0.2)6 = 0.6$	14.15	0.833	11.79
3	$4 \times 0.7 = 2.8$	66.02	2.0	132.04
4	$6 \times 2.6 = 15.6$	280.80	2.7	758.16
5	$\frac{1}{2}(2.6)(0.458) = 0.595$	10.71	3.13	33.52
		$P_v = 28.29$	4.0	113.16
		$\Sigma V = 470.71$		$1130.02 = \sum M_R$

Note: $\gamma_{Concrete} = 23.58 \, kN/m^3$

$$F.S_{Overturning} = \frac{\sum M_R}{\sum M_o} = \frac{1130.02}{382.79} = 2.95 > 2$$
 O.K.

2) Factor of safety against Sliding:

$$F.S_{Sliding} = \frac{(\Sigma V) \times \tan \delta + B C_a + P_p}{P_a \times \cos \beta}$$

$$P_p = \frac{1}{2} \times \gamma \times H^2 \times K_p + 2C\sqrt{K_p}$$

$$K_p = \frac{1 + \sin \phi_2}{1 - \sin \phi_2} = \frac{1 + \sin 20}{1 - \sin 20} = 2$$

$$P_p = \frac{1}{2} \times 19 \times (1.5)^2 \times 2 + (2 \times 40) \times \sqrt{2}$$

$$P_p = 43.61 - 171.39 = 215 \, kN/m$$

$$F.S_{Sliding} = \frac{470.71 \times \tan(\frac{2}{3} \times 20) + 4 \times (\frac{2}{3} \times 40) + 215}{160.43} = 2.7 > 2 \quad ... \text{O.K.}$$

3) Factor of safety against bearing capacity failure:

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_o}{\sum V}$$

$$e = \frac{4}{2} - \frac{1130.02 - 382.79}{470.71} = 0.411 < \frac{B}{6}$$

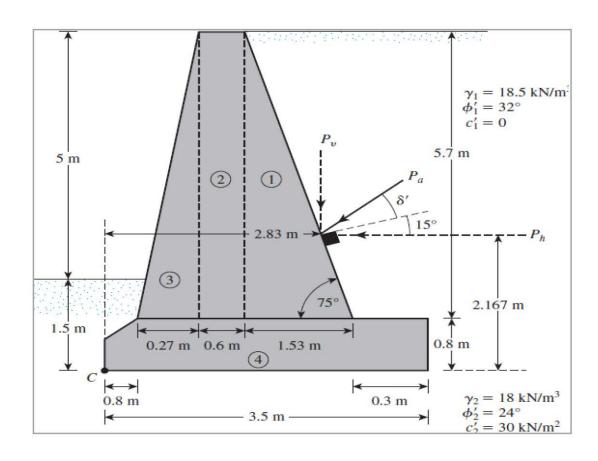
$$\begin{split} q_{max.} &= \frac{\Sigma V}{B} \left(1 + \frac{6e}{B} \right) \\ q_{max.} &= \frac{470.71}{4} \left(1 + \frac{6\times0.411}{4} \right) = 190.2 \ kN/m^2 \ (\text{Toe}) \\ q_{min.} &= \frac{\Sigma V}{B} \left(1 - \frac{6e}{B} \right) = \frac{470.71}{4} \left(1 - \frac{6\times0.411}{4} \right) = 45.13 kN/m^2 \ (\text{Heel}). \\ F. S_{Bearing\ Capacity} &= \frac{q_u}{q_{max}}. \end{split}$$

$$F.S_{Bearing\ Capacity} = \frac{560}{190.2} = 2.98 < 3$$
 Not O.K.

Note: F.S_(bearing capacity) is less than 3. Some re-proportioning will be needed.

Example (2): A gravity retaining wall is shown in below Figure. Use $\delta = \frac{2}{3}\phi$ and Coulomb's active earth pressure theory; determine:

- a. The factor of safety against overturning.
- b. The factor of safety against sliding.
- c. The pressure on the soil at the toe and heel.



Solution: Calculation of stability of a retaining wall.

$$H = H_1 + H_2 = 5 + 1.5 = 6.6 m$$

Coulomb's active force is:

$$P_a = \frac{1}{2} \times \gamma \times H^2 \times K_a$$

With
$$\beta = 0$$
, $\alpha = 90^{\circ} - 75^{\circ} = 15^{\circ}$, $\delta^{\circ} = \frac{2}{3}\phi$, and $\phi = 32^{\circ}$

$$K_{\alpha} = \frac{\sin^{2}(\alpha + \phi)}{\sin^{2}\alpha \cdot \sin(\alpha - \delta) \left[1 + \sqrt{\frac{\sin(\phi + \delta) \cdot \sin(\phi - \beta)}{\sin(\alpha - \delta) \cdot \sin(\alpha + \beta)}}\right]^{2}}$$

$$K_a = 0.402$$

$$P_{a} = \frac{1}{2} \times 18.5 \times (6.5)^{2} \times 0.402 = 157.22 \ kN/m$$

$$P_{h} = P_{a} \times \cos(\alpha + \delta)$$

$$= 157.22 \times \cos\left(15 + \frac{2}{3}\phi\right) = 157.22 \times \cos 36.33 = 126.65 \ kN/m$$

$$P_{v} = P_{a} \times \sin(\alpha + \delta) = 157.22 \times \sin 36.33 = 93.14 \ kN/m$$

1) Factor of safety against overturning (Rotation):

		N	n	
Area no.	Area (m²)	Weight* (kN/m)	from C (m)	Moment (kN-m/m)
1	$\frac{1}{2}(5.7)(1.53) = 4.36$	102.81	2.18	224.13
2	(0.6)(5.7) = 3.42	80.64	1.37	110.48
3	$\frac{1}{2}(0.27)(5.7) = 0.77$	18.16	0.98	17.80
4	$\approx (3.5)(0.8) = 2.8$	66.02	1.75	115.54
		$P_v = 93.14$	2.83	263.59
		$\Sigma V = 360.77 \text{kN/m}$		$\Sigma M_R = 731.54 \text{ kN-m/r}$

$$\gamma_c = 23.58 \, kN/m^3$$

Note that the weight of the soil above the back face of the wall is not taken into account in the preceding table.

2) Factor of safety against Sliding:

$$F.S_{Sliding} = \frac{(\sum V) \times tan \ \delta^+ B \ C_a + P_p}{P_h}$$

3) Factor of safety against bearing capacity failure:

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_o}{\sum V}$$

$$e = \frac{3.5}{2} - \frac{731.54 - 274.45}{360.77} = \mathbf{0.483} < \frac{B}{6}$$

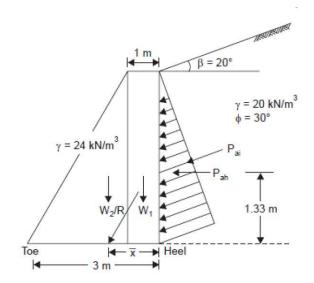
$$q_{max} = \frac{\sum V}{B \times 1} \left(1 + \frac{6e}{B} \right)$$

$$q_{max} = \frac{360.77}{3.5} \left(1 + \frac{6 \times 0.483}{3.5} \right) = 188.43 \ kN/m^2$$

$$q_{min} = \frac{\sum V}{B \times 1} \left(1 - \frac{6e}{B} \right)$$

$$q_{min} = \frac{360.77}{3.5} \left(1 - \frac{6 \times 0.483}{3.5} \right) = 17.73 \ kN/m^2$$

Example (3): A trapezoidal masonry retaining wall 1 m wide at top and 3 m wide at its bottom is 4 m high. The vertical face is retaining soil ($\varphi = 30^{\circ}$) at a surcharge angle of 20° with the horizontal. Determine the maximum and minimum intensities of pressure at the base of the retaining wall. Unit weights of soil and masonry are 20 kN/m3 and 24 kN/m3 respectively. Assuming the coefficient of friction at the base of the wall as 0.45, determine the factor of safety against sliding. Also determine the factor of safety against overturning.



Solution:

The Rankine active force per unit length of wall = P_a

$$P_a = \frac{1}{2} \times \gamma \times H^2 \times K_a$$

$$K_a=\frac{\cos\beta-\sqrt{(\sin\phi)^2-(\sin\beta)^2}}{\cos\beta+\sqrt{(\sin\phi)^2-(\sin\beta)^2}}$$
 , For $\beta=20$ and $\phi=30^o$

$$K_a = 0.414$$

$$P_a = \frac{1}{2} \times 20 \times (4)^2 \times 0.414$$

$$P_a = 66.24 \, kN/m$$

$$P_V = P_a \times \sin \beta = 22.66 \, kN/m$$

$$P_h = P_a \times \cos \beta$$

$$P_h = 62.25 \, kN/m$$

1) Factor of safety against overturning (Rotation):

$$\sum M_o = P_h \times \left(\frac{H}{3}\right) = 62.25 \times \frac{4}{3} = 83 \text{ kN. m/m}$$

The following table can now be prepared for determining the resisting moment M_R :

Section	Area	Weight	Moment	M_R
			arm	
1.	$1 \times 4 = 4$	$4 \times 24 = 96$	2.5	240
2.	$\frac{1}{2} \times 2 \times 4 = 4$	$4 \times 24 = 96$	1.334	128
3.		$P_V = 22.66$	3	67.98
		$\sum V = 214.66$		$\sum M_R = 436$

$$F. S_{Overturning} = \frac{\sum M_R}{\sum M_o} = \frac{436}{83} = 5.25 >> 2$$
 O.K.

2) Factor of safety against Sliding:

$$F.S_{Sliding} = \frac{(\sum V) \times \tan \delta + B C_a + P_p}{P_a \times \cos \beta}$$

$$P_p = 0$$

$$\tan \delta = 0.45$$

$$C_a = 0$$
 because $C = \theta$

$$F. S_{Sliding} = \frac{214.66 \times 0.45}{62.25} = 1.55 < 2$$
 Not O.K.

- 1. Use a key to the base slab.
- 2. Use a *dead-man anchor* at the stem of the retaining wall.

3) Factor of safety against bearing capacity failure:

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_o}{\sum V}$$

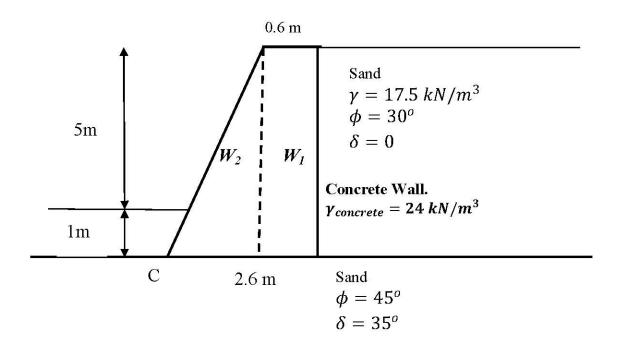
$$e = \frac{3}{2} - \frac{436 - 83}{214.66} = 0.144 \ m < \frac{B}{6} = 0.5 \ m$$

$$q_{max} = \frac{\sum V}{B} \left(1 + \frac{6e}{B} \right)$$

$$q_{max} = \frac{214.66}{3} \left(1 + \frac{6 \times 0.144}{3} \right) = 92 \ kN/m^2 \ \text{(Toe)}$$

$$q_{min} = \frac{\sum V}{B} \left(1 - \frac{6e}{B} \right) = \frac{214.66}{3} \left(1 - \frac{6 \times 0.144}{3} \right) = 51 \ kN/m^2 \ \text{(Heel)}.$$

Example (4): Check the concrete retaining wall shown in below Figure, considering rotating, sliding, and pressure of wall base.



Solution:

The active force per unit length of wall = P_a

$$P_a = \frac{1}{2} \times \gamma \times H^2 \times K_a$$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}, \text{ For } \phi = 30^\circ$$

$$K_a = 0.334$$

$$P_a = \frac{1}{2} \times 17.5 \times (6)^2 \times 0.334$$

$$P_a = 105 \text{ kN/m}$$

$$P_V = 0$$

$$P_h = P_a = 105 \text{ kN/m}$$

1) Factor of safety against overturning (Rotation):

$$\sum M_o = P_h \times \left(\frac{H}{3}\right) = 105 \times \frac{6}{3} = 210 \text{ kN. m/m}$$

The following table can now be prepared for determining the resisting moment M_R :

Section	Area	Weight	Moment	M_R
			arm	
1.	$0.6 \times 6 = 3.6$	$3.6 \times 24 = 86.4$	2.3	198.72
2.	$\frac{1}{2} \times 2 \times 6 = 6$	$6 \times 24 = 144$	1.334	192
3.		$P_V=0$	~	1
		$\sum V = 230.4$		$\sum M_R = 391$

$$F.S_{Overturning} = \frac{\sum M_R}{\sum M_o} = \frac{436}{83} = 1.86 > 1.5$$
 O.K.

2) Factor of safety against Sliding:

$$F.S_{Sliding} = \frac{(\Sigma V) \times \tan \delta + B C_a + P_p}{P_a \times \cos \beta}$$

$$P_p = \frac{1}{2} \times \gamma \times H^2 \times K_p$$

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi}, \text{ For } \phi = 30^o$$

$$K_p = 3$$

$$P_p = \frac{1}{2} \times 17.5 \times (1)^2 \times 3 = 26.25 \text{ kN/m}$$

$$\tan \delta = \tan 35 = 0.7$$

$$C_a = 0 \text{ because } C = \theta$$

$$F.S_{Sliding} = \frac{230.4 \times 0.7 + 26.25}{105} = 1.78 < 1.5$$
 O.K.

3) Factor of safety against bearing capacity failure:

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_o}{\sum V}$$

$$e = \frac{2.6}{2} - \frac{391 - 210}{230.4} = 0.514 \, m > \frac{B}{6} = 0.433 \, m$$

This state means that, the pressure at the heel is tension, therefore:

$$q_{max}$$
 = $\frac{2 \times \sum V}{3B}$, $R' = \frac{B}{2} - e = \frac{2.6}{2} - 0.514 = 0.786 m$ q_{max} = $\frac{2 \times 230.4}{3 \times 0.786} = 195 \, kN/m^2$ (Toe) q_{min} = 0 (Heel).

H.W: A retaining wall is battered away from the fill from bottom to top at an angle of 15° with the vertical. Height of the wall is 6 m. The fill slopes upwards at an angle 20° away from the rest of the wall. The friction angle is 30° and wall friction angle is 20°. Using Coulomb's wedge theory, determined the total active and passive thrusts on the wall, and Check the concrete retaining wall shown in below Figure, considering rotating, sliding, and pressure of wall base. Assuming $\gamma_{\text{Soil}} = 20 \text{ kN/m}^3$ and $\gamma_{\text{Concrete}} = 24 \text{ kN/m}^3$

