Physics of atom

Lecture Two / Practical

Half – life ($T_1/_2$) concepts

First stage

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Introduction

The **half-life ($T_{1/2}$)** is the time interval during which **half** of a given number of **radioactive nuclei decay** and **half remain undecayed** for radioactive substance. This concept is crucial in nuclear physics, chemistry, and applications like radiometric dating and medical imaging.

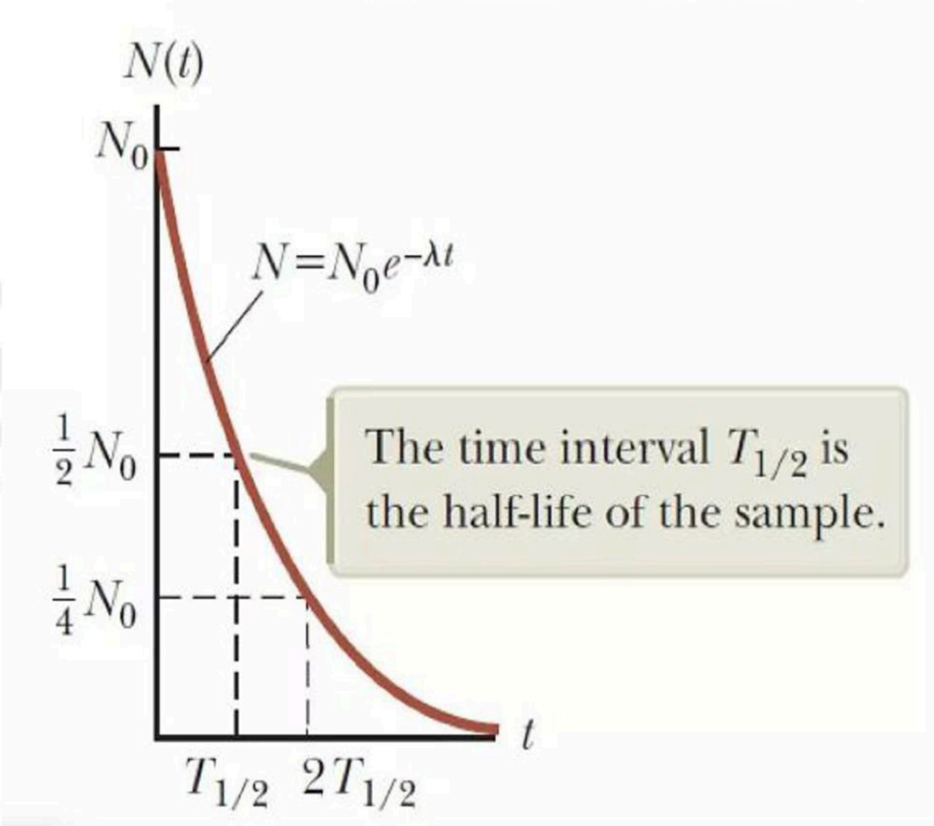
Each radioactive nuclide has its own half-life. Half-lives can be a **short** as a fraction of a second or as **long** as billions of years.

Examples:

- **Short Half-Life : Iodine-131 (8 days)** used in medicine for quick decay.
- **Long Half-Life : Uranium-238 (4.5 billion years)** used in geological dating.

Exponential Radioactive decay mathematics and Plot

Exponential decay describes a process where a quantity **decreases** by a **constant percentage over equal time intervals**.



The Mathematical Formula

The formula for exponential decay is:

$$N(t) = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}}$$

(t): Number of undecayed atoms at time t.

N0: Initial number of atoms (at t=0).

t: Elapsed time. (**units:** Same as T1/2 for calculations)

T1/2: Half-life (time for half the atoms to decay), (**units:** seconds (s), minutes (min) , hours (hr), days, years (yr) , millennia)

λ: Decay Constant (**units:** inverse time (e.g., s^{-1} , yr^{-1}) and is determined by the half-life of the isotope).

$$T_{1/2} = \frac{0.693}{\lambda}$$

Problem 1: **The half-life of Cesium-137 is 30 years . Calculate its decay constant (λ) in yr^{-1}?**

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$$T_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{T_{1/2}}$$

$$\lambda = \frac{0.693}{30} = 0.0231\, y^{-1}$$

Problem 2: **Iodine-131 (half-life = 8.0 days) is used in medical treatments. A sample initially contains 5.00×10^{23} undecayed atoms. How many atoms will remain undecayed after 30 days? Sol/**

$$N(t) = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}}$$

$$\frac{30}{8.0} = 3.75$$

$$N(t) = 5.00\times10^{23}\left(\frac{1}{2}\right)^{\frac{30}{8.0}}$$

$$N(t) = (5.00\times10^{23})\times(0.074)$$

$$N(t) = 0.371\times10^{23}$$

Problem 3: A sample of Carbon-14 (half-life = 5730 years) decays for 10,000 years, leaving 1.20×10^{22} undecayed atoms. What was the initial number of atoms in the sample?

$$N(t) = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}}$$

$$\frac{10{,}000}{5730} = 1.745$$

$$1.20\times10^{22} = N_0\left(\frac{1}{2}\right)^{\frac{10{,}000}{5730}}$$

$$1.20\times10^{22} = N_0\left(\frac{1}{2}\right)^{1.745}$$

$$1.20\times10^{22} = (N_0)\,(0.298)$$

$$N_0 = \frac{1.20\times10^{22}}{0.298}$$

$$N_0 = 4.026\times10^{22}$$

Problem 4: A radioactive isotope has a half-life of 87.7 years. A sample initially contains 5.00×10^{23} undecayed atoms. How long will it take for the number of undecayed atoms to decay to 1.00×10^{23}?

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$$N(t) = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}}$$

$$1.00\times10^{23} = 5.00\times10^{23}\left(\frac{1}{2}\right)^{\frac{t}{87.7}}$$

$$\frac{1.00\times10^{23}}{5.00\times10^{23}} = \left(\frac{1}{2}\right)^{\frac{t}{87.7}}$$

$$0.20 = \left(\frac{1}{2}\right)^{\frac{t}{87.7}}$$

By taking the ln of both sides

$\ln(a^b) = b\cdot\ln(a)$

$$\ln(0.20) = \frac{t}{87.7}\ln\left(\frac{1}{2}\right)$$

$$87.7(-1.609) = t(-0.693)$$

$$t = \frac{141.1093}{0.693} = 203.620 \text{ years}$$

Measures of Radioactivity

A frequently used unit of activity is the **curie (Ci)** **(Non - Sl unit)**, defined as

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decay/s}$$

This value was originally selected because it is the approximate activity of 1 g of radium.

The **Sl unit** of activity is the **Becquerel (Bq)**:

$$1 \text{ Bq} = 1 \text{ decay/s}$$

Therefore,

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

Common subunits include:

Millicurie (mCi) : 10^{-3} Ci = 3.7×10^{7} Bq

Microcurie (μCi) : 10^{-6} Ci = 3.7×10^{4} Bq

Problem 5: **Convert 5.0 Ci to becquerels (Bq).**

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$$1\ \text{Ci} = 3.7 \times 10^{10}\ \text{Bq}$$

$$\rightarrow 5.0\ \text{Ci} = 5.0 \times 3.7 \times 10^{10}\ \text{Bq} = \mathbf{1.85 \times 10^{11}\ Bq}$$

Problem 6: **Convert 250 μCi to millicuries (mCi)?**

$$1\ \mu\text{Ci} = 10^{-3}\ \text{mCi}$$

$$\rightarrow 250\ \mu\text{Ci} = 250 \times 10^{-3}\ \text{mCi} = \mathbf{0.25\ mCi}$$

Problem 7: **A thyroid scan uses 10 mCi of iodine-131. How many becquerels (Bq) is this?**

$$1\ \text{mCi} = 10^{-3}\ \text{Ci} = 3.7 \times 10^{7}\ \text{Bq}$$

$$\rightarrow 10\ \text{mCi} = 10 \times 3.7 \times 10^{7}\ \text{Bq} = \mathbf{3.7 \times 10^{8}\ Bq}$$

Problem 8: **A soil sample contains 0.5 μCi of cesium-137 per kilogram. How many becquerels (Bq) is this?**

$$1\ \mu Ci = 3.7 \times 10^4\ Bq$$

$$\rightarrow 0.5\ \mu Ci = 0.5 \times 3.7 \times 10^4\ Bq = \mathbf{1.85 \times 10^4\ Bq}$$