Composition of Function :-

From the last lecture, we got known how to take derivatives of algebraic Functions, like for example, the polynomials in standard Form or even a transcendental Functions that we had studied last course (we go through later), and the simple products and quotients Functions. But unfortunately, Functions can get more complicated than this, such as multiple operations happening at on equation, in order to take its derivative, we will have to use something called "a chain Rule".

we call the Functions/equations that have multiple operations, "the composite Functions"

Examples of the Composite Functions :-

① $y = f(u)$, $u = f(t)$, $t = f(x) \Longrightarrow y = f(f(f(x)))$

② $h(t) = t^{17}$, $t(x) = x^2+1 \Longrightarrow h(t(x)) = (x^2+1)^{17}$

③ $y = u^5$, $u = x^3 - 2x^2 + 4 \Longrightarrow y = (x^3 - 2x^2 + 4)^5$

④ $y = (z^3 + 2z^2 - 6z - 1)^{-6}$, $z = (2t^2 - 3)^{1/2}$, $t = x^{1/2} - 7$

$$\Longrightarrow y = \left(\left((2(x^{1/2}-7)^2-3)^{1/2}\right)^3 + 2\left((2(x^{1/2}-7)^2-3)^{1/2}\right)^2 - 6(2(x^{1/2}-7)^2-3)^{1/2} - 1\right)^{-6}$$

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As we said previously, to drive the composite Function we use the chain Rule

The chain Rule :-

If $y = F(w)$, $w = F(x)$ then

$$\frac{dy}{dx} = \frac{dy}{dw} * \frac{dw}{dx}$$

And If $y = f(w)$, $w = f(u)$, $u = f(x)$, then

$$\frac{dy}{dx} = \frac{dy}{dw} * \frac{dw}{du} * \frac{du}{dx}$$

Examples

① $y = u^{17}$ & $u = x^2 + 1$ Find $\frac{dy}{dx}$

Sol.)

∵ The Function is composite one, then we use "chain Rule" to drive it

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = 17u^{16} * 2x$$

next step is to remove "u" from the result by substituting x^2+1

$$\therefore \frac{dy}{dx} = 17(x^2+1)^{16} * 2x = \boxed{34x(x^2+1)^{16}}$$ Ans

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② $y = u^{-6}$, $u = x^3 + 4x^2 - 3x - 3$ Find $\frac{dy}{dx}$?

Sol.

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$$

$$\frac{dy}{dx} = -6\,u^{-7} * (3x^2 + 8x - 3)$$

$$= -6(x^3 + 4x^2 - 3x - 3)^{-7}(3x^2 + 8x - 3) = \boxed{\frac{-18x^2 - 48x + 18}{(x^3 + 4x^2 - 3x - 3)^7}}$$

Ans

③ $y = u^3$, $u = 3x^2 - 5$, Find $\frac{dy}{dx}$?

Sol.

$$\frac{dy}{dx} = \underbrace{3u^2}_{dy/du} * \underbrace{6x}_{dy/dx} = 3(3x^2 - 5)^2 * 6x$$

$$\frac{dy}{dx} = \boxed{18x(3x^2 - 5)^2}$$ Ans

④ $y = u^2$; $u = 3t^2 - 5$, $t = x^2$, Find $\frac{dy}{dx}$?

Sol.

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dt} * \frac{dt}{dx}$$

$$\therefore \frac{dy}{dx} = 2u * 6t * 2x$$

$$= 2(3t^2 - 5) * 6x^2 * 2x$$

$$= 2(3(x^2)^2 - 5) * 6x^2 * 2x$$

$$= 2 * 2 * 6 * x^2 * x\,(3x^4 - 5)$$

$$= \boxed{24x^3(3x^4 - 5)}$$ Ans

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Questions for Discussions : اسئلة المناقشة

Differentiate the following Functions using the "Chain Rule" :

a- $y = u^2 \;,\; u = 2x+3$

b- $y = w^{1/2} \;,\; w = x^2+2x+1$

c- $y = z^{21} \;,\; z = 3-x$

d- $y = u^{-3-8} \;,\; u = t^3 - \sqrt{t}$

e- $y = \frac{1}{\sqrt{u}} \;,\; u = 2-x^4$

f- $g = w^{3/7} \;,\; w = x + \frac{1}{x}$

g- $y = (x+2)(x+3)^2$

h- $y = (2x-1)^2(x+3)^3$

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Higher Derivatives :- مشتقات عليا

All we studied previously were just 1st derivative that can be written as;

$$y = f(x) \Longrightarrow \frac{dy}{dx} = \frac{df}{dx} = y' = f'(x) = y_x = f_x$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

- For 2nd derivative :-

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = \frac{d^2f}{dx^2} = y'' = f''(x) = y_{xx} = f_{xx}$$

- For 3rd derivative :-

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3} = \frac{d^3f}{dx^3} = y''' = f'''(x) = y_{xxx} = f_{xxx}$$

And so on with higher derivatives;

Ex) Find y'' if $y = x^3 + \sqrt{x}$?

Sol.)

$$y = x^3 + x^{\frac{1}{2}} \Longrightarrow y' = \frac{dy}{dx} = 3x^2 + \frac{1}{2}x^{-\frac{1}{2}}$$

$$y'' = \frac{d^2y}{dx^2} = 3 \times 2x + \frac{1}{2}\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= 6x - \frac{1}{4}x^{-\frac{3}{2}}$$

$$= \boxed{6x - \frac{1}{4\sqrt{x^3}}} \quad \underline{Ans}$$

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HW #1

1. Drive $f(x) = (x^4 - 1)^{50}$

2. Drive $f(x) = \left(\frac{x-1}{x+1}\right)^2$

3. Drive $f(x) = (x+3)^2 (x^2 - 4)^3$

4. Drive $f(u) = u^7 \, , \; u(x) = 3x^4 + 5x^2$

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