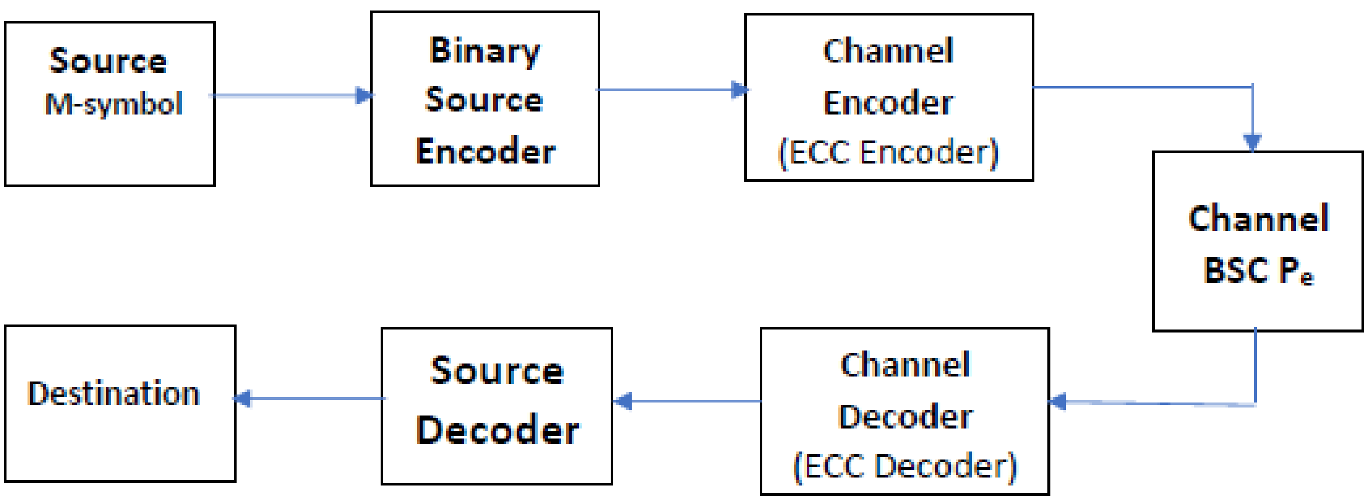
# Error Control Coding (ECC)

## 1-Basic Idea of ECC

ECC is also known as channel coding in relation to source coding. The channel coding deals with the reduction of errors caused by the channel $P_e$. Source coding is used to match the channel alphabet and to provide source compression. The following figure shows where in the information system we used channel coding and decoding.



To simplify the model, we are going to consider the channel encoder only with binary input and binary output as follows:

Input block or word of k message bits at a time. Called message block M

$M = [\, m_1 \;\; m_2 \;\; m_3 \; . \; . \; . \; . \;\; m_k \,]$

→

**Channel Encoder**
(ECC Encoder)
ECC rate is $R_c = k/n$
Add (n-k) check bits to the message bits

→

Output codeword of n coded bits C for each k input bits with n > k

$C = [\, c_1 \;\; c_2 \;\; c_3 \; . \; . \; . \; . \;\; c_n \,]$

The decoder similarly can be modeled as:

Input received block of n bits called received word R

$R = [\, r_1 \;\; r_2 \;\; r_3 \; . \; . \; . \; . \;\; r_n \,]$

→

**Channel Decoder**
(ECC Decoder)
Use the check bits to correct errors caused by channel

→

Output decoded word $\widetilde{C}$ or message word of k bits

$\widetilde{M} = [\, \widetilde{m_1} \;\; \widetilde{m_2} \;\; . \; . \; . \; . \;\; \widetilde{m_k} \,]$

In above model $m_i$, $c_i$, $r_i$ and $\widetilde{m_i}$ are binary values "0" or "1".

## 2- Rate and Bandwidth Expansion Factor

The ECC rate is given by: $R_c = k/n$ (often called channel coding efficiency). Since we have (n-k) check bits, then the less the rate the more check bits being used. Extra check bits improve correction and imply the wasting of transmission resources or reducing the effective message bit rate (in bps). In other word there is a waste of bandwidth equivalent to the ratio $n/k$. This ratio in % is called the bandwidth expansion factor, or;

$$\text{Bandwidth Expansion Factor} = \frac{n}{k}.100\% = \frac{1}{Rc}.100\%$$

## 3- Categorization of ECC

Error Control coding may be divided according to different properties into:

| According to Encoder Circuit | According to Correction Capability | According to Error Type |
| --- | --- | --- |
| -Block (without memory)<br>or<br>-Convolutional (with memory) | -Forward (**FEC**)<br>or<br>-Feedback (**ARQ**)<br>or<br>-Hybrid (**HARQ**) | -Random Errors<br>or<br>-Burst Errors |

**FEC**= Forward Error Correction (FEC is used in almost all digital communication systems as in digital satellite TV broadcast)

**ARQ**=Automatic Repeat on reQuest (also called error detection code) and some times called as CRC= Cyclic Redundancy Check. (ARQ is used in packet networks as in Internet, where the frame or packets when received with error the receiver node ask the transmitter to retransmit the given packet or frame again using ACK and NACK message, as in WLAN IEEE802.11).

**HARQ**= Hybrid ARQ and FEC (this is used in 4G and 5G cellular and wireless network system which provides an optimum error detection and correction capabilities)

## 4-The ECC Code Design

The main task of the encoder is to determine the values of the check bits as a function of the message bit. The task of the decoder algorithm, on the other hand, is how to use the added check bits by the encoder to distinguish weather the received word is correct or not. In FEC, the decoder should also correct the erroneous received bits. The ECC Design should cover and solve the followings:

a) Encoder parameters (n, k, $R_c$ )
b) Encoder algorithm (usually logic circuit or Look-up table)
c) Decoder algorithm (decoding steps may be logic circuit, or mathematical steps)
d) The code performance ($BW_{expansion}$ , $P_{e\text{-}coded} < P_{e\text{-}uncoded}$ )
e) Complexity of both the encoder and decoder.

## 5- Relation Between Capacity and ECC

ECC is usually reduced the probability of error in transmission thus it provides better Quality of Service (QoS). In addition to better QoS, ECC also improved the channel capacity.

From lecture#8 -B (page-4) it is shown that the capacity of BSC with error probability of $P_e$ is given by:

$$C = 1 + [p_e Log\, p_e + (1 - p_e). Log\, (1 - p_e)] \qquad in\ bits/symbol$$

For $P_e < 0.5$, the value of C is inversely proportional with $P_e$, the use of ECC will reduce $P_e$ and hence increase the capacity C. Thus, ECC can be seen as a method of increasing the capacity of the system. Thus, as a matter of fact the increased bit rate of 4G compared to 3G is achieved when good ECC is used. This is also happened when moving from 4G to 5G.

## Example-1

Consider a binary source with rate $R_x$=100 kbps is transmitted over BSC with $P_e$=0.1 then:

a- Find the capacity rate.

b- Find the new capacity rate when $P_e$ is reduced to 0.05 by using FEC with $R_c$=1/2.

c- What is the cost paid for the increased capacity rate in b?

**Solution:**

a- Using the relation of channel capacity over BSC channel as above for $P_e$=0.1 gives:

$$C = 1 + [p_e Log\, p_e + (1 - p_e).Log\,(1 - p_e)]$$

$$C = 1 + [0.1xLog\; 0.1 + 0.9xLog\; 0.9] = 0.531 \text{ bits/symbol}$$

Using $C_r = R_x C = 53.1$ kbps

b- For $P_e$=0.05 :

$$C = 1 + [0.05xLog\; 0.05 + 0.95xLog\; 0.95] = 0.7136 \text{ bits/symbol}$$

Then: $C_r = R_x C = 71.136$ kbps

c- Since $R_c$=1/2, then the bandwidth expansion factor is given by:

The Bandwidth Expansion Factor $= \frac{n}{k}.100\% = \frac{1}{Rc}.100\% = \frac{2}{1}.100\%$=200%

Thus, although the error probability is reduced (meaning better QoS) and the capacity rate is increased, the bandwidth is doubled. This is the cost paid for improved performance.