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Mathematics and Biostatistics

First Stage

LECTURE 4
Derivatives

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OUTLINE

- Line tangent and derivatives
- Differentiation rules
- Derivative of trigonometric function
- Practice exercises

1. Line Tangent and Derivatives

- **Tangent Line Definition:** A tangent line to a curve at a given point is a straight line that just "touches" the curve at that point without crossing it (locally). Its slope is given by the derivative of the function at that point.

Example:

Given $f(x) = x^2$, find the equation of the tangent line at $x = 2$.

1. Derivative: $f'(x) = 2x$
2. Slope at $x = 2$: $f'(2) = 4$
3. Point on the curve: $(2, f(2)) = (2, 4)$
4. Equation of tangent: $y - 4 = 4(x - 2)$, simplified to $y = 4x - 4$.

Find the equation of the tangent line to $f(x) = x^2 + 3x - 5$ at $x = 2$.

Solution:

1. **Find $f'(x)$:**

   $f(x) = x^2 + 3x - 5$

   $f'(x) = 2x + 3$

2. **Find the slope at $x = 2$:**

   $f'(2) = 2(2) + 3 = 4 + 3 = 7$

3. **Find the point on the curve:**

   $f(2) = (2)^2 + 3(2) - 5 = 4 + 6 - 5 = 5$

   So the point is $(2, 5)$.

4. **Find the equation of the tangent line:**

   Using the point-slope form $y - y_1 = m(x - x_1)$:

   $y - 5 = 7(x - 2)$

   Simplify: $y = 7x - 14 + 5$, so $y = 7x - 9$.

Tangent Line: $y = 7x - 9$.

2. Differentiation Rules

- Constant Rule: $\frac{d}{dx}[c] = 0$
- Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$
- Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
- Product Rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
- Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

$\frac{d}{dx}[x^n] = nx^{n-1}$

Example: Differentiate $f(x) = x^4$.

$f'(x) = 4x^{4-1} = 4x^3$.

$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Example: Differentiate $h(x) = x^3 + 2x^2 - x + 7$.

$$h'(x) = \frac{d}{dx}[x^3] + \frac{d}{dx}[2x^2] - \frac{d}{dx}[x] + \frac{d}{dx}[7]$$

$$h'(x) = 3x^2 + 4x - 1 + 0 = 3x^2 + 4x - 1.$$

$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Example: Differentiate $f(x) = (x^2)(\sin x)$.

$$f'(x) = \frac{d}{dx}[x^2] \cdot \sin x + x^2 \cdot \frac{d}{dx}[\sin x]$$

$$f'(x) = 2x \cdot \sin x + x^2 \cdot \cos x.$$

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Example: Differentiate $f(x) = \frac{x^2}{\sin x}$.

$$f'(x) = \frac{\frac{d}{dx}[x^2] \cdot \sin x - x^2 \cdot \frac{d}{dx}[\sin x]}{(\sin x)^2}$$

$$f'(x) = \frac{2x \cdot \sin x - x^2 \cdot \cos x}{\sin^2 x}.$$

3. Derivatives of Trigonometric Functions

- $\frac{d}{dx}[\sin x] = \cos x$
- $\frac{d}{dx}[\cos x] = -\sin x$
- $\frac{d}{dx}[\tan x] = \sec^2 x$
- $\frac{d}{dx}[\csc x] = -\csc x \cot x$
- $\frac{d}{dx}[\sec x] = \sec x \tan x$
- $\frac{d}{dx}[\cot x] = -\csc^2 x$

Example: Differentiate $f(x) = \sin x \cos x$.

Use the product rule:

$$f'(x) = \frac{d}{dx}[\sin x] \cdot \cos x + \sin x \cdot \frac{d}{dx}[\cos x]$$

$$f'(x) = (\cos x)(\cos x) + (\sin x)(-\sin x)$$

$$f'(x) = \cos^2 x - \sin^2 x.$$

Exercise 1: Tangent Line

Find the equation of the tangent line to $f(x) = x^3 + 2x - 4$ at $x = 1$.

Solution:

1. $f'(x) = 3x^2 + 2$.
2. $f'(1) = 3(1)^2 + 2 = 5$.
3. Point: $f(1) = (1)^3 + 2(1) - 4 = -1$.
4. Equation: $y - (-1) = 5(x - 1)$, or $y = 5x - 6$.

Exercise 2: Product Rule

Differentiate $f(x) = x^2 e^x$.

Solution:

$$f'(x) = \frac{d}{dx}[x^2] \cdot e^x + x^2 \cdot \frac{d}{dx}[e^x]$$

$$f'(x) = 2xe^x + x^2 e^x = e^x(2x + x^2).$$

Exercise 3: Trigonometric Derivatives

Differentiate $f(x) = \tan x + \sec x$.

Solution:

$$f'(x) = \frac{d}{dx}[\tan x] + \frac{d}{dx}[\sec x]$$

$$f'(x) = \sec^2 x + \sec x \tan x.$$

Homework exercises

1. Find the equation of the tangent line to $f(x) = 3x^2 - 5x + 2$ at $x = 1$.
2. Find the slope of the tangent line to $f(x) = \sqrt{x}$ at $x = 4$.
3. Compute the derivative of $f(x) = 5x^3 - 2x^2 + x - 7$.
4. Compute the derivative of $g(x) = \frac{2x+1}{x^2}$.
5. Differentiate $h(x) = \sin^2(x)$.
6. Compute $\frac{d}{dx}[\tan(x) + \sec(x)]$.
7. Compute $\frac{d}{dx}[(3x^2 + 1)(\sin x)]$.
8. Differentiate $f(x) = e^{x^2} \sin(x^3)$.

- Thanks for lessening ..

Any questions?