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Mathematics and Biostatistics **First Stage**

LECTURE 5
Integration

BY

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OUTLINE

- Indefinite integrals rules for indefinite integrals
- integration formulas for basic trigonometric function
- definite integrals
- properties of definite integrals
- practice exercises

1. Indefinite Integrals

An indefinite integral is a function $F(x)$ such that its derivative $F'(x)$ is equal to the integrand $f(x)$:

$$\int f(x)\,dx = F(x) + C$$

Where C is the constant of integration.

Rules for Indefinite Integrals

1. Linearity Rule:

$$\int [af(x) + bg(x)]\, dx = a\int f(x)dx + b\int g(x)dx$$

2. Power Rule:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

3. Constant Rule:

$$\int c\, dx = cx + C$$

4. **Basic Trigonometric Rules:**

- $\int \sin(x)dx = -\cos(x) + C$
- $\int \cos(x)dx = \sin(x) + C$
- $\int \sec^2(x)dx = \tan(x) + C$
- $\int \csc^2(x)dx = -\cot(x) + C$
- $\int \sec(x)\tan(x)dx = \sec(x) + C$
- $\int \csc(x)\cot(x)dx = -\csc(x) + C$

Indefinite Integrals

1. $\int(3x^2+2x+1)dx$
2. $\int \frac{1}{x^3}dx$
3. $\int \cos(2x)dx$
4. $\int e^x \sin(x)dx$ *(Hint: Use integration by parts)*

(a) $\int (3x^2 + 2x + 1)dx$

Using the **linearity rule** and the **power rule**:

$$\int (3x^2 + 2x + 1)dx = \int 3x^2 dx + \int 2x dx + \int 1 dx$$

$$= 3 \cdot \frac{x^{2+1}}{2+1} + 2 \cdot \frac{x^{1+1}}{1+1} + x + C$$

$$= x^3 + x^2 + x + C$$

(b) $\int \frac{1}{x^3} dx$

Rewriting the integrand: $\frac{1}{x^3} = x^{-3}$. Use the **power rule:**

$$\int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + C = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$$

(c) $\int \cos(2x)dx$

Using the formula for integrating $\cos(kx)$:

$$\int \cos(kx)dx = \frac{\sin(kx)}{k} + C$$

Here, $k = 2$:

$$\int \cos(2x)dx = \frac{\sin(2x)}{2} + C$$

(d) $\int e^x \sin(x)dx$

This requires **integration by parts**. Let $I = \int e^x \sin(x)dx$.

Using the formula:

$$\int udv = uv - \int vdu$$

Let $u = \sin(x)$ and $dv = e^x dx$:

$$u = \sin(x), \quad du = \cos(x)dx, \quad v = e^x$$

Substitute into the integration by parts formula:

$$I = e^x \sin(x) - \int e^x \cos(x)dx$$

Now apply integration by parts again to $\int e^x \cos(x)dx$: Let $u = \cos(x)$, $dv = e^x dx$:

$$u = \cos(x), \quad du = -\sin(x)dx, \quad v = e^x$$

$$\int e^x \cos(x)dx = e^x \cos(x) - \int e^x(-\sin(x))dx$$

$$\int e^x \cos(x)dx = e^x \cos(x) + \int e^x \sin(x)dx$$

Substitute back:

$$I = e^x \sin(x) - (e^x \cos(x) + I)$$

$$I = e^x \sin(x) - e^x \cos(x) - I$$

$$2I = e^x(\sin(x) - \cos(x))$$

$$I = \frac{e^x(\sin(x) - \cos(x))}{2} + C$$

2. Definite Integrals

A definite integral computes the accumulation of a quantity between two limits a and b:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Where $F(x)$ is an antiderivative of $f(x)$.

Properties of Definite Integrals

1. **Linearity:**

$$\int_a^b [af(x) + bg(x)]\, dx = a \int_a^b f(x)dx + b \int_a^b g(x)dx$$

2. **Reversing Limits:**

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

3. **Additivity:**

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

4. **Zero Width Interval:**

$$\int_a^a f(x)dx = 0$$

5. **Comparison:** If $f(x) \geq g(x)$ on $[a, b]$:

$$\int_a^b f(x)dx \geq \int_a^b g(x)dx$$

Definite Integrals

1. $\int_0^1 (x^2 + 3x + 2)dx$
2. $\int_{-\pi/2}^{\pi/2} \sin(x)dx$
3. $\int_0^{\pi} \cos^2(x)dx$ *(Hint: Use the identity* $\cos^2(x) = \frac{1+\cos(2x)}{2}$*)*
4. $\int_1^2 \frac{1}{x}dx$

(a) $\int_0^1 (x^2 + 3x + 2)dx$

Find the antiderivative:

$$\int (x^2 + 3x + 2)dx = \frac{x^3}{3} + \frac{3x^2}{2} + 2x$$

Evaluate at the limits:

$$\int_0^1 (x^2 + 3x + 2)dx = \left[\frac{1^3}{3} + \frac{3 \cdot 1^2}{2} + 2 \cdot 1\right] - \left[\frac{0^3}{3} + \frac{3 \cdot 0^2}{2} + 2 \cdot 0\right]$$

$$= \left(\frac{1}{3} + \frac{3}{2} + 2\right) - 0 = \frac{1}{3} + \frac{9}{6} + \frac{12}{6} = \frac{1}{3} + \frac{21}{6} = \frac{23}{6}$$

(b) $\int_{-\pi/2}^{\pi/2} \sin(x)dx$

The antiderivative of $\sin(x)$ is $-\cos(x)$:

$$\int_{-\pi/2}^{\pi/2} \sin(x)dx = [-\cos(x)]_{-\pi/2}^{\pi/2}$$

$$= -\cos\left(\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right)$$

$$= -0 + 0 = 0$$

(c) $\int_0^\pi \cos^2(x)dx$

Using the identity $\cos^2(x) = \frac{1+\cos(2x)}{2}$:

$$\int_0^\pi \cos^2(x)dx = \int_0^\pi \frac{1+\cos(2x)}{2}dx$$

Split the integral:

$$= \frac{1}{2}\int_0^\pi 1dx + \frac{1}{2}\int_0^\pi \cos(2x)dx$$

Evaluate each term:

1. $\int_0^\pi 1dx = [x]_0^\pi = \pi - 0 = \pi$
2. $\int_0^\pi \cos(2x)dx = \left[\frac{\sin(2x)}{2}\right]_0^\pi = \frac{\sin(2\pi)}{2} - \frac{\sin(0)}{2} = 0$

So:

$$\int_0^\pi \cos^2(x)dx = \frac{1}{2}(\pi + 0) = \frac{\pi}{2}$$

(d) $\int_1^2 \frac{1}{x} dx$

The antiderivative of $\frac{1}{x}$ is $\ln|x|$:

$$\int_1^2 \frac{1}{x} dx = [\ln|x|]_1^2 = \ln(2) - \ln(1)$$

$$= \ln(2) - 0 = \ln(2)$$

- Thanks for lessening ..

Any questions?