Errors as a Function of System Type

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$

Using Eq. (4.33), these results can be summarized by the foll
equations:

$$K_p = \lim_{s\to 0} GD_{cl}(s), \qquad n = 0,$$

$$K_v = \lim_{s\to 0} sGD_{cl}(s), \qquad n = 1,$$

$$K_a = \lim_{s\to 0} s^2 GD_{cl}(s), \qquad n = 2.$$

Problem 5.18 The forward transfer function of a unity feedback type 1, second order system has a pole at -2. The nature of gain K is so adjusted that damping ratio is 0.4. The above equation is subjected to input $r_{(t)} = 1 + 4t$. Find the steady state error.

Solution

$$G(s) = \frac{K}{s(s+2)}$$

$$\frac{G(s)}{R(s)} = \frac{K}{s^2 + 2s + 2}$$

$$2\zeta \omega_n = 2$$

$$\therefore \quad \omega_n = \frac{1}{\zeta} = \frac{1}{0.4} = 2.5$$

Also $$K = \omega_n^2$$

$$= (2.5)^2 = 6.25$$

$$G(s) = \frac{6.25}{s(s+2)}$$

$$e_{ss} = \frac{1}{1+K_p} + \frac{4}{K_v}$$

$$K_p = \lim_{s \to \infty} G_{(s)} = \infty$$

$$K_v = \lim_{s \to \infty} s\, G_{(s)} = \frac{6.25}{2}$$

$$\therefore \quad e_{ss} = \frac{1}{1+\infty} + \frac{4 \times 2}{6.25} = 1.28.$$

Ans.

(b) Find the dynamic error using the dynamic error co-efficients

(a) $$G(s) = \frac{100}{s(s+10)}$$

$$K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{100}{s(s+10)} = \infty$$ Ans.

$$K_v = \lim_{s \to} s\,G(s) = \lim_{s \to 0} \frac{100}{s+10} = 10$$ Ans.

$$K_a = \lim_{s \to 0} s^2\,G(s) = \lim_{s \to 0} \frac{100\,s}{s+10} = 0$$ Ans.

$$r_{(t)} = P_0 + P_1 t + \frac{P_2}{2} t^2$$

$$e_{ss} = \frac{R_1}{1+K_p} + \frac{R_2}{K_v} + \frac{R_3}{K_a}$$

$$= \frac{P_0}{1+\infty} + \frac{P_1}{10} + \frac{P_2}{0}$$

$$= 0 + \frac{P_1}{10} + \infty$$

$$= \infty$$ Ans.