كلية المستقبل الجامعة
قسم هندسة تقنيات البناء والانشاءات

TWO WAY RIBBED SLAB

المرحلة الثالثة

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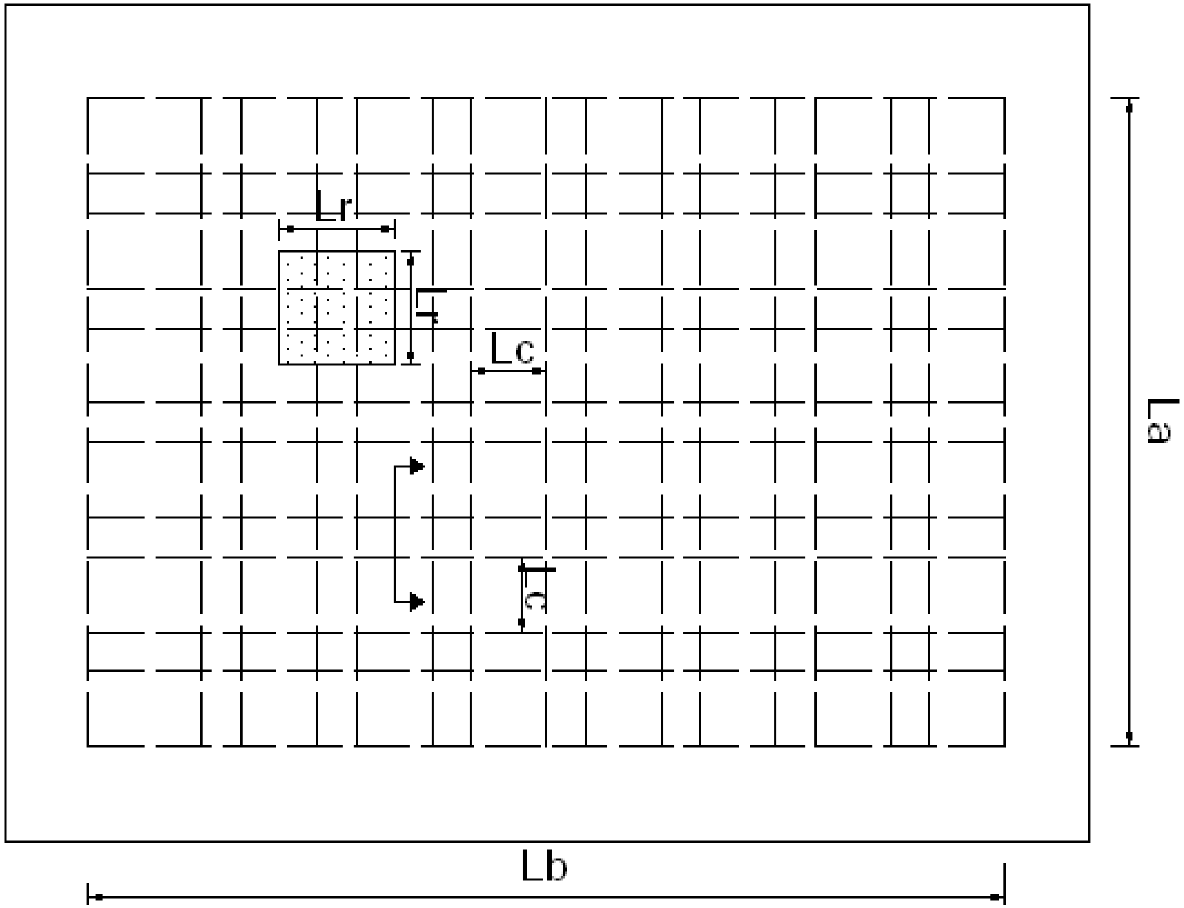
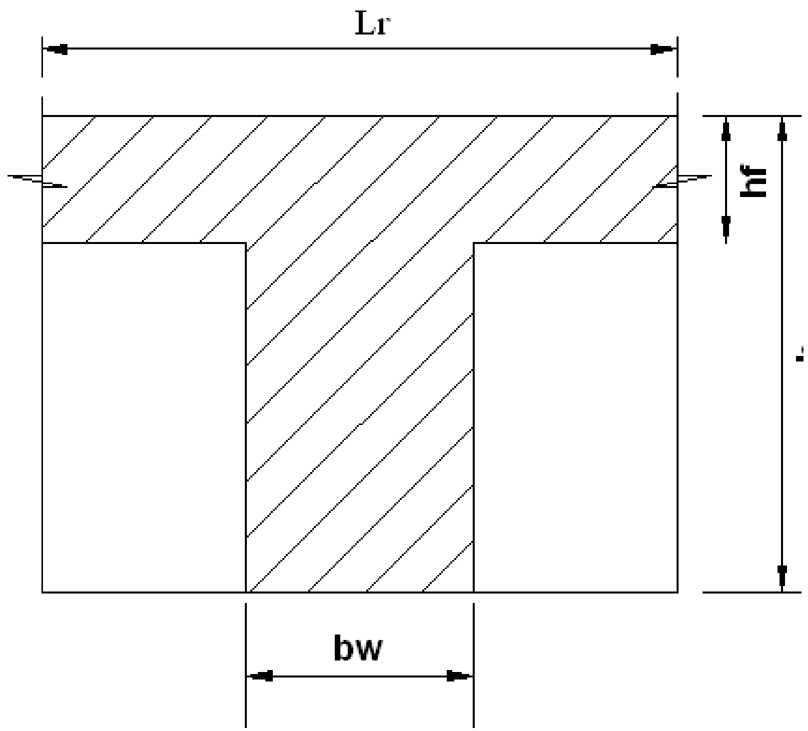
Two way ribbed slab

To be used for large spans and moderate loads.

method 3:

slab must be continuously supported on all four edges





ACI code limitation (ACI code 8.11)

$b_w \geq 100mm$

$h \leq 3.5b_w$

$L_c \leq 750\ mm \quad 8.11.3$

$$hf \geq max.\left(\frac{L_c}{12}, 50mm\right)$$

Note:-Section dimension may be obtained from manufactures catalogues

ACI 8.11.4:-joist construction not meeting these limitation shall be designed as slabs and beams

$$\text{If}\ \frac{L_b}{L_a} \geq 2.0 \text{ Design as one way ribbed slab}$$

$$\text{If}\ \frac{L_b}{L_a} < 2.0 \text{ Design as two way ribbed slab}$$

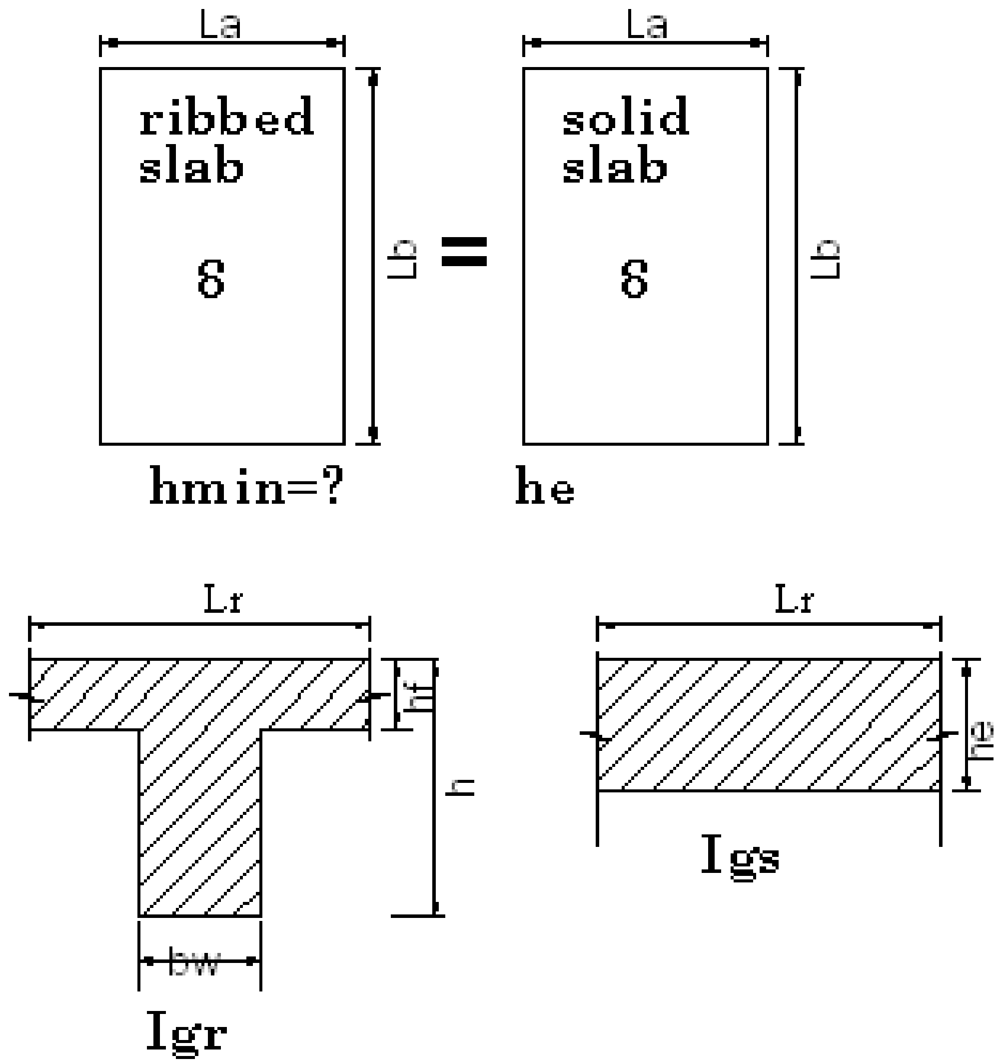
$$Wd: wt\ of\ typical\ unit\ (Lr, Lr) * \frac{1}{Lr^2}$$

$$Wd: \frac{kN}{m^2}$$

Minimum depth(h):

h, must satisfy :

1. Deflection requirement.
2. Shear requirement.
3. ACI code dimensions limitation.



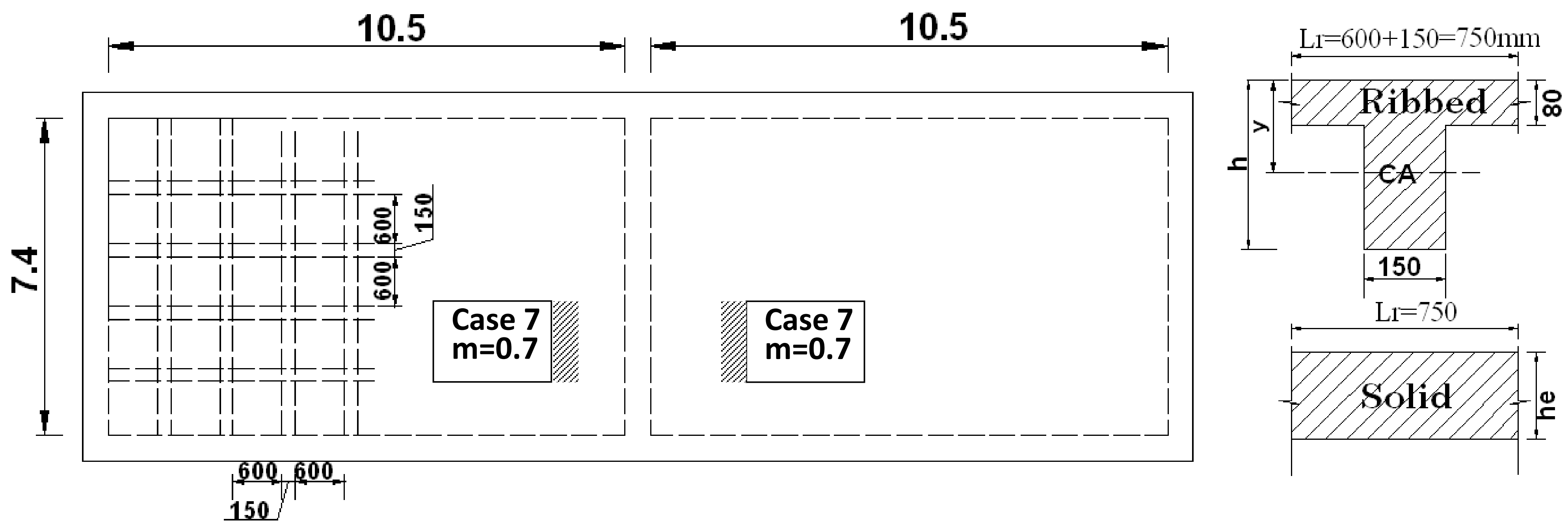
Deflection requirement for (h):

1. Assume suitable value for (h).
2. Find Ig_r.
3. Take $Ig_r = Ig_s = (bh_e^3)/12$ to find (h_e),(b=Lr)

4. Compute h_{min}=perimeter/180
5. Compare he with h_{min}
    - If $h_e \geq h_{min}$, assumed (h) O.K
    - If $h_e < h_{min}$ increase h gradually and repeat above steps.

Example :Design of two-way ribbed slab

WL=3.5kN/m2 (floor slab), bw=150mm, Lc=600mm, hf=80mm, fc'=25MPa,fy=350MPa.



Solution:

$$\frac{Lb}{La} = \frac{10.5}{7.4} = 1.4 < 2.0 \;\rightarrow two - way\ slab$$

Try h=300mm

$$let\ \alpha = \frac{b_w}{b},\ \ \gamma = \frac{h_f}{h}, m = \frac{\bar{y}}{h} = \frac{1}{2}\frac{(1-\alpha)\gamma^2 + \alpha}{(1-\alpha)\gamma + \alpha}$$

$$c = (1-\alpha)\gamma^3\left[1 + 3\left(\frac{2m}{\gamma} - 1\right)^2\right]$$

$$+ \alpha[1 + 3(2m-1)^2] \qquad \begin{cases} < 1.0 for\ T - sec \\ = 1.0\ for\ rect.sec \end{cases}$$

$$Ig_r = c\frac{bh^3}{12}$$

$$\alpha = \frac{b_w}{b} = \frac{150}{750} = 0.2, \;\; \gamma = \frac{h_f}{h} = \frac{80}{300} = 0.267,$$

$$m = \frac{\bar{y}}{h} = \frac{1}{2}\frac{(1-\alpha)\gamma^2 + \alpha}{(1-\alpha)\gamma + \alpha} = \frac{1}{2} * \frac{(1-0.2) * 0.267^2 + 0.2}{(1-0.2) * 0.267 + 0.2}$$

$$= 0.317$$

$$c = (1-0.2) * 0.267^3 \left[1 + 3\left(\frac{2 * 0.317}{0.267} - 1\right)^2\right] + 0.2$$

$$* \left[1 + 3(2 * 0.317 - 1)^2\right] = 0.385$$

$$Ig_r = c\frac{bh^3}{12} = 0.385 * \frac{0.75 * 0.3^3}{12} = 6.497 * 10^{-4}m^4$$

$$Ig_s = \frac{bh_e{}^3}{12} = \frac{0.75 * h_e{}^3}{12}$$

$$Ig_r = Ig_s \rightarrow 6.497 * 10^{-4}m^4 = \frac{0.75 * h_e{}^3}{12} \rightarrow h_e = 0.218m$$

$$h_{min} = \frac{perimeter}{180} = \frac{2(7.4 + 10.5)}{180} = 0.199m$$

$$h_e = 0.218m > h_{min} = 0.199m \;\; O.K\ (for\ deflection)$$

Check ACI code provisions for joist constructions

$$b_w = 150mm > 100\ mm\ o.k$$

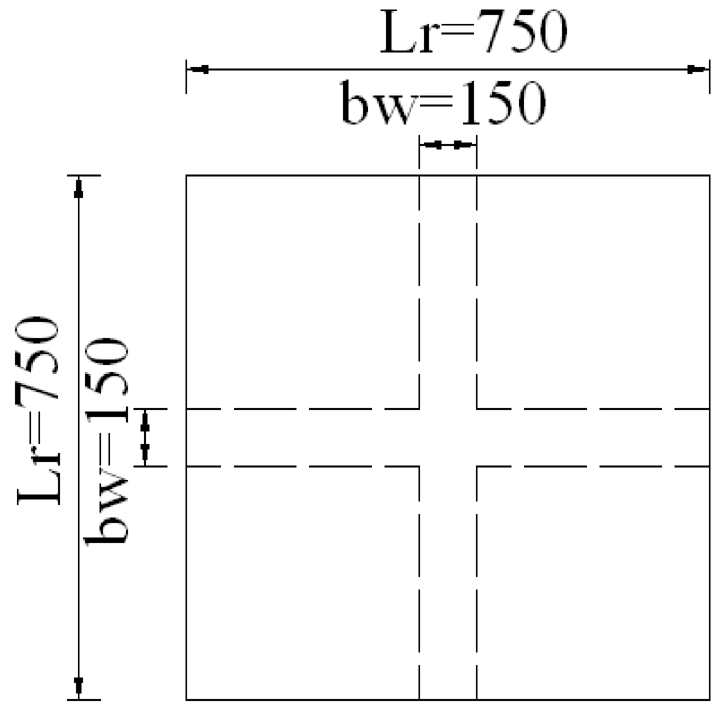
$$h = 300\ mm \leq 3.5\ b_w = 3.5 * 150 = 525mm\ o.k$$

$$L_c = 600 < 750\ mm\ o.k$$

$$hf = 80 > max.\left(\frac{L_c}{12} = \frac{600}{12} = 50\ mm, 50mm\right) o.k$$

Section dimensions are satisfied the ACI code limitation

$$W_{d1} = wt\ of\ typical\ unit * \frac{1}{{L_r}^2} = \frac{w}{{L_r}^2}$$



Typical unit

$$w = \left[h_f {L_r}^2 + b_w\left(h - h_f\right)L_r + b_w\left(h - h_f\right)(L_r - b_w)\right]\gamma_c \ \ by\ volume$$

$$OR\ \ w = {L_r}^2 h\gamma_c \underbrace{[\gamma + \alpha(1-\gamma)(2-\alpha)]}_{<1.0}$$

$$w = 0.75^2 * 0.3 * 24.5 \underbrace{[0.267 + 0.2(1 - 0.267)(2 - 0.2)]}_{=0.531<1.0}$$

$$= 2.195 \frac{kN}{typical\ unit}$$

$$W_{d1} = \frac{2.195}{0.75^2} = 3.902\,\frac{kN}{m^2}$$

$$tiling + morter = 0.98\,\frac{kN}{m^2}$$

$$minor\ pertitions = 0.7\,\frac{kN}{m^2}$$

$$w_d = \begin{pmatrix} w_{d1} = \frac{2.195}{0.75^2} = 3.902\,\frac{kN}{m^2} \\ tiling + morter = 0.98\,\frac{kN}{m^2} \\ minor\ pertitions = 0.7\,\frac{kN}{m^2} \end{pmatrix} = \sum w_d = 5.58\ \frac{kN}{m^2}$$

$$Wu = 1.2 * 5.58 + 1.6 * 3.5 = 12.3\ \frac{kN}{m^2}$$

Check for shear capacity:

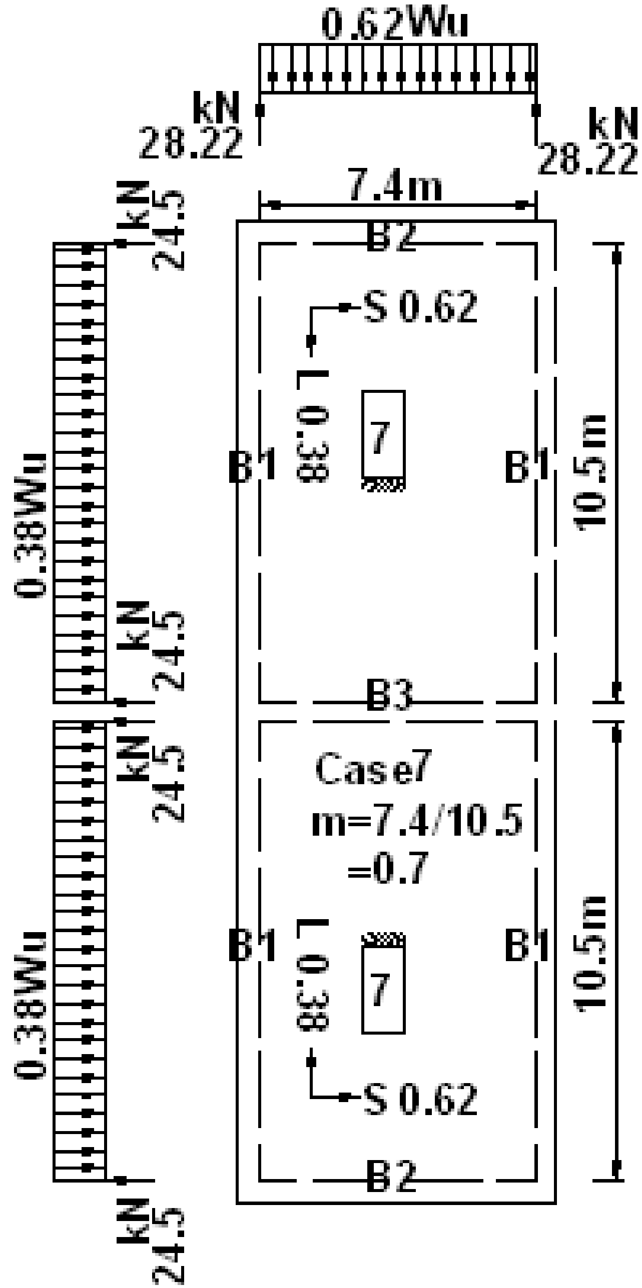
$$\max shear\ force = \frac{coeff.Wu.L_a}{2}$$

$$= \frac{0.62 * 12.3 * 7.4}{2} = 28.22\ \frac{kN}{m_{width}}$$

ds=300-25=275mm

Vud=28.22-0.62*12.3*0.275=

$$=26.12\ \frac{kN}{m_{width}}$$



$$\frac{Vud}{rib} = \overline{Vud} = L_r * Vud = 0.75 * 26.12 = 19.6 \frac{kN}{rib}$$

$$Vc = 0.17 * \sqrt{fc'} b_w d * \underbrace{1.1}_{ACI\ 8.11.8}$$

$$Vc = 0.17 * \sqrt{25} * 0.15 * 0.275 * \underbrace{1.1}_{ACI\ 8.11.8} * 1000 = 38.5 kN$$

$$\frac{\overline{Vud}}{\emptyset} = \frac{19.6}{0.75} = 26.12\ kN < Vc = 38.5\ kN \quad O.K$$

Design for flexure:

panel	direction	C^-	C_{+l}	C_{+D}	M-	M+	R-	R+	ω -	ω +	$\bar{\rho} = \omega * \frac{fc'}{f}$	ρ	As^- mm^2	As+ mm^2
Case 7 m= $\frac{7.4}{10.5}$ =0.7	Short L=7.4 m d_s=275 mm	-	0.058	0.063	-	30.44	-	0.0238	-	0.024	-	0.0017	-	353
	Long L=10.5 m d_s=265 mm	0.038	0.017	0.017	38.65	17.28	0.1631	0.0146	0.183	0.014	0.0131	0.001	520	199

$$M^-{}_{long} = c\_w_u Ln^2 = 0.038 * 12.3 * 10.5^2$$

$$= 51.5\ \frac{kN.m}{m} * \underbrace{0.75}_{Lr} = 38.65 \frac{kN.m}{rib}$$

$$M^{+}{}_{short} = [\, c_{+d} * 1.2w_d + c_{+l} * 1.6w_l \,] * Ln^2$$

$$= [0.058 * 1.2 * 5.58 + 0.063 * 1.6 * 3.5] * 7.4^2$$

$$= 40.58 \; \frac{kN.m}{m} * \underbrace{0.75}_{Lr} = 30.44 \frac{kN.m}{rib}$$

$$M^{+}{}_{long} = [0.017 * 1.2 * 5.58 + 0.017 * 1.6 * 3.5] * 10.5^2$$

$$= 23.04 \; \frac{kN.m}{m} * \underbrace{0.75}_{Lr}$$

$$= 17.28 \frac{kN.m}{rib}$$

Calculations of reinforcement:

Short direction:

d=t-25=300-25=275mm

M^{+}=30.44 kN.m [rectangular or T-sec]

Let Ø=0.9 to be check later

$$M_u f = \emptyset 0.85\, fc' L_r hf \left(d - \frac{hf}{2}\right)$$

$$= 0.9 * 0.85 * 25 * 10^3 * 0.75 * 0.08 * \left(0.275 - \frac{0.08}{2}\right)$$

$$= 269\ \mathrm{kN.m} > M_u^+ = 30.44\ \mathrm{kN.m}$$

$\therefore a < hf$ (Rectangular section with L_r width)→use $R = \frac{Mu}{\emptyset fc' bd^2}$

$$R^{+} = \frac{M_u^{+}}{\emptyset fc' L_r d^2} = \frac{30.44}{0.9 * 25 * 10^3 * 0.75 * (0.275)^2} = 0.0238$$

$$\omega^{+} = 0.024 < \ \omega_{max} = 0.364 * \beta_1 = 0.364 * 0.85 = 0.309$$

$$\rightarrow \rho < \rho_{max} \ \text{O.K}$$

$$\rho^{+} = \omega^{+} \frac{fc'}{fy} = 0.024 * \frac{25}{350} = 0.00171 > \rho_{min.}{}^{+}$$

$$= \max.\left(\frac{1.4}{fy} = 0.004, \frac{\sqrt{fc'}}{4fy} = 0.0036\right) * \frac{b_w}{b} = 0.004 * \frac{150}{750}$$

$$= 0.0008 \ \text{O.K}$$

$$As^{+} = \rho^{+} * L_r * d = 0.00171 * 750 * 275 = 353 \ \text{mm}^2$$

Long direction:

d=ds-10=275-10=265mm

for $M^{-} = 38.65\ kN.m$ (rec. sec bw * h)

$$R^{-} = \frac{M_u^{-}}{\emptyset fc' b_w d^2} = \frac{38.65}{0.9 * 25 * 10^3 * 0.15 * (0.265)^2} = 0.1631$$

$$\omega^{-} = 0.183 < \omega_{max} = 0.364 * \beta_1 = 0.364 * 0.85 = 0.309$$

$$\rightarrow \rho < \rho_{max}\ O.K$$

$$\rho^{-} = \omega^{-} \frac{fc'}{fy} = 0.183 * \frac{25}{350} = 0.0131 > \rho_{min.}^{-}$$

$$= max.\left(\frac{1.4}{fy} = 0.004, \frac{\sqrt{fc'}}{4fy} = 0.0036\right) = 0.004\ O.K$$

$$As^{-} = \rho^{-} * b_w * d = 0.0131 * 150 * 265 = 520\ mm^2$$

for $M^{+} = 17.28\ kN.m < M_u f = 269\ kN.m(\therefore rec.sec\ \ Lr * h)$

$$R^{+} = \frac{M_u^{+}}{\emptyset fc' L_r d^2} = \frac{17.28}{0.9 * 25 * 10^3 * 0.75 * (0.265)^2} = 0.0146$$

$$\omega^{+} = 0.014 < \ \omega_{max} = 0.364 * \beta_1 = 0.364 * 0.85 = 0.309$$

$$\rightarrow \rho < \rho_{max}\ \ O.K$$

$$\rho^{+} = \omega^{+} \frac{fc'}{fy} = 0.014 * \frac{25}{350} = 0.001 > \rho_{min.}{}^{+}$$

$$= max.\left(\frac{1.4}{fy} = 0.004, \frac{\sqrt{fc'}}{4fy} = 0.0036\right) * \frac{b_w}{b} = 0.004 * \frac{150}{750}$$

$$= 0.0008\ O.K$$

$$As^{+} = \rho^{+} * L_r * d = 0.001 * 750 * 265 = 199\ mm^2$$

checking Ø(redaction factor)

$$\rho_t = 0.85 * \beta_1 * \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} = 0.85 * 0.85 * \frac{25}{350} * \frac{3}{8}$$

$$= 0.0193$$

$$\text{all } \rho < \rho_t \rightarrow \ \emptyset = 0.9 \text{ o.k}$$

$$As_{min} = 0.004\text{bw.d} = 0.004 * 150 * 265 = 165 \text{ mm}^2$$

OR

$$As_{min} = 0.0008\text{L}_\text{r}.\text{d} = 0.0008 * 750 * 275 = 165 \text{ mm}^2$$

Note: As must ≥ As_{min} in column and middle strips at both directions for positive and negative sections.

Arrangement of reinforcement:

1. Long direction

a. Middle strip:

$$As^{+} = 199mm^2 \rightarrow \left(\begin{matrix} 1\emptyset 12b \\ \underbrace{1\emptyset 12s}_{\geq \frac{As^{+}}{3}\ continue\ to\ support} \end{matrix} \right) = 226\ mm^2$$

$$As^{-} = 520mm^2 - \underbrace{2\emptyset 12b}_{=226}\ each\ side = 294\ mm^2 add$$

$$\rightarrow use 2\emptyset 14 add = 308mm^2 > 294\ mm^2$$

$at\ discontinuous\ edge$:

$$1Ø12b = 113mm^2 \begin{cases} \geq \dfrac{As^+}{3} = \dfrac{199}{3} = 66mm^2 \\ < As_{min} = 165mm^2 \rightarrow use1Ø10 \\ = 79mm^2 add > (165 - 113 = 52mm^2) \end{cases}$$

b. Column strip:

$$As^{+}{}_{C.S} = \frac{2}{3} As^{+}{}_{M.S} = \frac{2}{3} * 199 = 133mm^2 < As_{min}$$

$$= 165mm^2 \rightarrow$$

$$use(\underbrace{1\emptyset 10s}_{\geq \frac{As^{+}}{3}} + 1\emptyset 12b) = 192mm^2 > 165mm^2$$

$$As^{-}{}_{C.S} = \frac{2}{3} As^{-}{}_{M.S} = \frac{2}{3} * 520 = 347mm^2 > As_{min}$$

$$= 165mm^2$$

$$347-\underbrace{226}_{2\emptyset 12b(1each\ side)}=121mm^2 add[\ use 2\emptyset 10=157mm^2$$

$$>121mm^2]$$

at discontinuous edge:

$$1\emptyset 12b=113mm^2\begin{cases}\geq \frac{As^{+}{}_{c.s}}{3}=\frac{133}{3}=44mm^2\\ < As_{min}=165mm^2\rightarrow use 1\emptyset 10\\ =79add>(165-113=52mm^2)\end{cases}$$

2. Short direction:

a. Middle strip:

$$As^{+} = 353mm^2 \rightarrow \left(\begin{matrix} 1\text{Ø}16b \\ \underbrace{1\text{Ø}14s} \\ \geq \frac{As^{+}}{3} \text{ continue to support} \end{matrix} \right) = 355\ mm^2$$

at discontinuous edge:

$$1\text{Ø}16b = 201mm^2 \begin{cases} \geq \dfrac{As^{+}}{3} = \dfrac{353}{3} = 118mm^2 \\ > As_{min} = 165mm^2\ O.K \end{cases}$$

b. Column strip:

$$As^{+}{}_{C.S} = \frac{2}{3} As^{+}{}_{M.S} = \frac{2}{3} * 353 = 235mm^2 > As_{min}$$

$$= 165mm^2 \rightarrow$$

$$use(\underbrace{1\text{Ø}12s}_{\geq \frac{As^{+}{}_{C.S}}{3}} + 1\text{Ø}14b) = 267mm^2 > 235mm^2$$

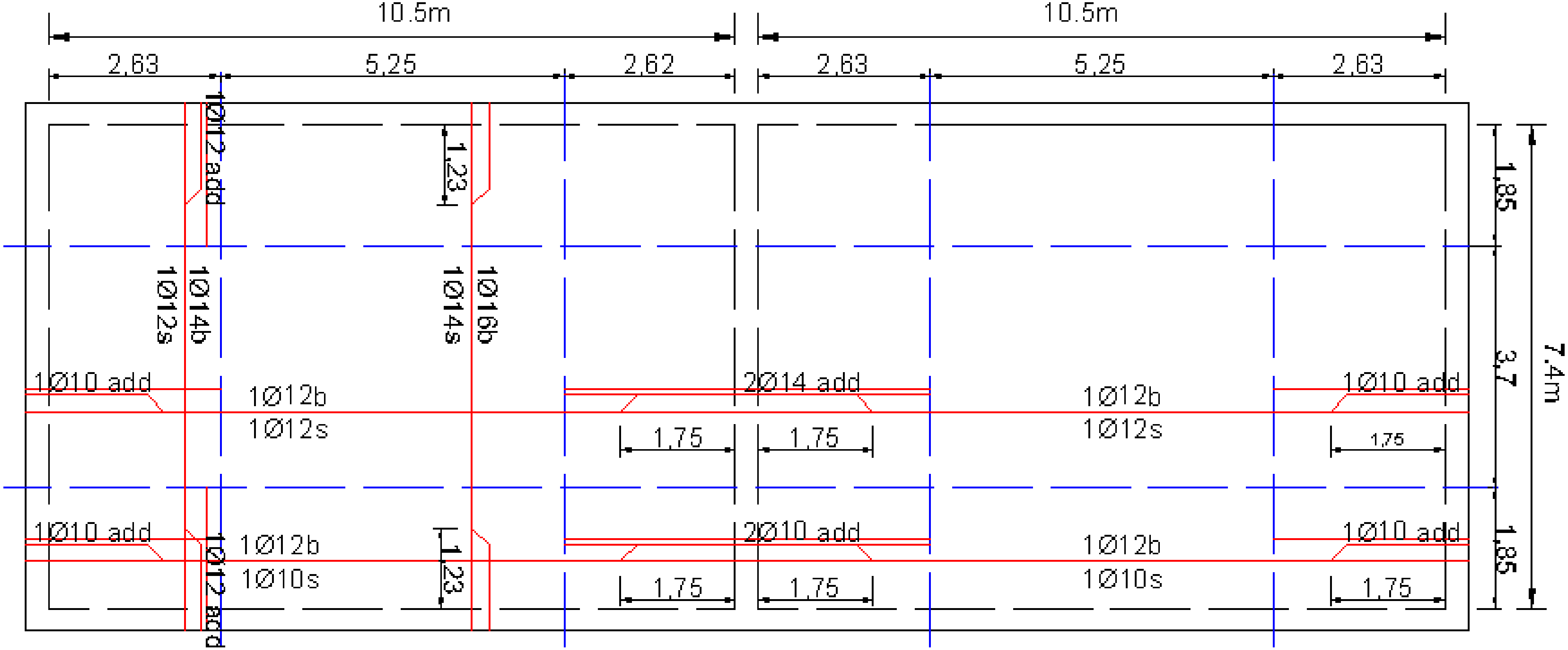
at discontinuous edge:

$$1\text{Ø}14b = 154mm^2 \begin{cases} \geq \dfrac{As^{+}{}_{C.S}}{3} = \dfrac{235}{3} = 78mm^2 \\ < As_{min} = 165mm^2 \rightarrow use 1\text{Ø}12 \\ = 113add > (165 - 154 = 11mm^2) \end{cases}$$

$$As_{\min\, for\ slab} = 0.002 * b * hf = 0.002 * 1000 * 80$$

$$= 160\ \frac{mm^2}{m}$$

Use Ø10@175< s_{max} welded wire mesh ($161mm^2$) $\rightarrow s_{max} =$ $\min(5t = 5 * 80 = 400mm, 500mm) = 400mm$


10.5m
10.5m
2,63
5,25
2,62
2,63
5,25
2,63
1Ø12 add
1,23
1Ø12s
1Ø14b
1Ø14s
1Ø16b
1,85
3,7
7.4m
1Ø10 add
1Ø12b
1Ø12s
2Ø14 add
1Ø12b
1Ø12s
1Ø10 add
1,75
1,75
1,75
1Ø10 add
1Ø12b
1Ø10s
1,23
1Ø12 add
2Ø10 add
1Ø12b
1Ø10s
1Ø10 add
1,85
1,75
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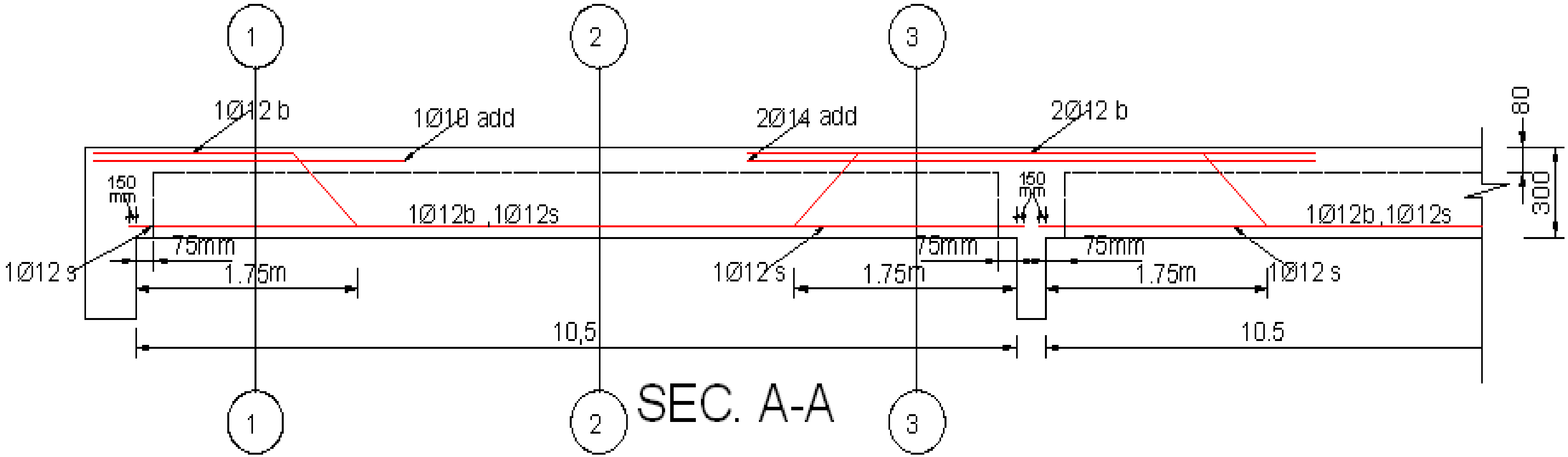
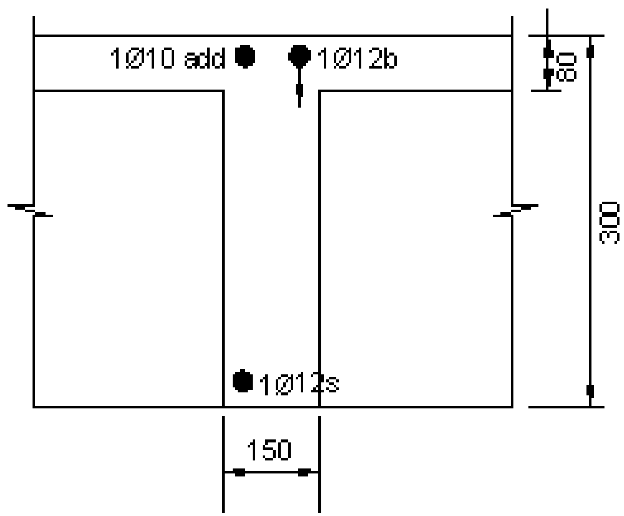
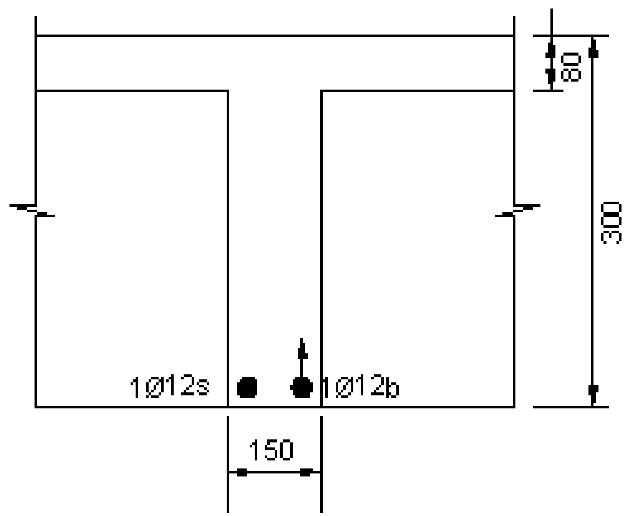


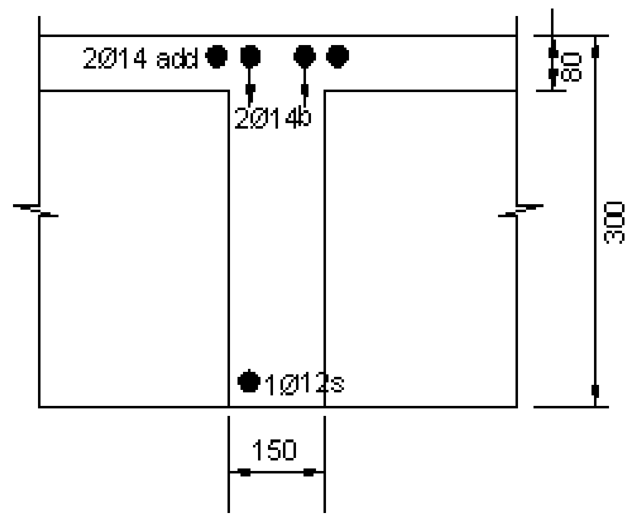
SEC. A-A



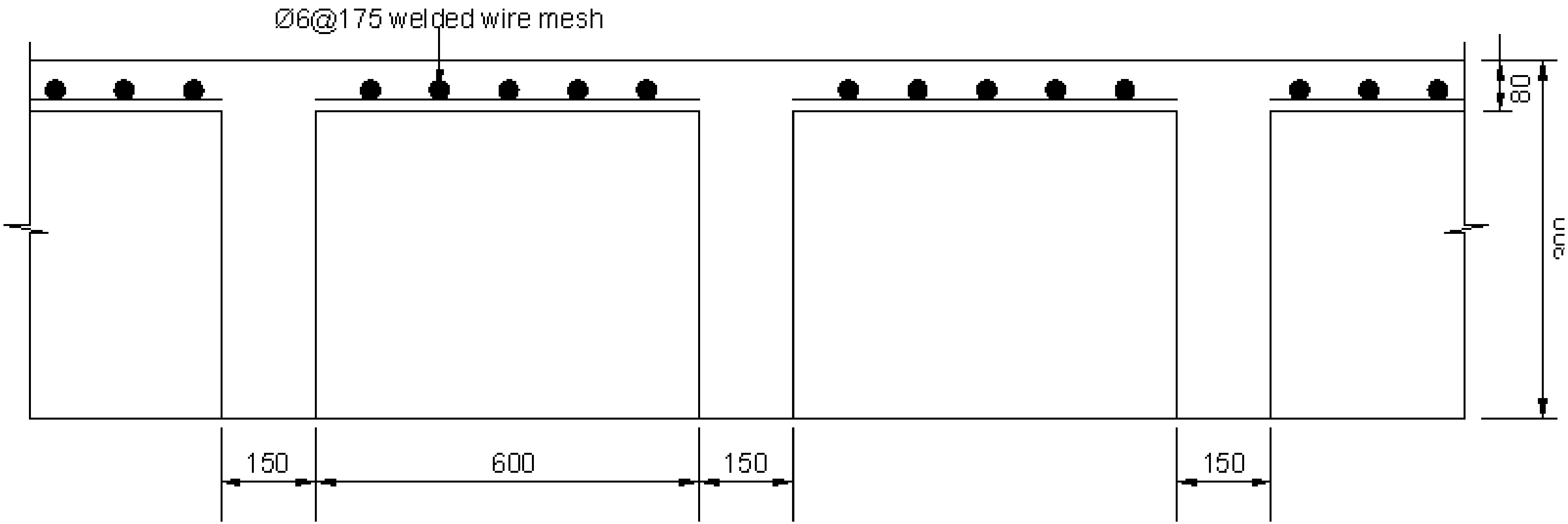
SEC 1-1



SEC 2-2



SEC 3-3



Flange reinforcement

No. of ribs :

Short direction:

No. of ribs $= L_a/L_r = 7.4*0.75 = 9.87 \approx 10$

Span $= n*L_c + (n-1)b_w = 10*0.6 + (10-1)*0.15 = 7.35m$

Excess $= (7.4-7.35)/2 = 0.025m < 100mm$

Long direction:

No. of ribs $= L_b/L_r = 10.5*0.75 = 14$

Span $= n*L_c + (n-1)b_w = 14*0.66 + (14-1)*0.15 = 10.35m$

Excess $= (10.5-10.35)/2 = 0.075m < 100mm$

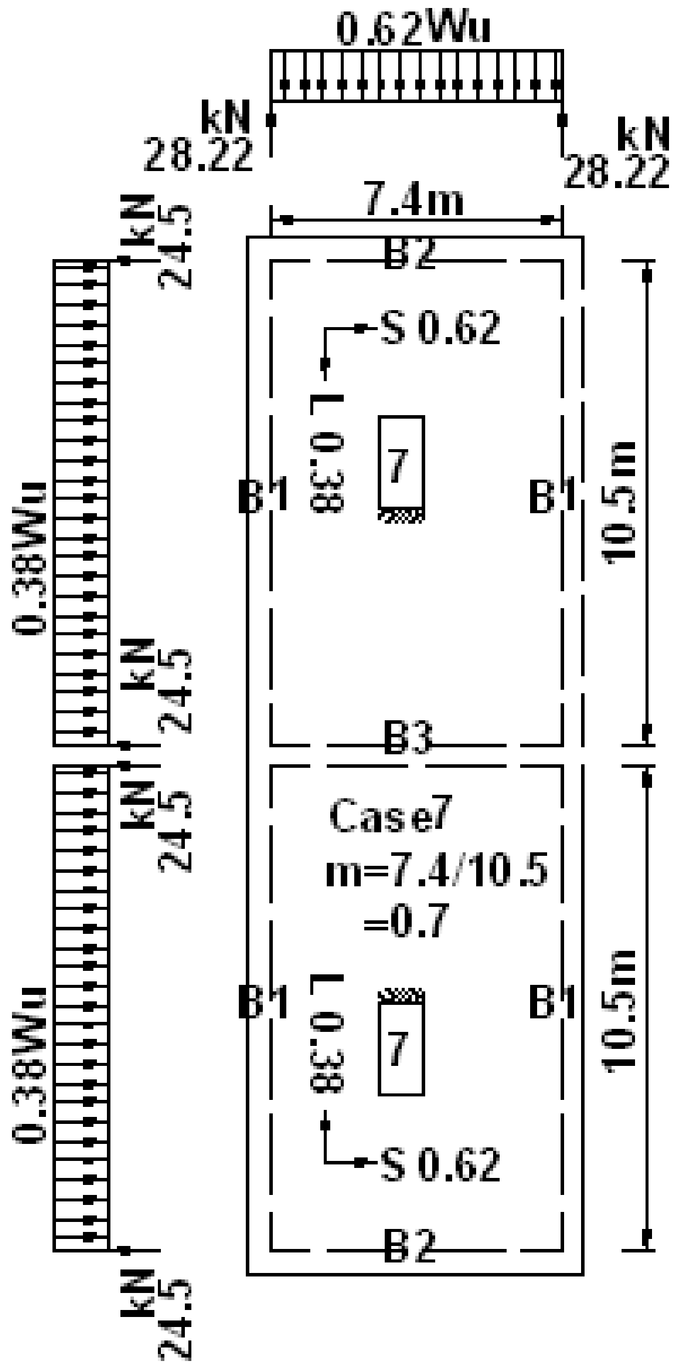
Loads transferred to beams:

a. Elastic stage:

B2: Wu=24.5 →30.34 kN/m+ 1.2*any additional dead load

B1: Wu=28.22 →30.07 kN/m+ 1.2*any additional dead load

B3: Wu=2*24.549.08→2*30.34 =60.68 k/m+ 1.2*any additional dead load



b. Failure stage:

B2: $We = \frac{Wu.L_a}{3} = \frac{12.3*7.4}{3} = 30.34\frac{kN}{m}$

B1: $We = \frac{Wu.L_a}{3}(\frac{3-m^2}{2}) = \frac{12.3*7.4}{3}(\frac{3-0.7^2}{2}) = 30.07\frac{kN}{m}$

B3: $We = 2*30.34 = 60.68\frac{kN}{m}$