SLAB DESIGN

Reading Assignment

Chapter 9 of Text and, Chapter 13 of ACI318-02

Introduction

ACI318 Code provides two design procedures for slab systems:

13.6.1 Direct Design Method (DDM) For slab systems with or without beams loaded only by gravity loads and having a fairly regular layout meeting the following conditions:

13.6.1.1 There must be three or more spans in each directions.

13.6.1.2 Panels should be rectangular and the long span be no more than twice the short span.

13.6.1.3 Successive span lengths center-to-center of supports in each direction shall not differ by more than 1/3 of the longer span.

13.6.1.4 Columns must be near the corners of each panel with an offset from the general column line of no more 10% of the span in each direction.

13.6.1.5 The live load should not exceed 3 time the dead load in each direction. All loads shall be due gravity only and uniformly distributed over an entire panel.

13.6.1.6 If there are beams, there must be beams in both directions, and the relative stiffness of the beam in the two directions must be related as follows:

$$0.2 \le \frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} \le 5.0$$

where

$$\alpha = \frac{E_{cb} I_b}{E_{cs} I_s}$$

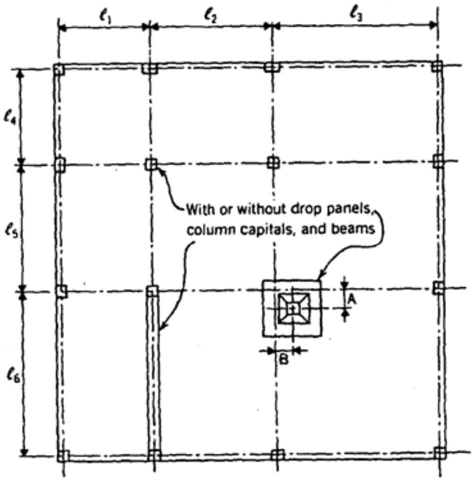
is the ratio of flexural stiffness of beam sections to flexural stiffness of a width of slab bounded laterally by center lines of adjacent panels (if any) on each side of the beam.

1. Three continuous spans in each direction, minimum.
2. Rectangular panels with aspect ratio ≤ 2.0. e.g., $\frac{\ell_6}{\ell_1} \le 2.0$.
3. Span lengths differ by $\frac{1}{3}$ or less of longer span. e.g., $\frac{\ell_5}{\ell_6} \ge 0.67$.
4. Column offset a maximum at 10%. e.g., $A \le 0.1\,\ell_6$.

5A. Gravity loads only, uniformly distributed, not lateral loads.

5B. Live load ≤ 3 × dead load.

6. Beam stiffness: $0.2 \le \frac{\alpha_1 \ell_2^2}{\alpha_2 \ell_1^2} \le 5.0$.
7. No moment redistribution as permitted by ACI Code Section 8.4.



For slab systems loaded by horizontal loads and uniformly distributed gravity loads, or not meeting the requirement of the section 13.6.2, the Equivalent Frame Method (EFM) of Sect. 13.7 of ACI code may be used. Although Sect. 13.7 of the ACI code implies that the EFM may be satisfactory in cases with lateral as well horizontal loads, the Commentary cautions that additional factors may need to be considered. The method is probably adequate when lateral loads are small, but serious questions may be raised when major loads must be considered in addition to the vertical loads.

The direct design method gives rules for the determination of the total static design moment and its distribution between negative and positive moment sections. The EFM defines an equivalent frame for use in structural analysis to determine the negative and positive moments acting on the slab system. Both methods use the same procedure to divide the moments so found between the middle strip and column strips of the slab and the beams (if any).

Section 13.3.1 of the Code could be viewed as an escape clause from the specific requirements of the code. It states: “A slab may be designed by any procedure satisfying conditions for equilibrium and geometrical compatibility if shown that the design strength at every section is at least equal to the required strength considering Secs. 9.2 and 9.3 (of the ACI code), and that all serviceability conditions, including specified limits on deflections, are met.” The methods of elastic theory moment analysis such as the Finite Difference procedure satisfies this clause. The limit design methods, for example the yield line theory alone do not satisfy these requirements, since although the strength provisions are satisfied, the serviceability conditions may not be satisfied without separate checks of the crack widths and deflections at service load levels.

The thickness of a floor slab must be determined early in design because the weight of the slab is an important part of the dead load of the structure. The minimum thickness can be determined by many factors:

- Shear strength of beamless slabs (usually a controlling factor); slab must be thick enough to provide adequate shear strength
- Flexural moment requirement (less often a governing factor)
- Fire resistance requirements
- Deflection control (most common thickness limitations)

Section 9.5.3 of ACI gives a set of equations and other guides to slab thickness, and indicates that slabs which are equal to or thicker than the computed limits should have deflections within acceptable range at service load levels.

ACI code direct design method and equivalent methods can be conveniently discussed in terms of a number of steps used in design. The determination of the total design moment in concerned with the safety (strength) of the structure. The remaining steps are intended to distribute the total design moment so as to lead to a serviceable structure in which no crack widths are excessive, no reinforcement yields until a reasonable overload is reached, and in which deflections remain within acceptable limits. These steps are discussed as we go along.

Equivalent frame method may be used in those cases where:

- slab layout is irregular and those not comply with the restrictions stated previously
- where horizontal loading is applied to the structure
- where partial loading patterns are significant because of the nature of the loading
- high live load/dead load ratios.

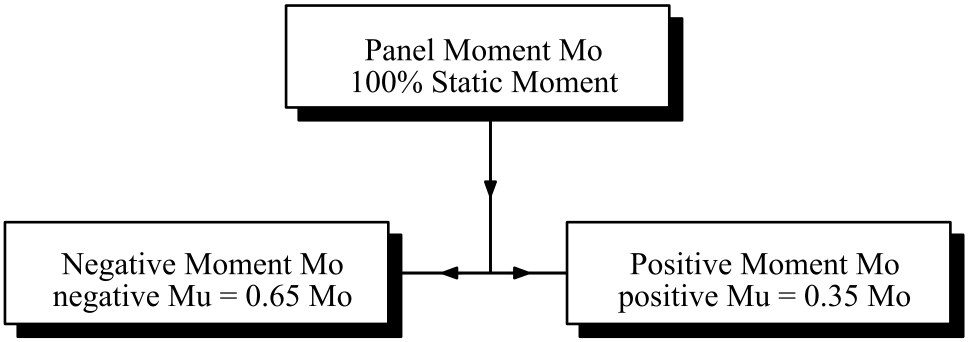
Design Procedure

The basic design procedure of a two-way slab system has five steps.

1. Determine moments at critical sections in each direction, normally the negative moments at supports and positive moment near mid-span.
2. Distribute moments transverse at critical sections to column and middle-strip and if beams are used in the column strip, distribute column strip moments between slab and beam.
3. Determine the area of steel required in the slab at critical sections for column and middle strips.
4. Select reinforcing bars for the slab and concentrate bars near the column, if necessary
5. Design beams if any, using procedures you learned in CIVL 4135.

Positive and Negative Distribution of Moments

For interior spans, the total static moment is apportioned between critical positive and negative bending sections as **(See ACI 318-02 Sect. 13.6.3)**:



As was shown, the critical section for negative bending moment is taken at the face of rectangular supports, or at the face of an equivalent square support.

For the Case of End Span

The apportionment of M_o among three critical sections (interior negative, positive, and exterior negative) depends on

1. Flexural restraint provided for slab by the exterior column or the exterior wall.
2. Presence or absence of beams on the column lines.

See ACI 318-02

Sect. 13.6.3.3 of ACI

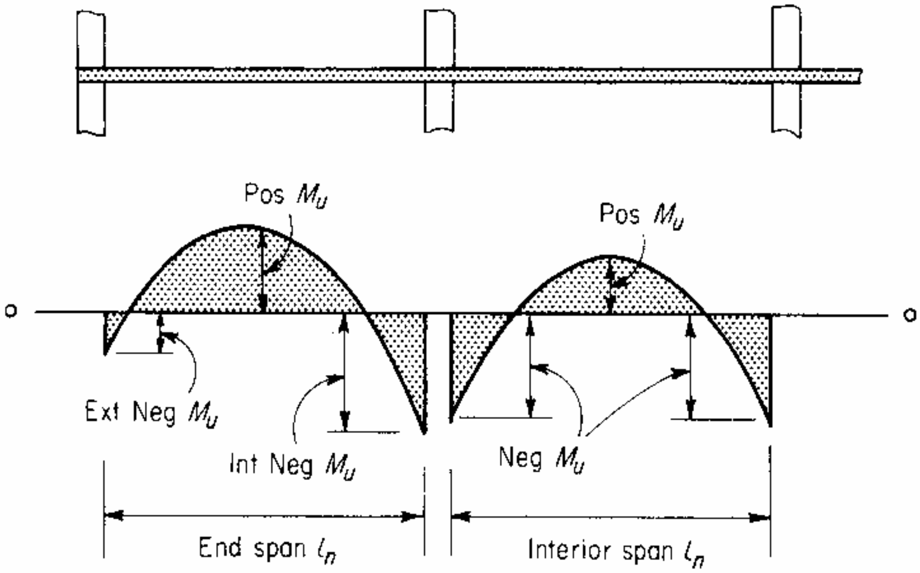


FIGURE 13.4
Distribution of total static moment M_o to critical sections for positive and negative bending.

Distribution factors applied to static moment M_o for positive and negative moments in end span

	(a)	*(b)*	*(c)*	*(d)*	*(e)*
	Exterior edge unrestrained	**Slab with beams between all supports**	**Slab without beams between interior supports**		**Exterior edge fully restrained**
			Without edge beam	**With edge beam**	
Interior negative moment	0.75	0.70	0.70	0.70	0.65
Positive moment	0.63	0.57	0.52	0.50	0.35
Exterior negative moment	0	0.16	0.26	0.30	0.65

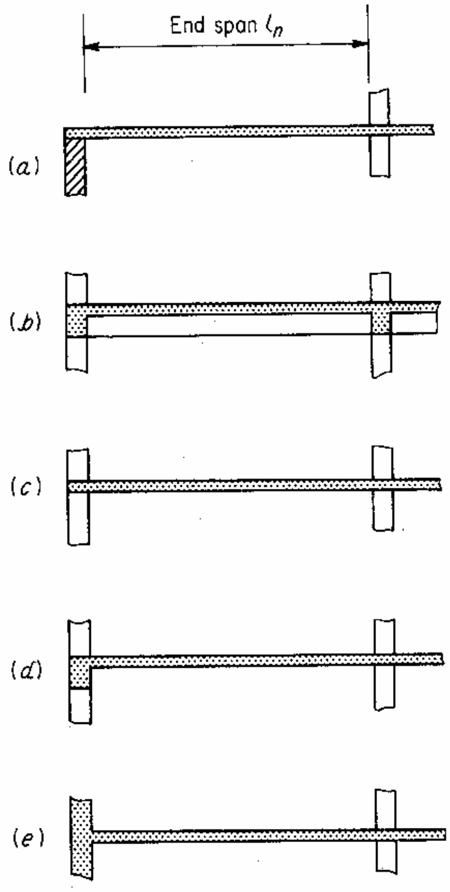


FIGURE 13.5
Conditions of edge restraint considered in distributing total static moment M_o to critical sections in an end span: (*a*) exterior edge unrestrained, e.g., supported by a masonry wall; (*b*) slab with beams between all supports; (*c*) slab without beams, i.e., flat plate; (*d*) slab without beams between interior supports but with edge beam; (*e*) exterior edge fully restrained, e.g., by monolithic concrete wall.

Lateral Distribution of Moments

Here we will study the various parameters affecting moment distribution across width of a cross-section. Having distributed the moment $\boldsymbol{M_o}$ to the positive and negative moment sections as just described, we still need to distribute these design moments across the width of the critical sections. For design purposes, we consider the moments to be constant within the bounds of a middle or column strip unless there is a beam present on the column line. In the latter case, because of its greater stiffness, the beam will tend to take a larger share of the column-strip moment than the adjacent slab. For an interior panel surrounded by similar panels supporting the same distributed loads, the stiffness of the supporting beams, relative to slab stiffness is the controlling factor.

The distribution of total negative or positive moment between slab middle strip, column strip, and beams depends on:

- the ratio of l_2/l_1,
- the relative stiffness of beam and the slab,
- the degree of torsional restraint provided by the edge beam.

The beam relative stiffness in direction 1 is:

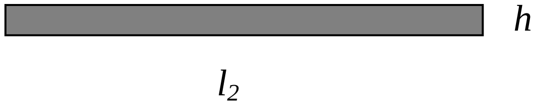
$$a_1 = \frac{E_{cb} I_{b1}}{E_{cs} I_s}$$

where

$E_{cb}I_{b1}$ = Flexural rigidity of beam in direction 1

$E_{cs}I_s$ = Flexural rigidity of slabs of width l_2

$= bh^3/12$ where b = width between panel centerlines on each side of beam.



similarly

$$a_2 = \frac{E_{cb} I_{b2}}{E_{cs} I_s}$$

in general

$$0 < a < \infty$$

$a = \infty$ → Supported by walls

$a = 0$ → no beams

for beam supported slabs

$$a < 4 \text{ or } 5$$

Note:
Values of a are ordinarily calculated using uncracked gross section moments of inertia for both slab and beam.

Beams cross section to be considered in calculating I_{b1} and I_{b2} are shown below. (**see ACI sect. 13.2.4)**

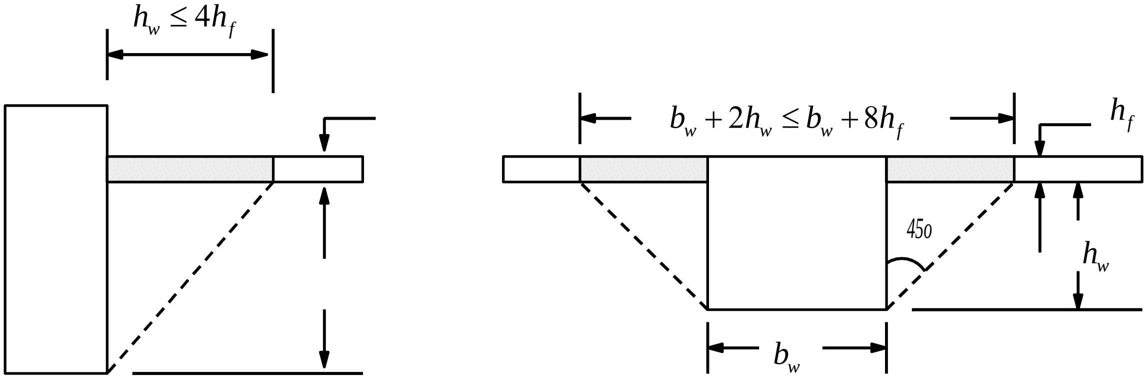


Fig 13.2.4 of ACI
Examples of the portion of slab to be included
with the beam under 13.2.4

The relative restraint provided by the torsional resistance of the effective transverse edge beam is reflected by parameter β_t such as:

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$$

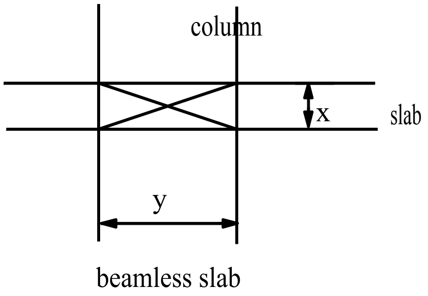
where

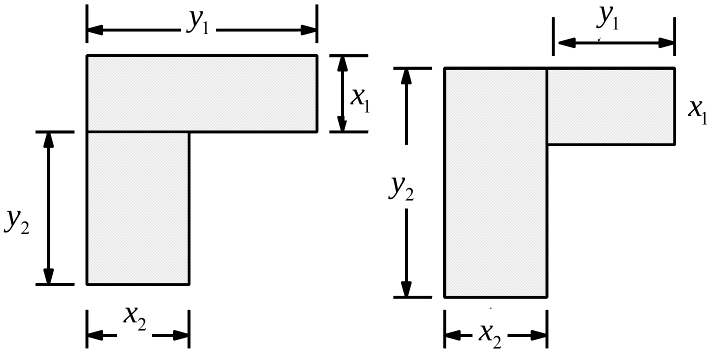
E_{cb} = Muduls of Elasticity of Beam Concrete

C = Torsional Constant of the Cross–section

The constant C is calculated by dividing the section into its rectangles, each having smaller dimension x and larger dimension y:

$$C = \sum (1 - 0.63\frac{x}{y})\frac{x^3 y}{3}$$





See Section 13.6.4 of ACI for factored moments in column strips.

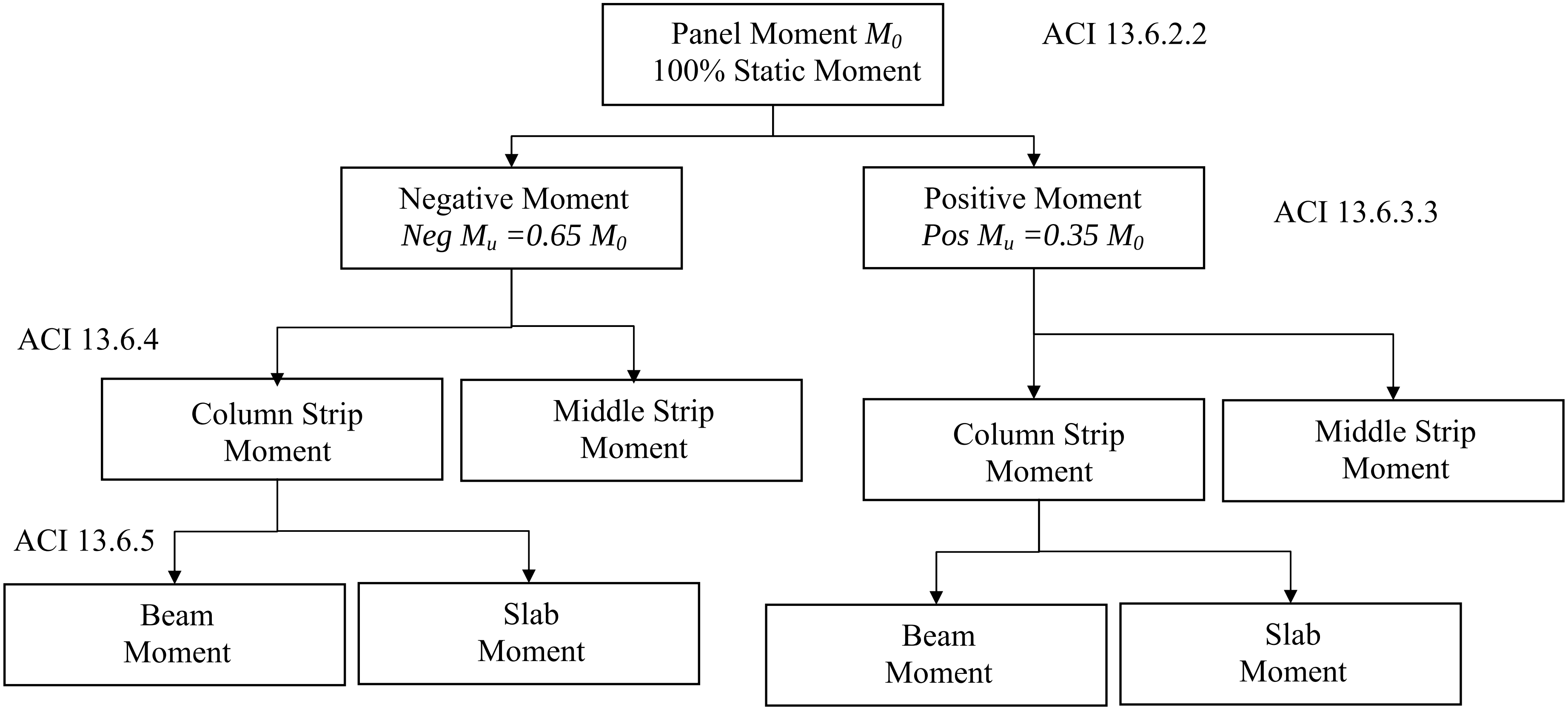

Panel Moment M_0
100% Static Moment
ACI 13.6.2.2
Negative Moment
Neg M_u =0.65 M_0
Positive Moment
Pos M_u =0.35 M_0
ACI 13.6.3.3
ACI 13.6.4
Column Strip Moment
Middle Strip Moment
Column Strip Moment
Middle Strip Moment
ACI 13.6.5
Beam Moment
Slab Moment
Beam Moment
Slab Moment

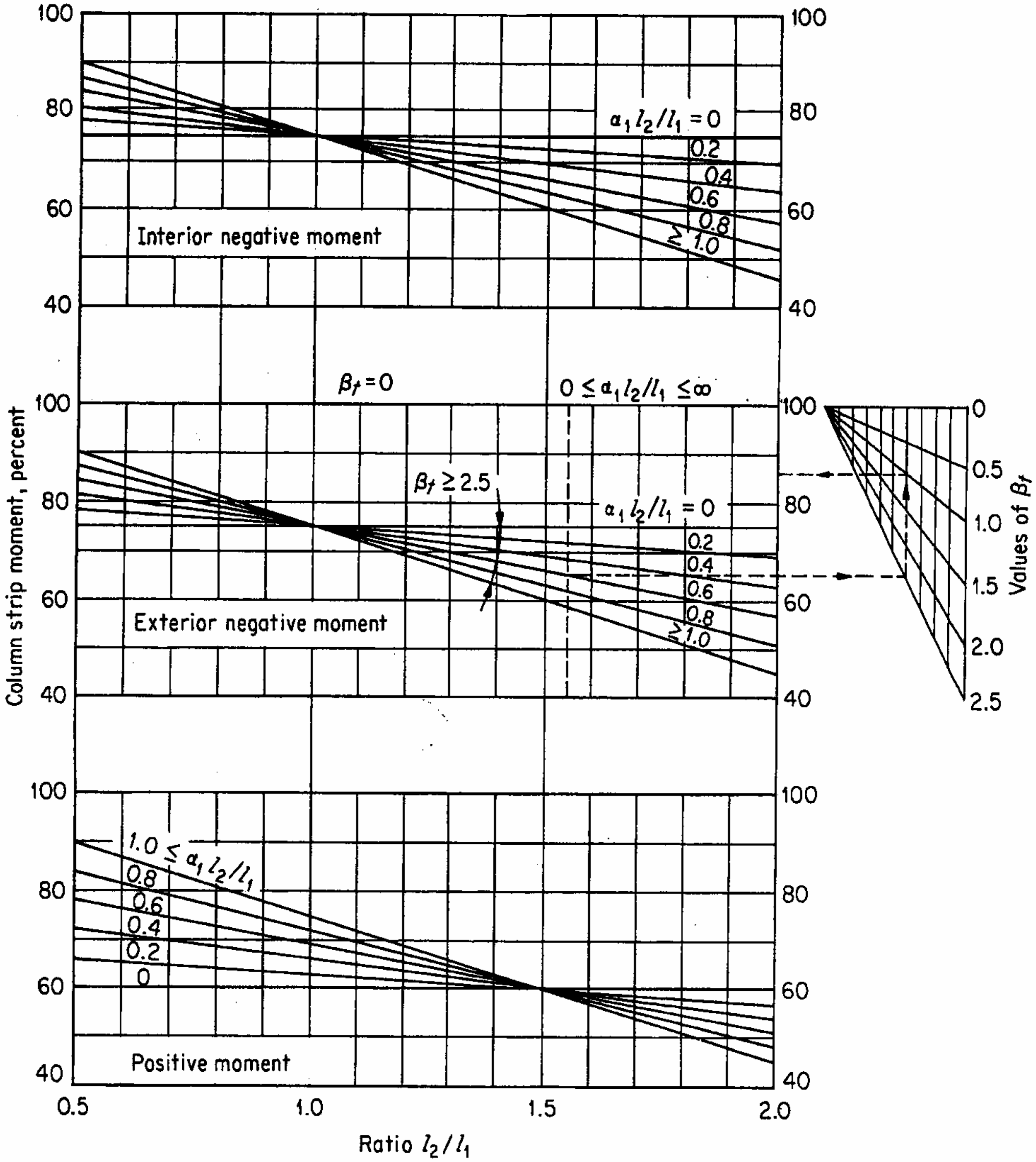
TABLE 13.2

Column-strip moment, percent of total moment at critical section

		l_2/l_1		
		0.5	**1.0**	**2.0**
Interior negative moment				
$\alpha_1 l_2/l_1 = 0$		75	75	75
$\alpha_1 l_2/l_1 \geq 1.0$		90	75	45
Exterior negative moment				
$\alpha_1 l_2/l_1 = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$\alpha_1 l_2/l_1 \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45
Positive moment				
$\alpha_1 l_2/l_1 = 0$		60	60	60
$\alpha_1 l_2/l_1 \geq 1.0$		90	75	45

Implementation of these provisions is facilitated by the interpolation charts of Graph A.4 of App. A. Interior negative- and positive-moment percentages can be read directly from the chart for known values of l_2/l_1 and $\alpha_1 l_2/l_1$. For exterior negative moment, the parameter β_t requires an additional interpolation, facilitated by the auxiliary diagram on the right side of the chart. To illustrate its use for $l_2/l_1 = 1.55$ and $\alpha_1 l_2/l_1 = 0.6$, the dotted line indicates moment percentages of 100 for $\beta_t = 0$ and 65 for $\beta_t = 2.5$. Projecting to the right as indicated by the arrow to find the appropriate vertical scale of 2.5 divisions for an intermediate value of β_t, say 1.0, then upward and finally to the left, one reads the corresponding percentage of 86 on the main chart.

The column-line beam spanning in the direction l_1 is to be proportioned to resist 85 percent of the column-strip moment if $\alpha_1 l_2/l_1$ is equal to or greater than 1.0. For values between one and zero the proportion to be resisted by the beam may be obtained by linear interpolation. Concentrated or linear loads applied directly to such a beam should be accounted for separately.



GRAPH A.4
Interpolation charts for lateral distribution of slab moments.

ACI Two-Slabs Depth Limitation

- Serviceability of a floor system can be maintained through deflection control and crack control
- Deflection is a function of the stiffness of the slab as a measure of its thickness, a minimum thickness has to be provided irrespective of the flexural thickness requirement.
- Table 9.5(c) of ACI gives the minimum thickness of slabs without interior beams.
- Table 9.5(b) of ACI gives the maximum permissible computed deflections to safeguard against plaster cracking and to maintain aesthetic appearance.
- Could determine deflection analytically and check against limits
- Or alternatively, deflection control can be achieved indirectly to more-or-less arbitrary limitations on minimum slab thickness developed from review of test data and study of the observed deflections of actual structures. This is given by ACI.

For a_m greater than 0.2 but not greater than 2.0, the thickness shall not be less than

$$h = \frac{l_n\left(0.8 + \frac{f_y}{200,000}\right)}{36 + 5\beta\left[a_m - 0.20\right]} \qquad \text{Eq. 9-12 of ACI}$$

and not less than 5.0 inches.

For a_m greater than 2.0, the thickness shall not be less than

$$h = \frac{l_n\left(0.8 + \frac{f_y}{200,000}\right)}{36 + 9\beta} \qquad \text{Eq. 9-13 of ACI}$$

and not less than 3.5 inches.

. β = Ratio of clear span in long direction to clear span in short direction

α_m = Average value of α for all beams on edges of panel.

In addition, the thickness h must not be less than (ACI **9.5.3.2**):

For slabs without beams or drop panels	5	inches
For slabs without beams but with drop panels	4	inches

Read Section 9.5.3.3 (d) for 10% increase in minimum thickness requirements.

DESIGN AND ANALYSIS PROCEDURE- DIRECT DESIGN METHOD

Operational Steps

Figure 11.9 gives a logic flowchart for the following operational steps.

1. Determine whether the slab geometry and loading allow the use of the direct design method as listed in DDM.
2. Select slab thickness to satisfy deflection and shear requirements. Such calculations require a knowledge of the supporting beam or column dimensions A reasonable value of such a dimension of columns or beams would be 8 to 15% of the average of the long and short span dimensions, namely $(l_1 + l_2)/2$. For shear check, the critical section is at a distance $d/2$ from the face of the'! support. If the thickness shown for deflection is not adequate to carry the shear, use one or more of the following:

   (a) Increase the column dimension.

   (b) Increase concrete strength.

   (c) Increase slab thickness.

   (d) Use special shear reinforcement.

   (e) Use drop panels or column capitals to improve shear strength.

3. Divide the structure into equivalent design frames bound by centerlines of panels on each side of a line of columns.
4. Compute the total statical factored moment $M_0 = \dfrac{w_u l_2 l_n^2}{8}$
5. Select the distribution factors of the negative and positive moments to the exterior and interior columns and spans and calculate the respective factored moments.
6. Distribute the factored equivalent frame moments from step 4 to the column and middle strips.
7. Determine whether the trial slab thickness chosen is adequate for moment-shear transfer in the case of flat plates at the interior column junction computing that portion of the moment transferred by shear and the properties of the critical shear section at distance d/2 from column face.
8. Design the flexural reinforcement to resist the factored moments in step 6.
9. Select the size and spacing of the reinforcement to fulfill the requirements for crack control, bar development lengths, and shrinkage and temperature stresses.