Lines in Space

Angle between Two Lines

Intersection of Two lines

Shortest Distance from a Point to a Line

Planes in Space

Intersection of Two Planes

Angle between Two Planes

Angle between a Line and a Plane

Shortest Distance from a Point to a Plane

Basic Concepts

What is scalar?

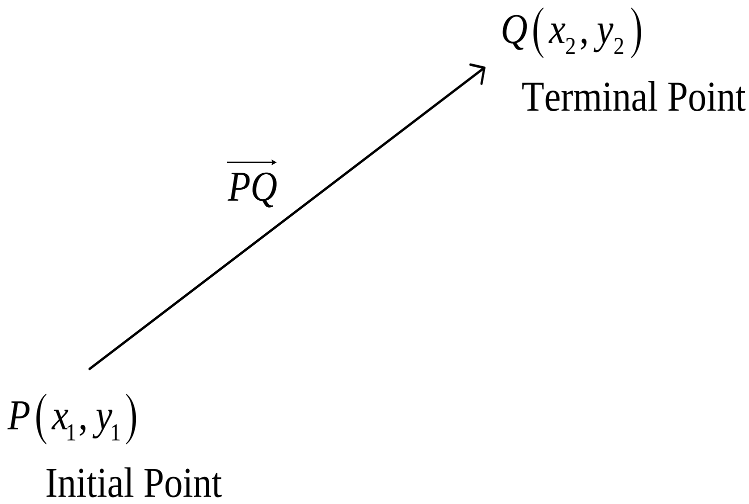
- ✓ a quantity that has only magnitude

What is vector?

- ✓ a quantity that has magnitude and direction

A *vector* can be represented by a *directed line segment* where

- ✓ length of the line segment
  - the magnitude of the vector
- ✓ direction of the line segment
  - the direction of the vector



- ✓ A vector can be written as $\overrightarrow{PQ}$, or $\boldsymbol{a}$. The order of the letters is important. $\overrightarrow{PQ}$ means the vector is from *P* to *Q* or the position vector *Q* relative to *P*, $\overrightarrow{QP}$ means vector is from *Q* to *P* or the position vector *P* relative to *Q*.

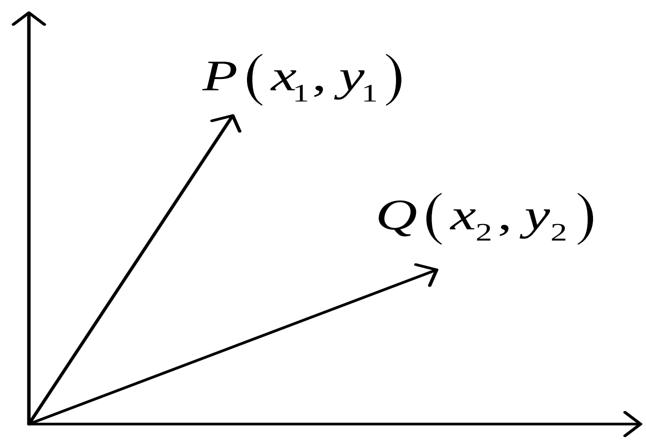
- ✓ If *P* (x_1, y_1) is the initial point and $Q(x_2, y_2)$ is the terminal point of a directed line segment, $\overrightarrow{PQ}$ then **component form** of vector **v** that represents $\overrightarrow{PQ}$ is

$$\langle v_1, v_2 \rangle = \langle x_x - x_1, y_2 - y_1 \rangle = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

and the **magnitude** or the **length** of **v** is

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{c_1^2 + c_2^2}$$



$$\overrightarrow{OP} = \langle x_1 - 0, y_1 - 0 \rangle = \langle x_1, y_1 \rangle$$

$$\overrightarrow{OQ} = \langle x_2 - 0, y_2 - 0 \rangle = \langle x_2, y_2 \rangle$$

-Note-

Any vector that has magnitude of 1 unit = unit vector.

Example:

Find the component form and length of the vector **v** that has initial point $(3,-7)$ and terminal point $(-2,5)$.

Solution:

$$\bar{v} = \langle -2-3, 5+7 \rangle = \langle -5, 12 \rangle$$

$$|\bar{v}| = \sqrt{(-5)^2 + (12)^2} = \sqrt{25+144} = 13$$

Example:

Given $\mathbf{v} = \langle -2, 5 \rangle$ and $\mathbf{w} = \langle 3, 4 \rangle$, find each of the following vectors:

a) $\frac{1}{2}\mathbf{v}$ b) $\mathbf{w} - \mathbf{v}$ c) $\mathbf{v} + 2\mathbf{w}$

Answer:

a) $\langle -1, 5/2 \rangle$ b) $\langle 5, -1 \rangle$ c) $\langle 4, 13 \rangle$

<u>Theorem:</u>

If $\boldsymbol{a}$ is a non-null vector and if $\hat{\boldsymbol{a}}$ is a **unit vector** having the same direction as $\boldsymbol{a}$, then

$$\hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

-Note-

To verify that magnitude is 1, $|\hat{\boldsymbol{a}}| = 1$

Example:

Find a unit vector in the direction of

$\mathbf{v} = \langle -2,5 \rangle$ and verify that it has length 1.

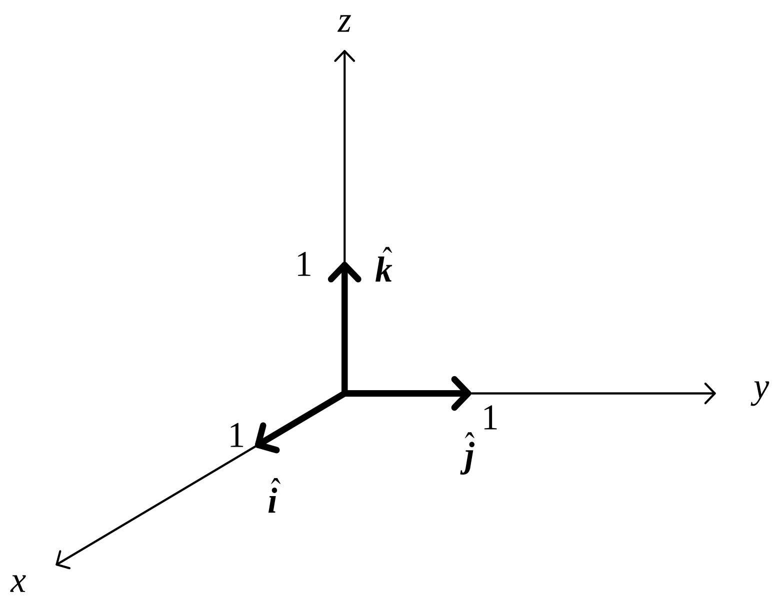
Solution:

$$\bar{v} = \frac{\langle -2,5 \rangle}{\sqrt{(-2)^2 + 5^2}} = \frac{1}{\sqrt{29}} \langle -2,5 \rangle$$

$$|\bar{v}| = \sqrt{\left(-2/\sqrt{29}\right)^2 + \left(5/\sqrt{29}\right)^2} = \sqrt{1} = 1$$

Standard Unit Vectors

- ✓ Three standard unit vectors are: **i**, **j** dan **k**



- ✓ Vectors **i**, **j** and **k** can be written in components form:

$\boldsymbol{i} = < 1, 0, 0 >$,

$\boldsymbol{j} = < 0, 1, 0 >$ and

$\boldsymbol{k} = < 0, 0, 1 >$

and can interpreted as

$$\boldsymbol{a} = <x, y, z>$$
$$= x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

✓ The vector $P\vec{Q}$ with initial point $P(x_1, y_1, z_1)$ and terminal point $Q(x_2, y_2, z_2)$ has the standard representation

$$P\overline{Q} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$

or

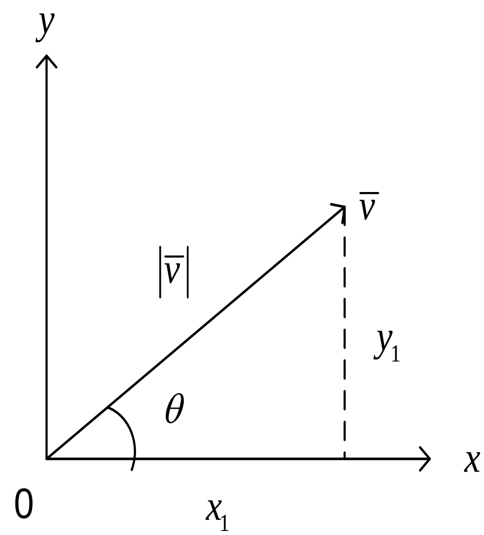
$$PQ = < x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 >$$

Example:

Let **u** be the vector with initial point (2,-5) and terminal point (-1,3), and let $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$. Write each of the following vectors as a linear combination of **i** and **j.**

a) **u**

b) $\mathbf{w} = 2\mathbf{u} - 3\mathbf{v}$



-Note-

If θ is the angle between $\bar{v}$ and the positive x – axis then we can write

$$x = |\bar{v}|\cos\theta \text{ and } y = |\bar{v}|\sin\theta \,;\, |\bar{v}| = \sqrt{x_1^{\ 2} + y_2^{\ 2}}\ .$$

Example:

The vector **v** has a length of 3 and makes an angle of $30^o = \dfrac{\pi}{6}$ with the positive *x*-axis. Write **v** as a linear combination of the unit vectors **i** and **j**.