Ministry of Higher Education and Scientific Research
Almustaqbal University, College of Engineering
And Engineering Technologies
Computer Technology Engineering Department

# Four week :

# Reviews for Complex Numbers and there mathematical operations

Course Name : Fundamentals of Electricity
Stage : One
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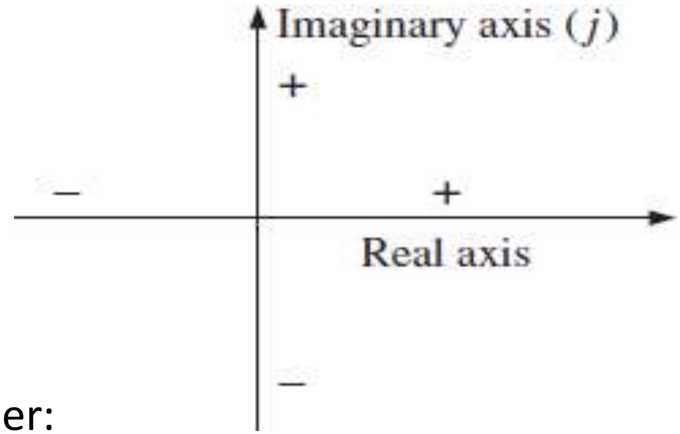
## 6. COMPLEX NUMBERS

A **complex number** represents a point in a two-dimensional plane located with reference to two distinct axes. This point can also determine a radius vector drawn from the origin to the point. The horizontal axis is called the real axis, while the vertical axis is called the imaginary axis.

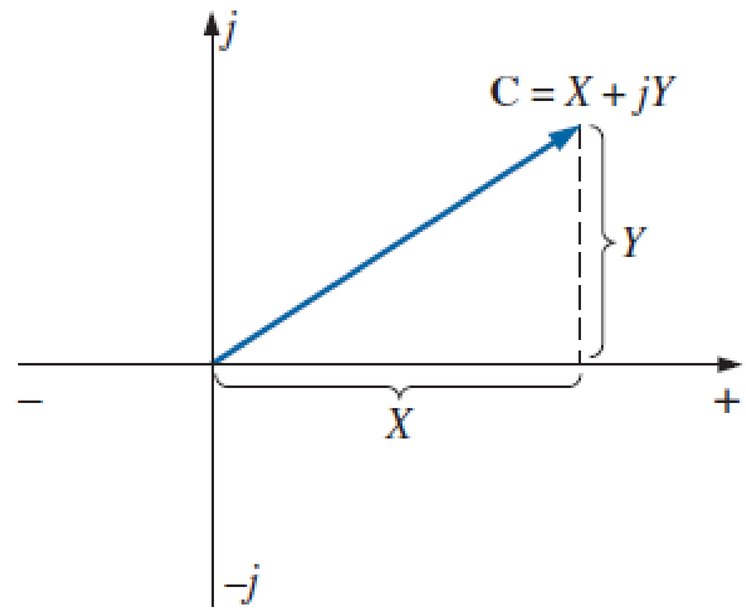



Two forms are used to represent a complex number: **rectangular and polar.** Each can represent a point in the plane or a radius vector drawn from the origin to that point.

## 7. RECTANGULAR FORM (CARTESIAN FORM)

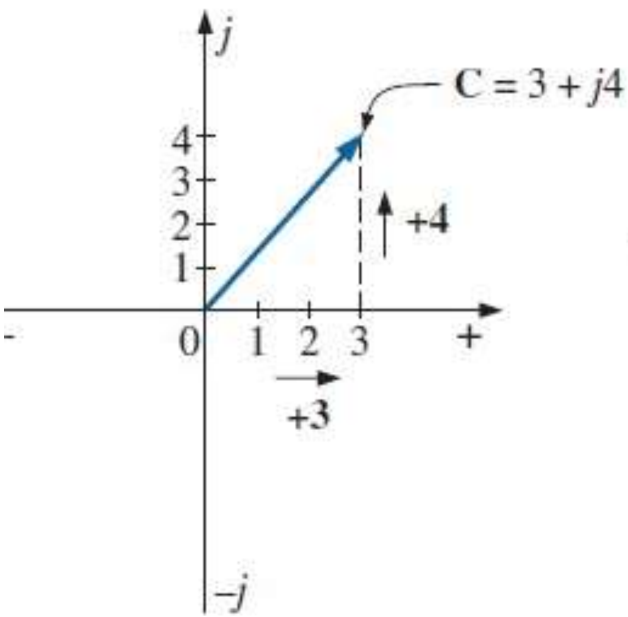
The format for the **rectangular form** is

$$\mathbf{C} = X + jY$$

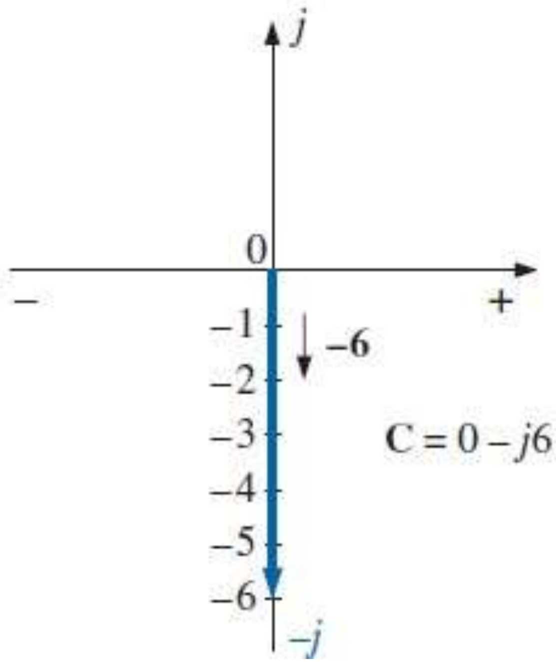


**EXAMPLE 14.13** Sketch the following complex numbers in the complex plane:

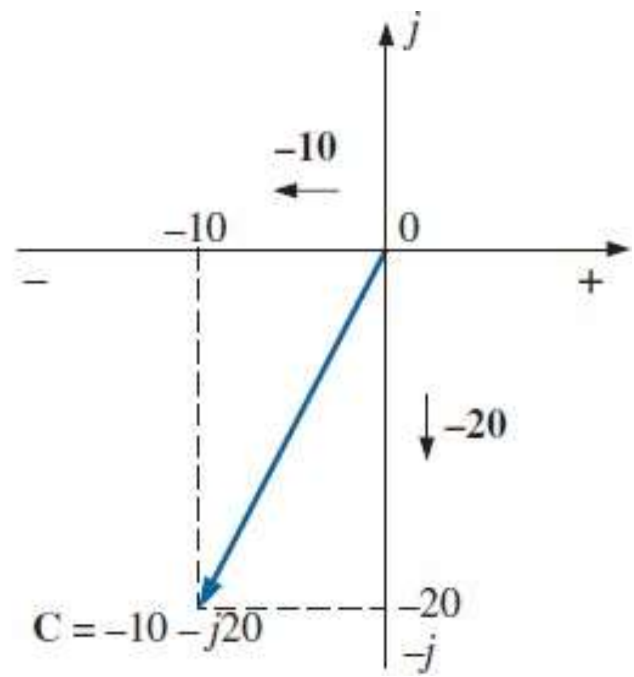
a. $\mathbf{C} = 3 + j4$
b. $\mathbf{C} = 0 - j6$
c. $\mathbf{C} = -10 - j20$




a. $\mathbf{C} = 3 + j4$




b. $\mathbf{C} = 0 - j6$

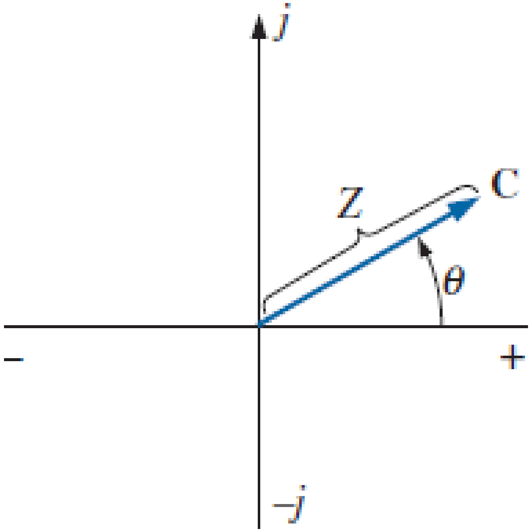



c. $\mathbf{C} = -10 - j20$

# 14.8 POLAR FORM

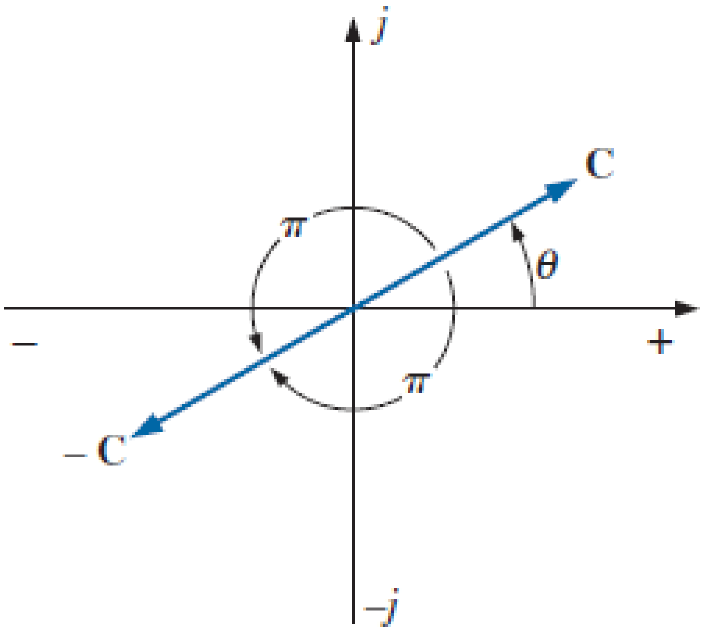
The format for the **polar form** is

$$\mathbf{C} = Z \angle \theta$$

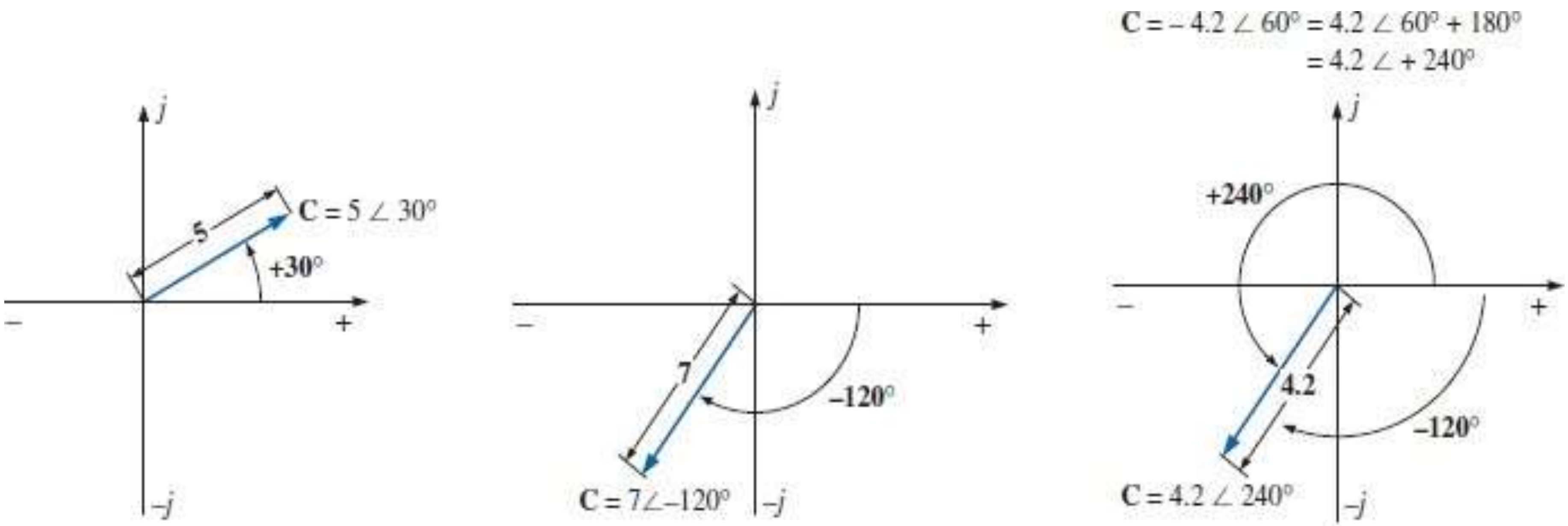



A negative sign in front of the polar form has the effect shown in Fig. Note that it results in a complex number directly opposite the complex number with a positive sign.

$$-\mathbf{C} = -Z \angle \theta = Z \angle \theta \pm 180^\circ$$



**EXAMPLE 14.14** Sketch the following complex numbers in the complex plane:




a. $\mathbf{C} = 5\angle 30^\circ$

b. $\mathbf{C} = 7\angle -120^\circ$

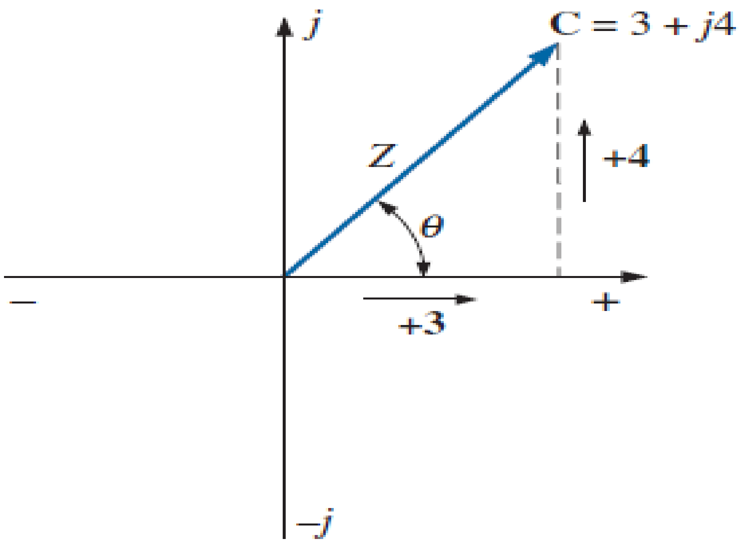
c. $\mathbf{C} = -4.2\angle 60^\circ$

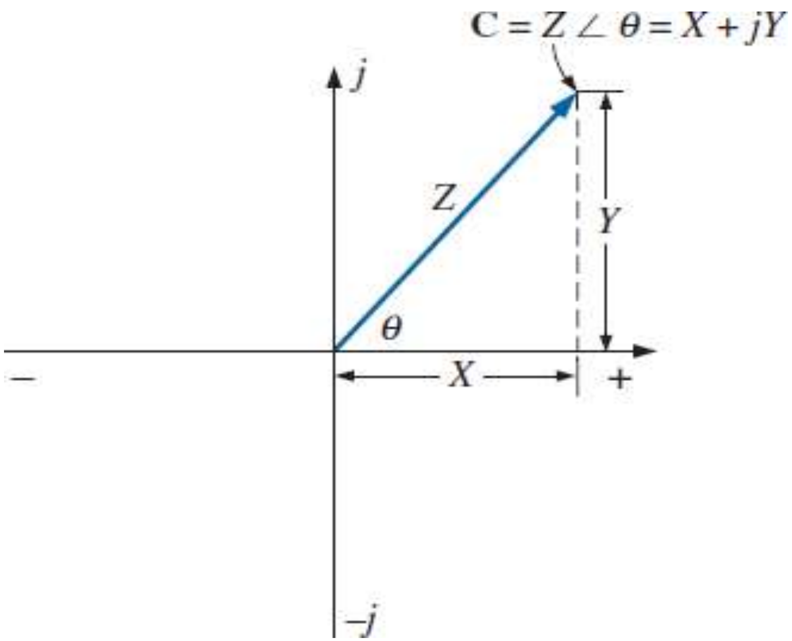
# 14.9 CONVERSION BETWEEN FORMS

## Rectangular to Polar

$$Z = \sqrt{X^2 + Y^2}$$

$$\theta = \tan^{-1} \frac{Y}{X}$$



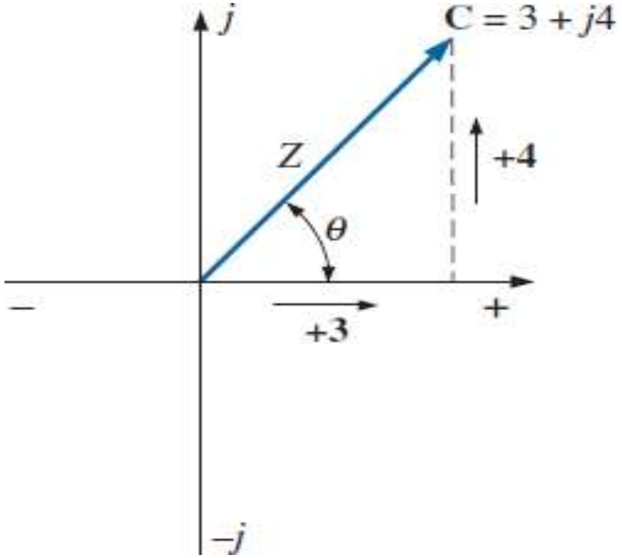

## Polar to Rectangular

$$X = Z \cos \theta$$

$$Y = Z \sin \theta$$

**EXAMPLE 14.15** Convert the following from rectangular to polar form:

$$\mathbf{C} = 3 + j4$$



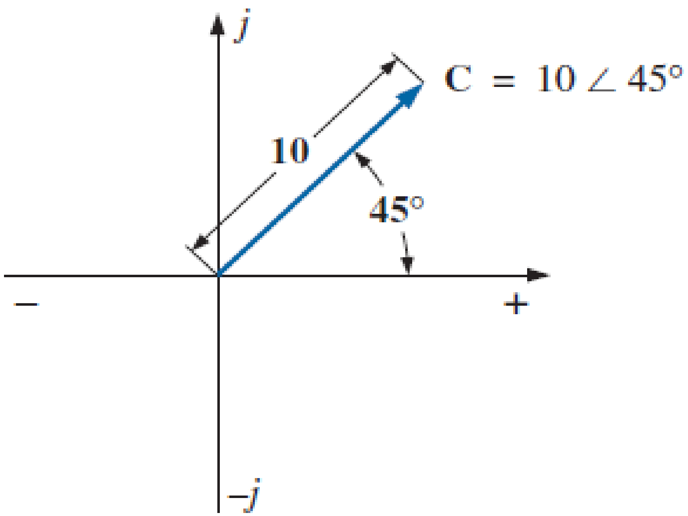

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

and $\mathbf{C} = \mathbf{5\angle 53.13^\circ}$

**EXAMPLE 14.16** Convert the following from polar to rectangular form:

$$\mathbf{C} = 10\,\angle 45^\circ$$




$$X = 10 \cos 45^\circ = (10)(0.707) = 7.07$$

$$Y = 10 \sin 45^\circ = (10)(0.707) = 7.07$$

and $\mathbf{C} = \mathbf{7.07 + j7.07}$

# 14.10 MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

$$j = \sqrt{-1}$$

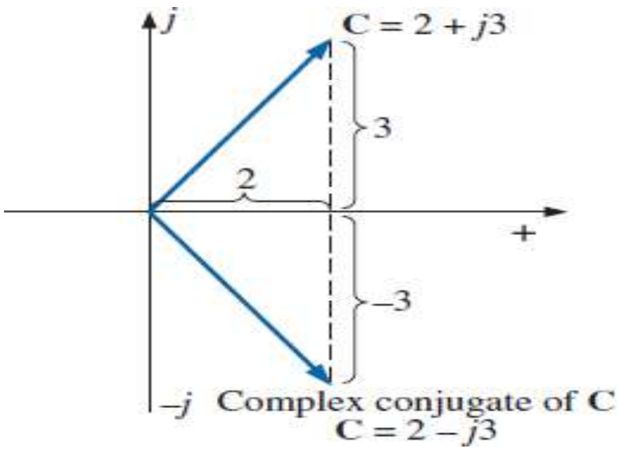
$$j^2 = -1$$

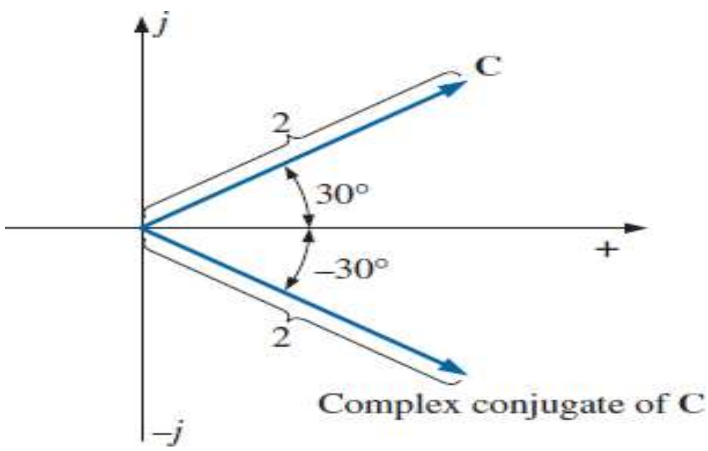
$$\frac{1}{j} = -j$$

$$\frac{1}{j} = (1)\left(\frac{1}{j}\right) = \left(\frac{j}{j}\right)\left(\frac{1}{j}\right) = \frac{j}{j^2} = \frac{j}{-1}$$

## Complex Conjugate

The **conjugate or complex conjugate** of a complex number can be found by simply changing the sign of the imaginary part in the rectangular form or by using the negative of the angle of the polar form.



## Addition

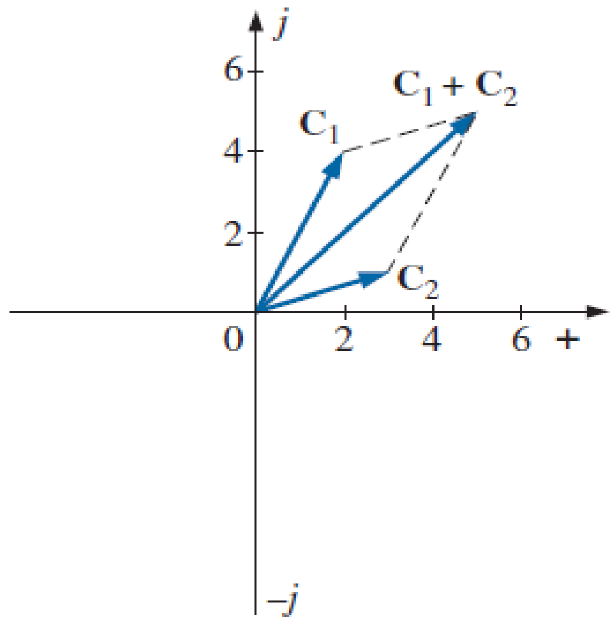
$$\mathbf{C}_1 = \pm X_1 \pm jY_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm jY_2$$

$$\boxed{\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)}$$

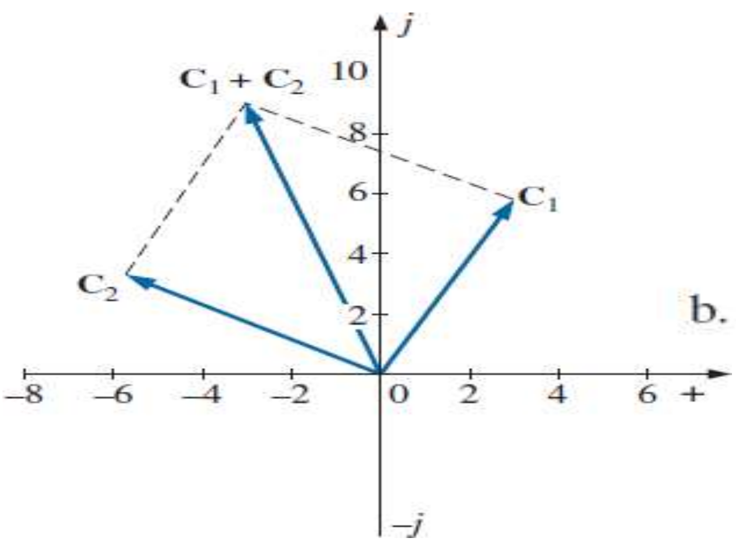
## EXAMPLE 14.19

a. Add $\mathbf{C}_1 = 2 + j4$ and $\mathbf{C}_2 = 3 + j1$.

b. Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.




a. $\mathbf{C}_1 + \mathbf{C}_2 = (2 + 3) + j(4 + 1) = \mathbf{5 + j5}$



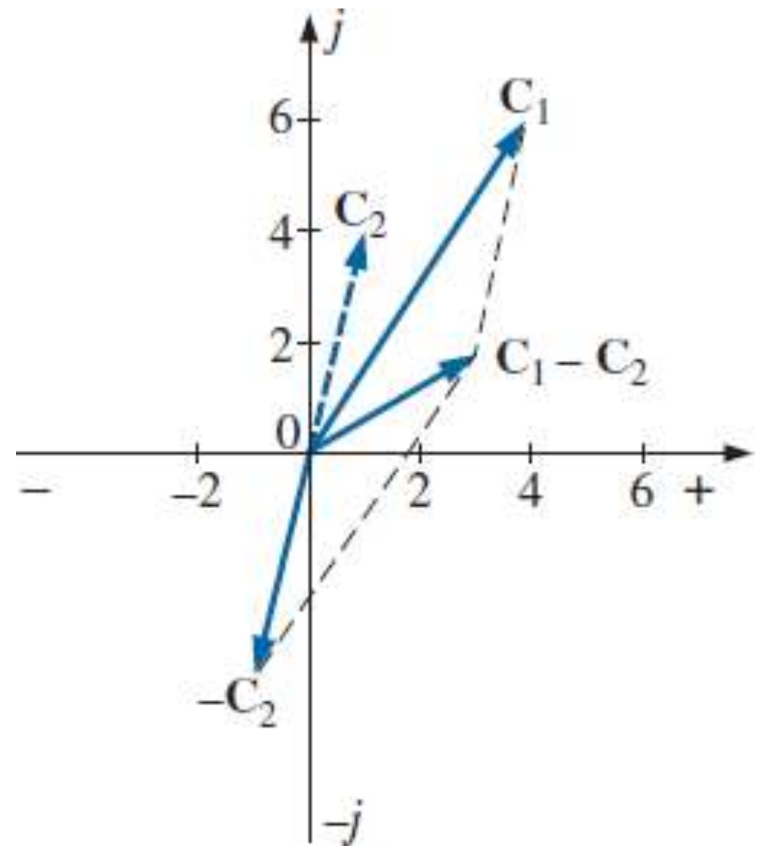

b. $\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + j9}$

**Subtraction** $\mathbf{C}_1 = \pm X_1 \pm jY_1$ and $\mathbf{C}_2 = \pm X_2 \pm jY_2$

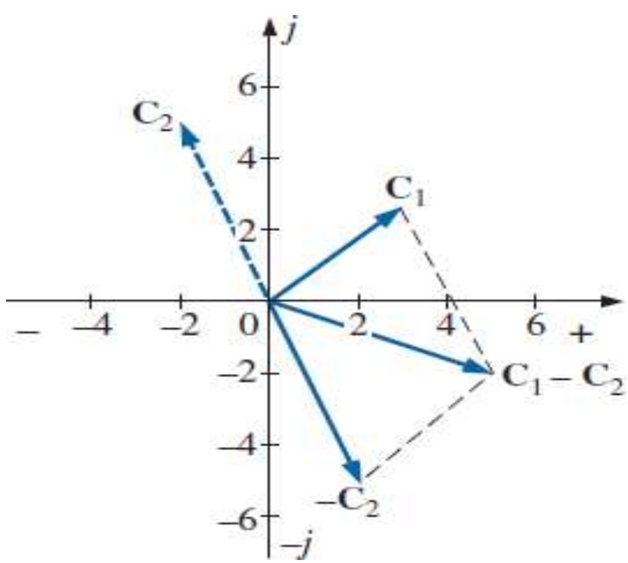
$$\mathbf{C}_1 - \mathbf{C}_2 = [\pm X_1 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)]$$

## EXAMPLE 14.20

a. Subtract $\mathbf{C}_2 = 1 + j4$ from $\mathbf{C}_1 = 4 + j6$.
b. Subtract $\mathbf{C}_2 = -2 + j5$ from $\mathbf{C}_1 = +3 + j3$.



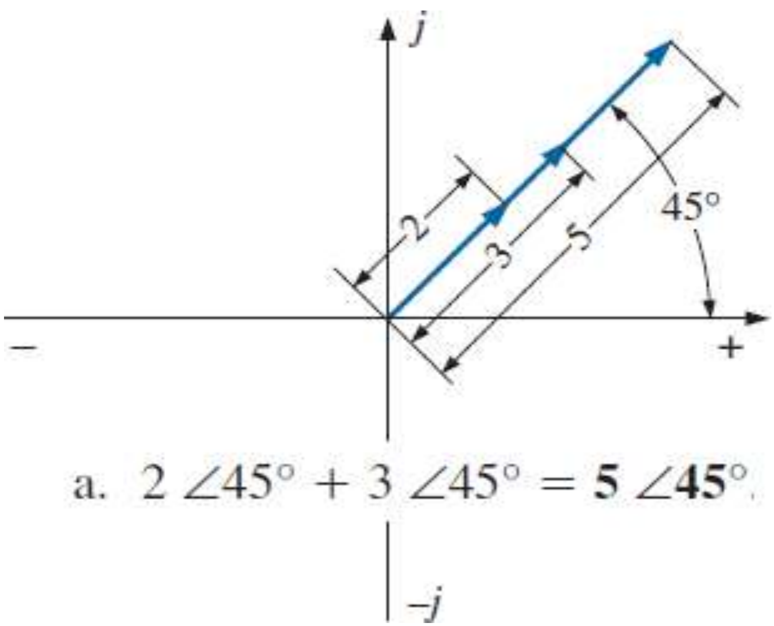

a. $\mathbf{C}_1 - \mathbf{C}_2 = (4 - 1) + j(6 - 4) = \mathbf{3 + j2}$

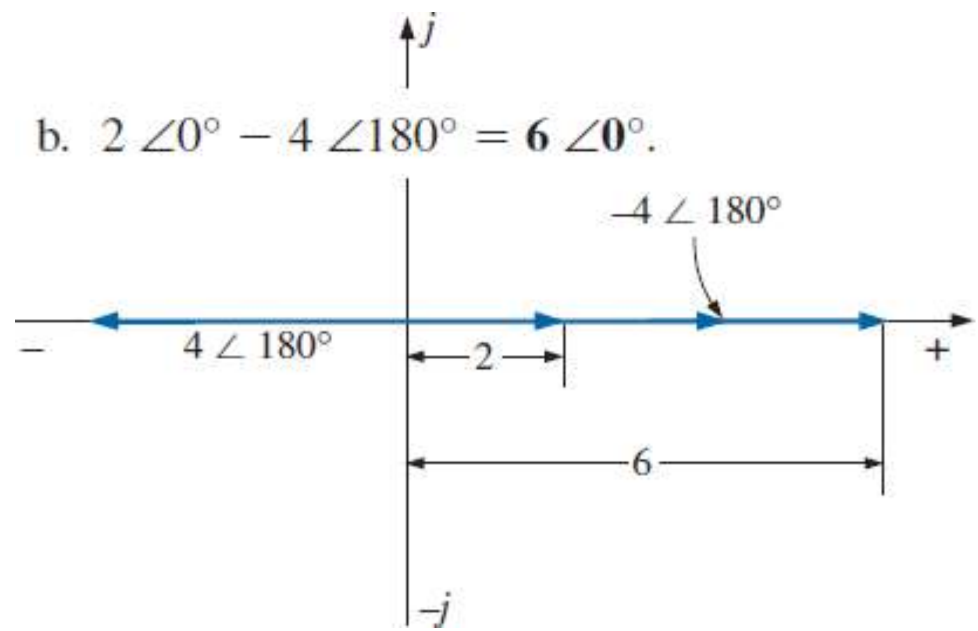



b. $\mathbf{C}_1 - \mathbf{C}_2 = [3 - (-2)] + j(3 - 5) = \mathbf{5 - j2}$

***Addition or subtraction cannot be performed in polar form unless the complex numbers have the same angle θ or unless they differ only by multiples of 180°.***

**EXAMPLE 14.21**



**Multiplication**

$$\mathbf{C}_1 = X_1 + jY_1 \quad \text{and} \quad \mathbf{C}_2 = X_2 + jY_2$$

then $\mathbf{C}_1 \cdot \mathbf{C}_2$:

$$\begin{array}{r} X_1 + jY_1 \\ \underline{X_2 + jY_2} \\ X_1X_2 + jY_1X_2 \\ \underline{\qquad\qquad + jX_1Y_2 + j^2Y_1Y_2} \\ X_1X_2 + j(Y_1X_2 + X_1Y_2) + Y_1Y_2(-1) \end{array}$$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)}$$

**EXAMPLE 14.22**

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if $\mathbf{C}_1 = 2 + j3$ and $\mathbf{C}_2 = 5 + j10$

a. Using the format above, we have

$$\begin{aligned} \mathbf{C}_1 \cdot \mathbf{C}_2 &= [(2)(5) - (3)(10)] + j[(3)(5) + (2)(10)] \\ &= \mathbf{-20 + j35} \end{aligned}$$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if $\mathbf{C}_1 = -2 - j3$ and $\mathbf{C}_2 = +4 - j6$

b. Without using the format, we obtain

$$\begin{array}{l} -2 - j3 \\ \underline{+4 - j6 \qquad\qquad} \\ -8 - j12 \\ \underline{\qquad + j12 + j^2 18 \quad} \\ -8 + j(-12 + 12) - 18 \end{array}$$

and $$\mathbf{C}_1 \cdot \mathbf{C}_2 = \mathbf{-26 = 26 \angle 180^\circ}$$

In *polar form, the magnitudes are multiplied and the angles added algebraically.* For example, for

$$\mathbf{C}_1 = Z_1 \angle\theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle\theta_2$$

we write

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1 Z_2 \angle \theta_1 + \theta_2}$$

**EXAMPLE 14.25**

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 15\angle 10°$ and $\mathbf{C}_2 = 2\angle 7°$.
b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 8\angle 120°$ and $\mathbf{C}_2 = 16\angle -50°$.

a. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{15\angle 10°}{2\angle 7°} = \frac{15}{2}\angle 10° - 7° = \mathbf{7.5\angle 3°}$$

b. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{8\angle 120°}{16\angle -50°} = \frac{8}{16}\angle 120° - (-50°) = \mathbf{0.5\angle 170°}$$

**EXAMPLE 14.26** Perform the following operations, leaving the answer in polar or rectangular form:

a. $$\frac{(2 + j3) + (4 + j6)}{(7 + j7) - (3 - j3)} = \frac{(2 + 4) + j(3 + 6)}{(7 - 3) + j(7 + 3)}$$

$$= \frac{(6 + j9)(4 - j10)}{(4 + j10)(4 - j10)}$$

**Complex Conjugate**

$$= \frac{[(6)(4) + (9)(10)] + j[(4)(9) - (6)(10)]}{4^2 + 10^2}$$

$$= \frac{114 - j24}{116} = \mathbf{0.98 - j0.21}$$

$$\text{b.}\quad \frac{(50\angle 30^\circ)(5+j5)}{10\angle -20^\circ} = \frac{(50\angle 30^\circ)(7.07\angle 45^\circ)}{10\angle -20^\circ} = \frac{353.5\angle 75^\circ}{10\angle -20^\circ}$$

$$= 35.35\,\underline{/75^\circ - (-20^\circ)} = \mathbf{35.35\angle 95^\circ}$$

$$\text{c.}\quad \frac{(2\angle 20^\circ)^2(3+j4)}{8-j6} = \frac{(2\angle 20^\circ)(2\angle 20^\circ)(5\angle 53.13^\circ)}{10\angle -36.87^\circ}$$

$$= \frac{(4\angle 40^\circ)(5\angle 53.13^\circ)}{10\angle -36.87^\circ} = \frac{20\angle 93.13^\circ}{10\angle -36.87^\circ}$$

$$= 2\,\underline{/93.13^\circ - (-36.87^\circ)} = \mathbf{2.0\angle 130^\circ}$$

$$\text{d.}\quad 3\angle 27^\circ - 6\angle -40^\circ = (2.673 + j1.362) - (4.596 - j3.857)$$

$$= (2.673 - 4.596) + j(1.362 + 3.857)$$

$$= \mathbf{-1.92 + j5.22}$$

# PROBLEMS

**SECTION 14.2 Derivative: 1, 3**

**SECTION 14.3 Response of Basic *R, L, and C* Elements to a Sinusoidal Voltage or Current:**
**4, 6, 8, 13, 15, 20**

**SECTION 14.4 Frequency Response of the Basic Elements: 22, 23, 25, 27**

**SECTION 14.5 Average Power and Power Factor: 30, 31, 34, 37, 38**

**SECTION 14.9 Conversion between Forms: 39, 40**

**SECTION 14.10 Mathematical Operations with Complex Numbers:**
**43, 44, 45, 46, 47**

THANK YOU