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Department of

Electrical Engineering Techniques

AC ELECTRICAL CIRCUITS

Asst. Lec.

ALI IMAD

AC Fundamentals

1- Sinusoidal AC Voltage:

The voltages of ac sources alternate in polarity and vary in magnitude and thus produce currents that vary in magnitude and alternate in direction.

The sinusoidal ac waveform has the shape shown in Figure 1. One complete variation is referred to as a **cycle.** Since the waveform repeats itself at regular intervals as in (b), it is called a **periodic** waveform.

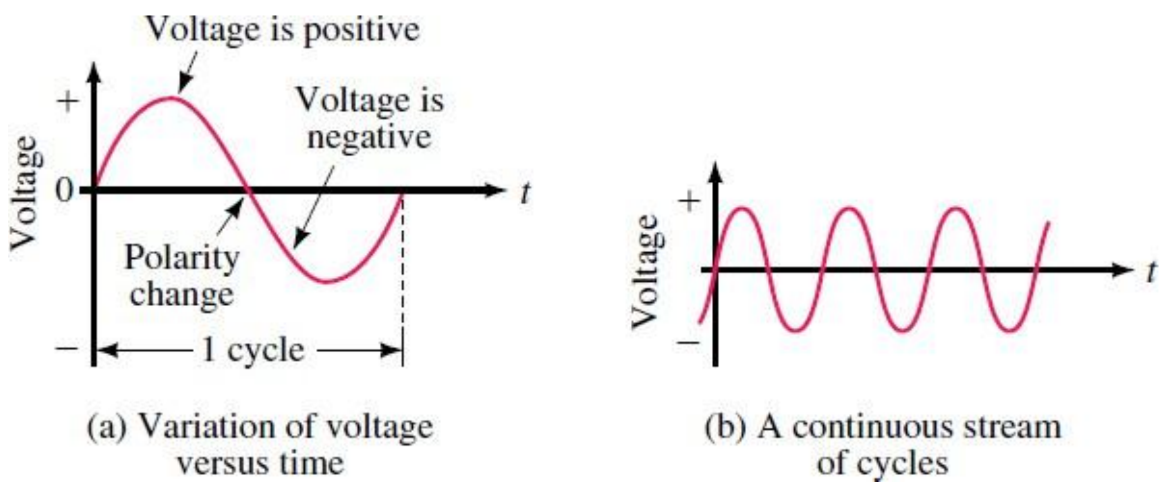


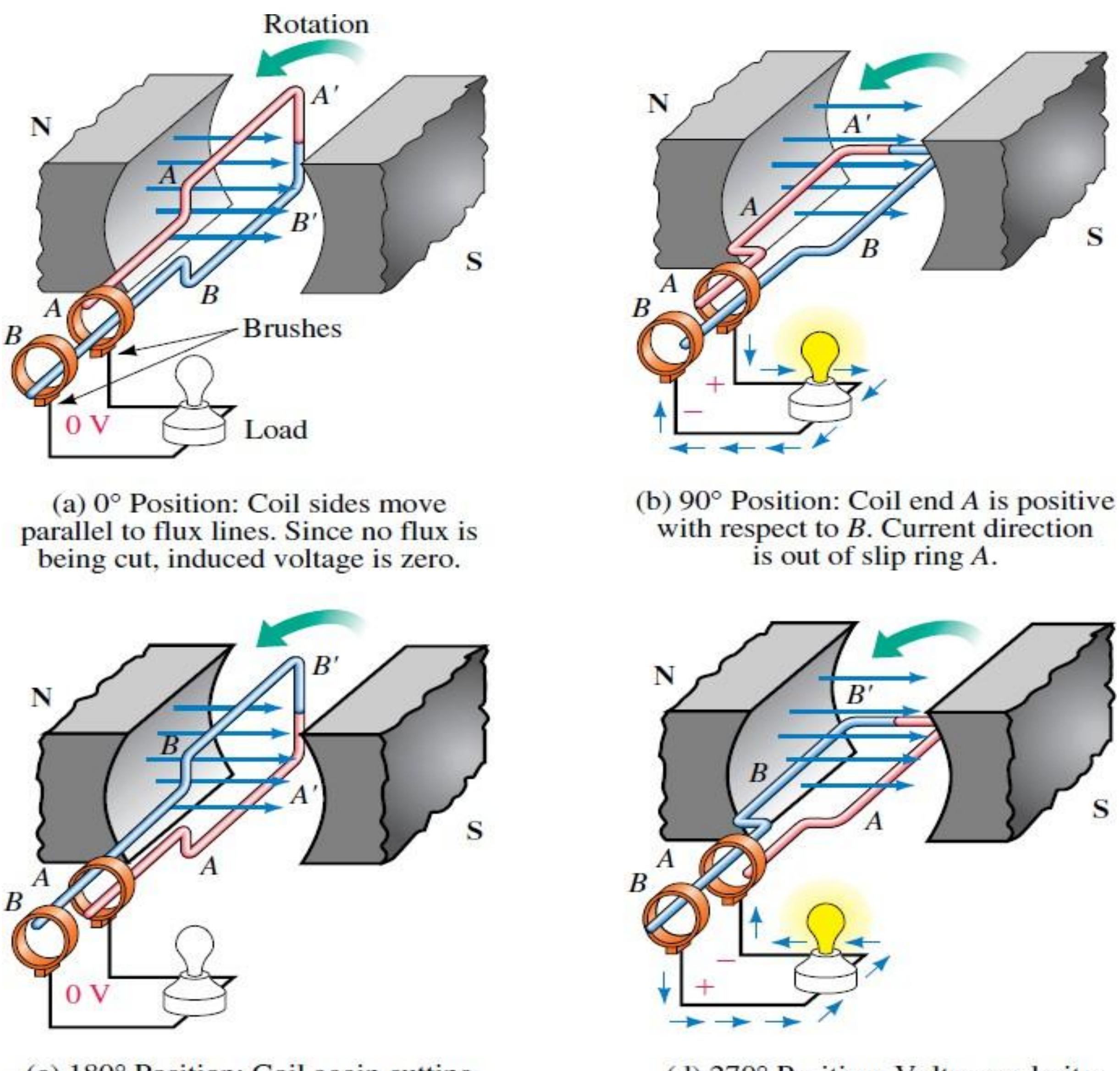
FIGURE 1. sinusoidal ac waveforms. Values above the axis are positive while values below are negative.

2-Generating AC Voltages:

One way to generate an ac voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field, Figure 2. (Slip rings and brushes connect the coil to the load.) The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut

(Faraday's law), and its polarity is dependent on the direction the coil sides move through the field. Since the coil rotates continuously, the voltage produced will be a repetitive, periodic waveform as you saw in Figure 3.

The time for one revolution of 600 rpm is one tenth of a second, i.e., 100 ms.



(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.

(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.

(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.

(d) 270° Position: Voltage polarity has reversed, therefore, current direction reverses.

FIGURE 2. Generating an ac voltage. The 0° position of the coil is defined as in (a) where the coil sides move parallel to the flux lines

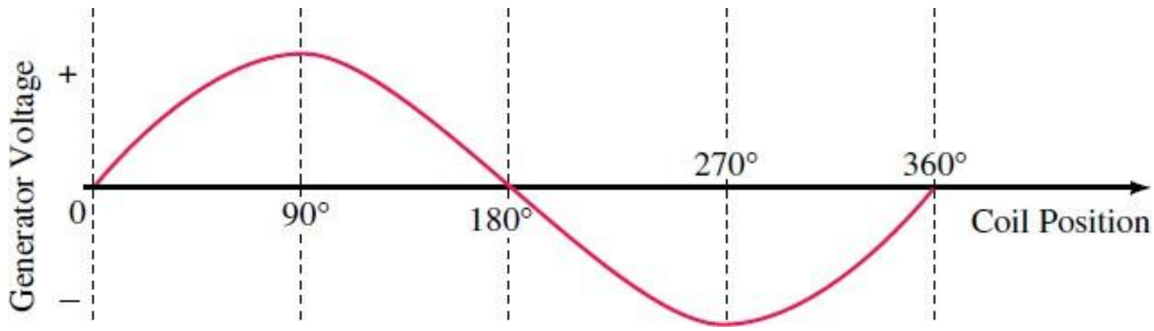


FIGURE 3. Coil voltage versus angular position.

As Figure 4 shows, the coil voltage changes from instant to instant. The value of voltage at any point on the waveform is referred to as its **instantaneous value.**

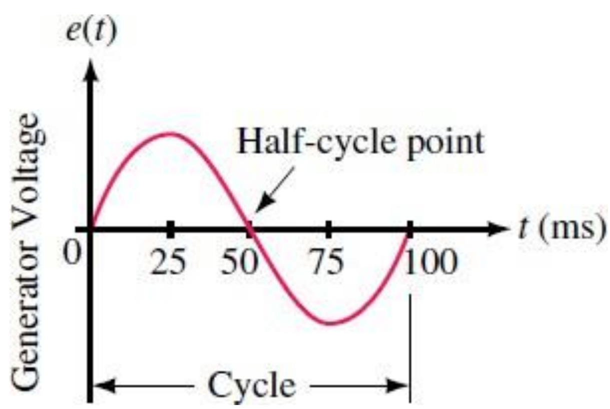


FIGURE 4 Cycle scaled in time. At 600 rpm, the cycle length is 100 ms.

3-Frequency, Period, Amplitude, and Peak Value:

The number of cycles per second of a waveform is defined as its **frequency.** its unit is the **hertz (Hz).**

1 Hz = 1 cycle per second.

The **period,** *T*, of a waveform, (Figure 15–15) is the duration of one cycle. It is the inverse of frequency.

$$T = \frac{1}{f} \ s$$

$$f = \frac{1}{T} \ \ Hz$$

Note that these definitions are independent of wave shape.

The **amplitude** of a sine wave is the distance from its average to its peak E_m. **Peak-to-peak voltage** is measured between minimum and maximum peaks denoted $V_{p\text{-}p}$.

For the case of DC voltage E added to the sine wave the peak value is $E+E_m$.

4- **Angular and Graphic Relationships for Sine Waves**

Consider again the generator of Figure 2, reoriented and redrawn in end view as Figure 5.

The voltage produced by this generator is

$$e = E_m sin\alpha$$

where E_m is the maximum coil voltage and a is the instantaneous angular position of the coil.

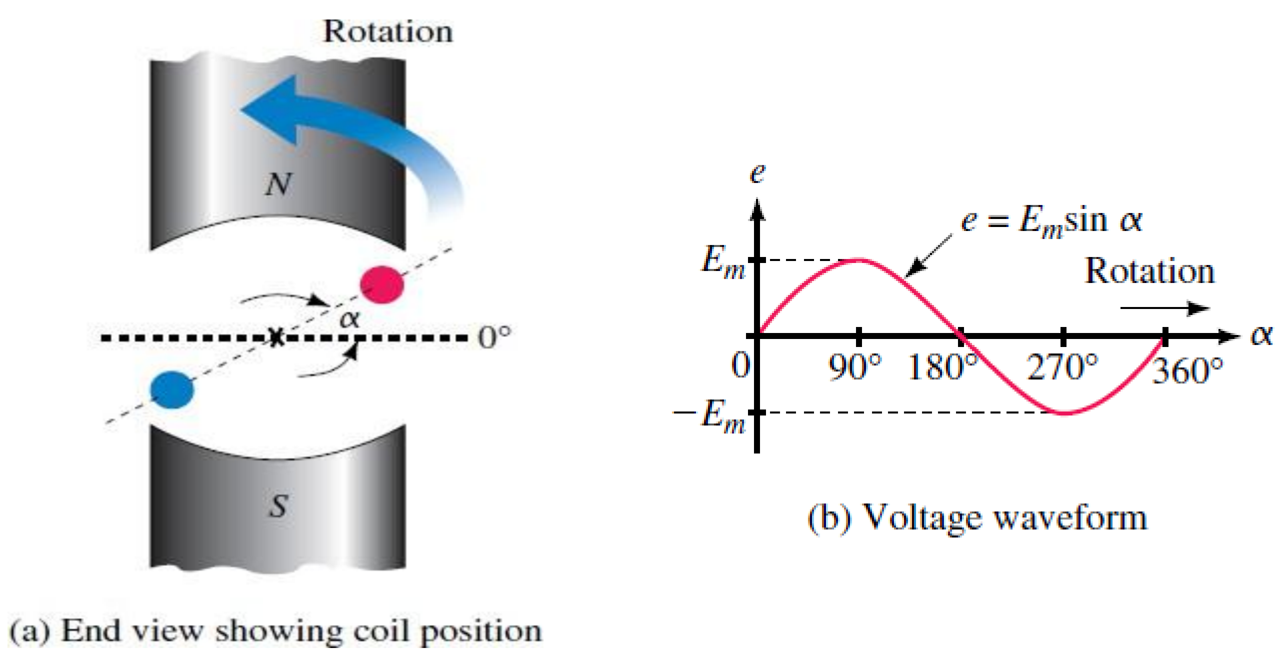


FIGURE 5 Coil voltage versus angular position.

Not that if E=100V, then e=0, 50, and -100V for α=0°, 30°, and 270°. The rate at which the generator coil rotates is called its **angular velocity,** denoted by the Greek letter *w* (omega), (Normally angular velocity is expressed in radians per second instead of degrees per second. In general,

$$\alpha = wt, \; and \; w = \frac{\alpha}{t}$$

$$2\pi \text{ radians} = 360^{\circ}$$

$$\alpha_{radians} = \frac{\pi}{180} \times \alpha_{degree}$$

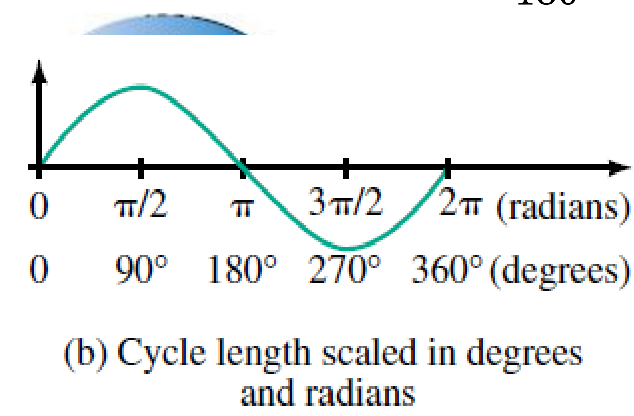


Figure 6

$$wT = 2\pi \quad (rad)$$

$$w = \frac{2\pi}{T} \quad (\frac{rad}{s})$$

Recall, $f = 1/T$ Hz.

$$w = 2\pi f \quad (\frac{rad}{s})$$

$$e = E_m \sin wt$$

$$v = V_m \sin wt$$

$$i = I_m \sin wt$$

If a sine wave does not pass through zero at t =0 s as in Figure 7, it has a **phase shift.**

(a) $v = V_m \sin(\omega t + \theta)$ (b) $v = V_m \sin(\omega t - \theta)$

Figure 7

5-Phasor Diagram:

A **phasor** is a rotating line whose projection on a vertical axis can be used to represent sinusoidally varying quantities.

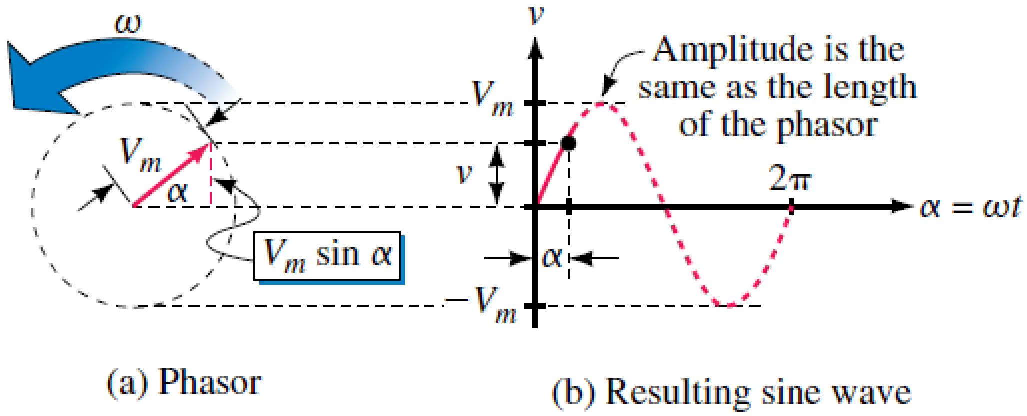


Figure 7 As the phasor rotates about the origin, its vertical projection creates a sine wave.

Phase difference refers to the angular displacement between different waveforms of the same frequency. Consider Figure 8. If the angular displacement is 0° as in (a), the waveforms are said to be **in phase;** otherwise, they are **out of phase.** When describing a phase difference, select one waveform as reference. Other waveforms then lead, lag, or are in phase with this reference. For example, in (b), for reasons to be discussed in the next paragraph, the current waveform is said to lead the voltage waveform, while in (c) the current waveform is said to lag.

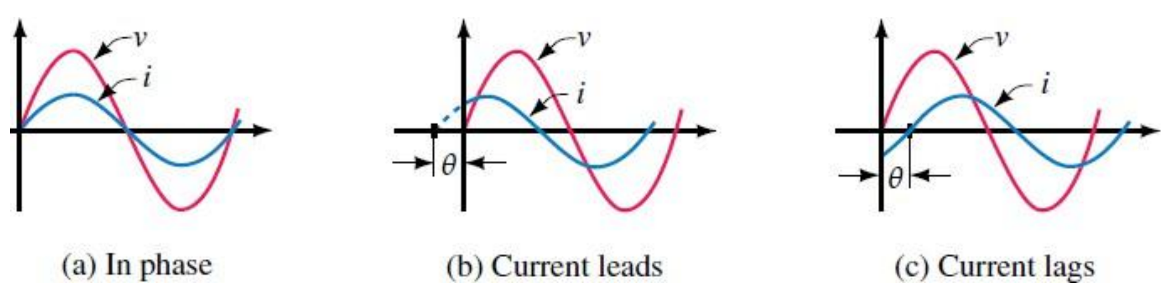


Figure 8

Sometimes voltages and currents are expressed in terms of *cos wt* rather than *sin wt*, a cosine wave is a sine wave shifted by +90°, or alternatively, a sine wave is a cosine wave shifted by -90°. For sines or cosines with an angle, the following formulas apply.

$$cos(wt+\theta)=sin(wt+\theta+90)$$

$$sin(wt+\theta)=cos(wt+\theta-90)$$

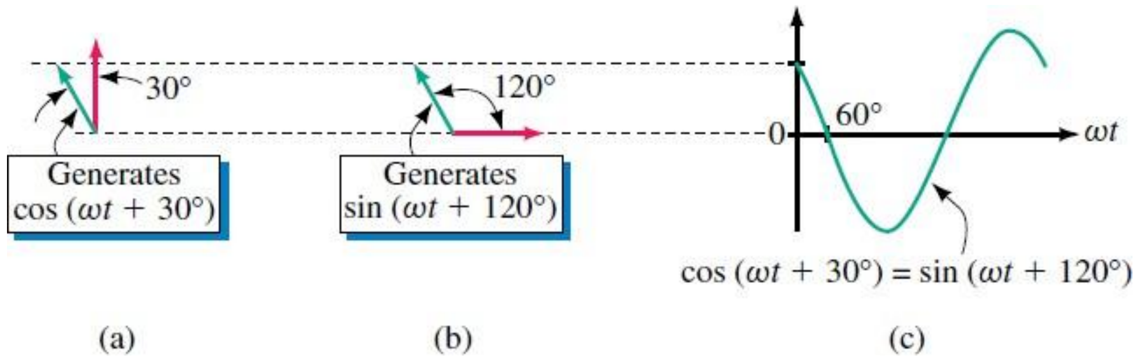
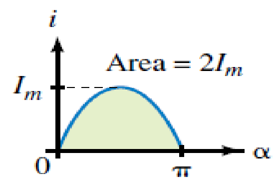


Figure 9

6-AC Waveforms and Average Value:

Ac quantities are generally described by a group of characteristics, including instantaneous, peak, average, and effective values. to find the average value of a waveform, divide the area under the waveform by the length of its base. Areas above the axis are counted as positive, while areas below the axis are counted as negative Because a sine wave is symmetrical, its area below the horizontal axis is the same as its area above the axis; thus, over a full cycle its net area is zero, independent of frequency and phase angle. Thus, the average of sin wt, sin(wt ± θ), sin 2wt, cos wt, cos(wt ± θ), cos 2wt, and so on are each zero. The average of half a sine wave, however, is not zero. Consider Figure 10. The area under the half-cycle may be found using calculus as



$$\text{area} = \int_0^{\pi} I_m \sin\alpha \, d\alpha = -I_m \cos\alpha \Big|_0^{\pi} = 2I_m$$

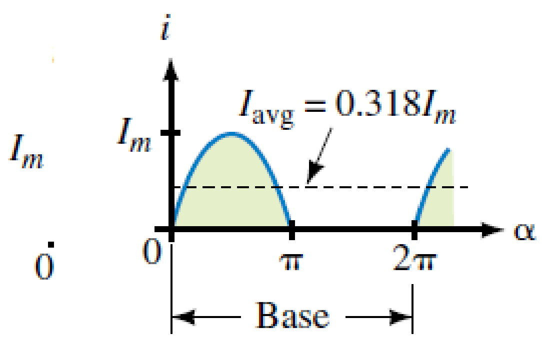
Figure 10

Two cases are important; full-wave average and half-wave average. The full-wave case is illustrated in Figure 11-a. The area from 0 to 2π is 2(2*Im*) and the base is 2π. Thus, the average is

$$I_{avg} = \frac{2(2I_m)}{2\,\pi} = \frac{2I_m}{\pi} = 0.637 I_m$$

For the half-wave case (Figure 11-b),

$$I_{avg} = \frac{2I_m}{2\,\pi} = \frac{I_m}{\pi} = 0.318 I_m$$



(a) **(b)**

Figure (11)

The corresponding expressions for voltage are

$$V_{avg} = 0.637 V_m \quad \text{(full-wave)}$$
$$V_{avg} = 0.318 V_m \quad \text{(half-wave)}$$

Sometimes ac and dc are used in the same circuit. Figure 12 shows superimposed ac and dc. it does not alternate in polarity since it never changes polarity to become negative.

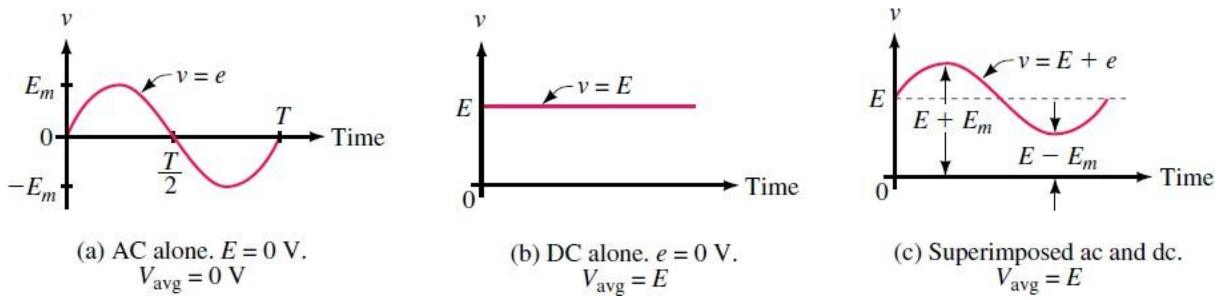


Figure 12

7-
EffectiveValues

An effective value is an equivalent dc value: it tells you how many volts or amps of dc that a time-varying waveform is equal to in terms of its ability to produce average power Effective values are also called **rms values** for reasons discussed shortly.

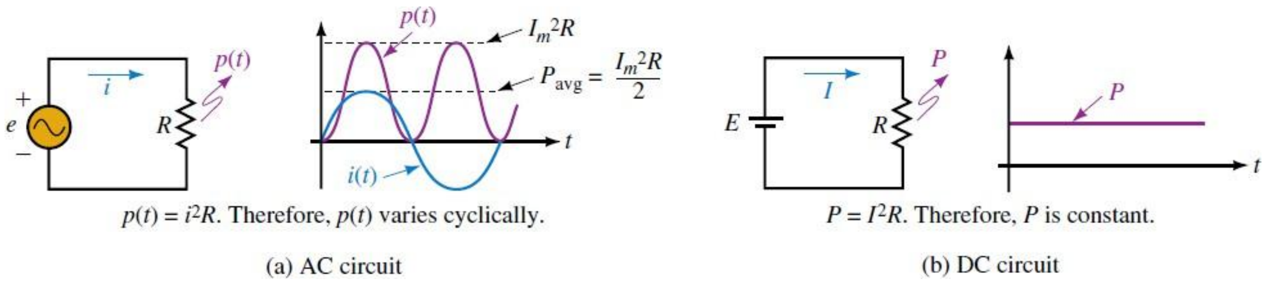


Figure 13

First, consider the dc case. Since current is constant, power is constant, and average power is

$$P_{avg} = P = I^2R$$

Now consider the ac case. Power to the resistor at any value of time is $p(t)=i^2R$, where i is the instantaneous value of current.

Since $i = I_m \sin wt$,

$$\begin{aligned} p(t) &= i^2R \\ &= (I_m \sin \omega t)^2R = I_m{}^2R \sin^2\omega t \\ &= I_m{}^2R\left[\frac{1}{2}(1 - \cos 2\omega t)\right] \end{aligned}$$

$$p(t) = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t$$

note that the average of cos 2w*t* is zero

$$P_{avg} = \text{average of } p(t) = \frac{I_m^2 R}{2}$$

Equate two eq.

$$I^2 = \frac{I_m^2}{2}$$

$$I = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$I_{eff} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$E_{eff} = \frac{E_m}{\sqrt{2}} = 0.707 E_m$$

$$V_{eff} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

The $\sqrt{2}$ relationship holds only for sinusoidal waveforms. For other waveforms, you need a more general formula. Using calculus, it can be shown that for any waveform

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

The integral of i^2 represents the area under the i^2 waveform. Thus,

$$I_{eff} = \sqrt{\frac{\text{area under the } i^2 \text{curve}}{\text{base}}}$$

To use this equation, we compute the root of the mean square to obtain the effective value. For this reason, effective values are called **root mean square** or **rms** values and **the terms *effective* and *rms* are synonymous.**