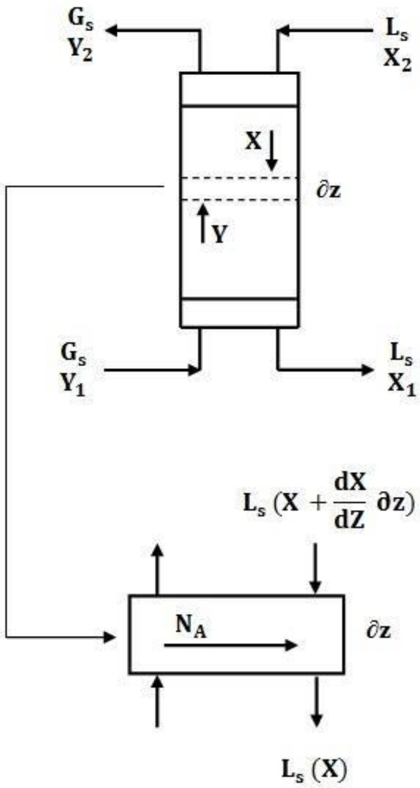
Calculations of the packing height based on liquid phase:

Overall material balance on the solute (A) over an element (∂z) based on liquid phase:

$$\mathbf{G_S\ dY = L_S\ dX = N_A .\ A}$$

$$\mathbf{N_A = L_s\left(X + \frac{dX}{dZ}\,\partial z\right) - L_s\,X = (KoL)(a\ \ S\,\partial z)(X^* - X)}$$



Where:

$$\mathbf{The\ interficial\ area\ for\ transfer = a\ dV = a\ \ S\,\partial z}$$

S: is the cross-sectional area of column (m^2).

a: is the surface area of interface per unit volume of column (m^2/m^3).

$$\mathbf{L_s\left(\frac{dX}{dZ}\,\partial z\right) = (KoL.a)(S\,.\,\partial z)(X^* - X)}$$

$$\mathbf{L_s\,\frac{dX}{dZ} = (KoL.a)(S\,.\,\partial z)(X^* - X)}$$

$$\mathbf{\int_0^Z dZ = \frac{L_S}{(KoL.a).S}\int_{X_2}^{X_1}\frac{dX}{(X^* - X)}}$$

$$\mathbf{Z = \frac{(L_S\,/S)}{KoL.a}\int_{X_2}^{X_1}\frac{dX}{(X^* - X)}}$$

$$\boxed{\mathbf{Z = \frac{L_S}{KoL.a}\int_{X_2}^{X_1}\frac{dX}{(X^* - X)}}}$$

Z = HOL * NOL = HTU * NTU

Where:

$$\mathbf{HOL} = \frac{\bar{L}_s}{KoL.a}$$: heiht of transfer unit (HTU) based on liquid phase, with the units of (m).

$$\mathbf{NOL} = \int_{X_2}^{X_1} \frac{dX}{(X^* - X)}$$: number of transfer unit (NTU) based on liquid phase, without units.

Calculation of Number of Transfer Unit (NOL):

A. For Linear Equilibrium Relationship $(Y = m\,X^*)$:

$$\mathbf{NOL} = \int_{X_2}^{X_1} \frac{dX}{(X^* - X)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\,.\,(1)$$

$$X^* = \frac{Y}{m} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$G_s\,(Y - Y_2) = L_s\,(X - X_2) \qquad \ldots\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\,.\,(3)$$

$$\Longrightarrow \quad Y = \frac{L_s}{G_s}(X - X_2) + Y_2$$

For pure liquid solvent used then, $\mathbf{X_2 = 0}$

$$Y = \frac{L_s}{G_s}X + Y_2 \qquad \ldots\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\,.\,(4)$$

Substitution Eq.(4) into Eq.(2) to get:

$$X^* = \frac{L_s}{mG_s}X + \frac{Y_2}{m} = \frac{1}{\phi}X + \frac{Y_2}{m} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Substitution Eq.(5) into Eq.(1) to get:

$$\mathbf{NOL} = \frac{}{(1}$$

$$\mathbf{NOL = \phi\ NOG}$$

Where: $\phi = \frac{mG_s}{L_s}$

B. For Non-linear Equilibrium Relationship:

In this case the integration [$NOL = \int_{X_2}^{X_1} \frac{dX}{(X^* - X)}$] will be solved using graphical method or

numerical method (Simpson rule) following steps below:

1. Draw the given equilibrium data.
2. Draw the operating line, from two points **(X_1, Y_1)** and **(X_2, Y_2)** or one point and slope of ($\frac{L_s}{G_s}$).
3. Create the table below by calculated (**X^***) from the plot as below:

X Assume points between (X_1 - X_2)	**X^*** Calculated from plot	$\frac{1}{(X^* - X)}$
X_1	- calculated	√ = f_0
- (assumed)	- calculated	√ = f_1
- (assumed)	- calculated	√ = f_2
- (assumed)	- calculated	√= f_3
X_2	- calculated	√ = f_n

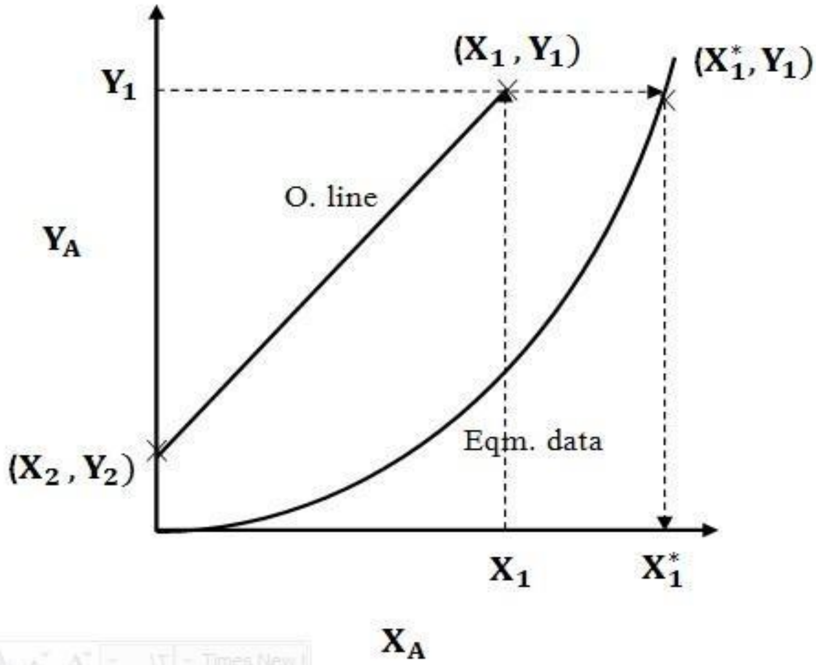
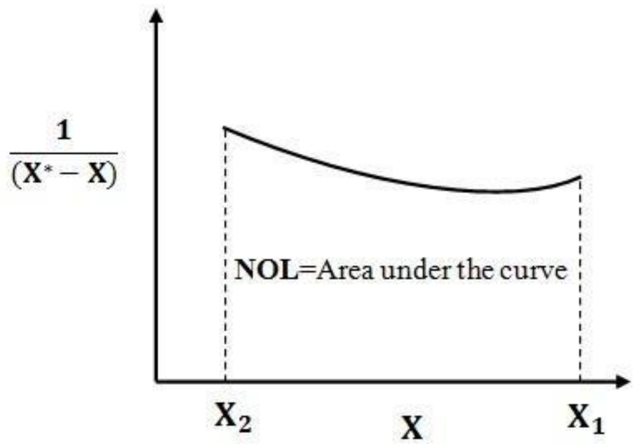


Figure: Calculation of (X^*) for packed column.

4. To calculate **NOL** we draw $[\frac{1}{(X^*-X)}]$ Vs. [X] to find the area under the curve:



Where:

NOL = Area under the curve

Simpson rule for calculation of NOL:

NOL = Area under the curve

$$NOL = \frac{h}{3} \quad f_0 + f_n + 2 \quad f_{even} + 4 \quad f_{odd}$$