## Calculation of Number of Transfer Unit (NOG):

A. For Linear Equilibrium Relationship $\left(\mathbf{Y}^{*}=\mathbf{m} \mathbf{X}\right)$ :
$\mathrm{NOG}=\int_{\mathbf{Y}_{2}}^{\mathrm{Y}_{1}} \frac{\mathrm{dY}}{\left(\mathbf{Y}-\mathbf{Y}^{*}\right)}$
$\mathbf{Y}^{*}=\mathbf{m X}$
$\mathbf{G}_{\mathbf{s}}\left(\mathbf{Y}-\mathbf{Y}_{2}\right)=\mathbf{L}_{\mathbf{s}}\left(\mathbf{X}-\mathbf{X}_{\mathbf{2}}\right)$
$\Longrightarrow \quad \mathbf{X}=\frac{\mathbf{G}_{\mathbf{s}}}{\mathbf{L}_{\mathbf{s}}}\left(\mathbf{Y}-\mathbf{Y}_{\mathbf{2}}\right)+\mathbf{X}_{\mathbf{2}}$
For pure liquid solvent used then, $\mathbf{X}_{\mathbf{2}}=\mathbf{0}$
$\mathbf{X}=\frac{\mathbf{G}_{\mathbf{s}}}{\mathbf{L}_{\mathbf{s}}}\left(\mathbf{Y}-\mathbf{Y}_{\mathbf{2}}\right)$
Substitution Eq.(4) into Eq.(2) to get:
$\mathbf{Y}^{*}=\frac{\mathbf{m} \mathbf{G}_{\mathbf{s}}}{\mathbf{L}_{\mathbf{s}}}\left(\mathbf{Y}-\mathbf{Y}_{2}\right)$
Substitution Eq.(5) into Eq.(1) to get:
$N O G=\int_{\mathbf{Y}_{\mathbf{2}}}^{\mathbf{Y}_{1}} \frac{\mathbf{d Y}}{\left(\mathbf{Y}-\frac{\mathbf{m} \mathbf{G}_{\mathbf{s}}}{\mathbf{L}_{\mathbf{s}}}\left(\mathbf{Y}-\mathbf{Y}_{2}\right)\right)}$
Let: $\quad \frac{\mathbf{m} \mathbf{G}_{\mathbf{s}}}{\mathbf{L}_{\mathbf{s}}}=\boldsymbol{\phi}=\frac{\text { Slope of equilibrium line }}{\text { Slope of operating line }}=\frac{\mathbf{m}}{\mathbf{L}_{\mathbf{s}} / \mathbf{G}_{\mathbf{s}}}<1.0$
$N O G=\int_{Y_{2}}^{\mathrm{Y}_{1}} \frac{\mathrm{dY}}{Y-\phi Y+\phi Y_{2}}$
$N O G=\int_{Y_{2}}^{\mathrm{Y}_{1}} \frac{\mathrm{dY}}{(1-\phi) Y+\phi Y_{2}}$
$N O G=\frac{1}{(1-\phi)} \ln \left[\frac{(1-\phi) Y_{1}+\phi Y_{2}}{(1-\phi) Y_{2}+\phi Y_{2}}\right]$

## B. For Non-linear Equilibrium Relationship:

In this case the integration [ $\mathrm{NOG}=\int_{\mathbf{Y}_{\mathbf{2}}}^{\mathbf{Y}_{\mathbf{1}}} \frac{\mathbf{d Y}}{\left(\mathbf{Y}-\mathbf{Y}^{*}\right)}$ ] will be solved using graphical method or
numerical method (Simpson rule) following steps below:

1. Draw the given equilibrium data.
2. Draw the operating line, from two points $\left(\mathbf{X}_{\mathbf{1}}, \mathbf{Y}_{\mathbf{1}}\right)$ and $\left(\mathbf{X}_{\mathbf{2}}, \mathbf{Y}_{\mathbf{2}}\right)$ or one point and slope of
$\left(\frac{L_{s}}{G_{s}}\right)$.
3. Create the table below by calculated $\left(\mathbf{Y}^{*}\right)$ from the plot as below:

| $\mathbf{Y}$ |  |  |
| :---: | :---: | :---: |
| Assume points between $\left(\mathrm{Y}_{1}-\mathrm{Y}_{2}\right)$ | $\mathbf{Y}_{*}$ |  |
| Calculated from plot | $\frac{\mathbf{1}}{\left(\mathbf{Y}-\mathbf{Y}^{*}\right)}$ |  |
| $\mathrm{Y}_{1}$ | - calculated | $\sqrt{ }=\mathrm{f}_{0}$ |
| - (assumed) | - calculated | $\sqrt{ }=\mathrm{f}_{1}$ |
| - (assumed) | - calculated | $\sqrt{ }=\mathrm{f}_{2}$ |
| - (assumed) | - calculated | $\sqrt{ }=\mathrm{f}_{3}$ |
| $\mathrm{Y}_{2}$ | - calculated | $\sqrt{ }=\mathrm{f}_{\mathrm{n}}$ |



Figure: Calculation of ( $\mathbf{Y}^{*}$ ) for packed column.
4. To calculate NOG we draw $\left[\frac{\mathbf{1}}{\left(\mathbf{Y}-\mathbf{Y}^{*}\right)}\right]$ Vs. [Y] to find the area under the curve:

Where:


Simpson rule for calculation of NOG:

Y
NOG = Area under the curve

NOG = Area under the curve
NOG $=\frac{h}{3} \quad f_{0}+f_{n}+2 \quad f_{\text {even }}+4 \quad f_{\text {odd }}$

Where:

$$
h=\frac{Y_{1}-Y_{2}}{n} \quad, \quad n=2,4,6,8, \ldots \ldots . \text { etc }
$$

## Notes:

* If the entering solute concentration is dilute ( $\mathrm{Y}<5 \%$ ), then:

$$
\mathbf{Y}_{\mathbf{A}}=\mathbf{y}_{\mathbf{A}} \quad, \quad \mathbf{X}_{\mathbf{A}}=\mathbf{x}_{\mathbf{A}} \quad, \quad \mathbf{G}_{\mathbf{s}}=\mathbf{G} \quad, \quad \mathbf{L}_{\mathbf{s}}=\mathbf{L}
$$

* If the tower type is not mention in the problem we can take it as a packed tower.


## Example (1):

Ammonia is to be removed from a 10 percent ammonia-air mixture by countercurrent scrubbing with water in a packed tower at 293 K so that 99 percent of the ammonia is removed when working at a total pressure of $101.3 \mathrm{kN} / \mathrm{m}^{2}$. If the gas rate is 0.95 $\mathrm{kg} / \mathrm{m}^{2}$.s of tower cross-section and the liquid rate is $0.65 \mathrm{~kg} / \mathrm{m}^{2}$. s , find the necessary height of the tower if the absorption coefficient KoG. $\mathrm{a}=0.008 \mathrm{kmol} / \mathrm{m}^{3} . \mathrm{s} . \mathrm{kPa}$., The equilibrium data are:
$\mathrm{Y}^{*}=0.8 \mathrm{X}$.

## Solution:

$\mathrm{y}_{2}=(1-$ recovery $) \mathrm{y}_{1}=(1-0.99)(0.1)=0.001$

## Convert mole fraction to mole ratio:

$\mathbf{Y}_{\mathbf{1}}=\frac{\mathrm{y}_{1}}{1-\mathrm{y}_{1}}=\frac{0.1}{1-0.1}=0.11$
$\mathbf{Y}_{2}=\frac{\mathrm{y}_{2}}{1-\mathrm{y}_{2}}=\frac{0.001}{1-0.001}=0.001$
We can see that at low conc. (mole ratio = mole fraction):

The gas mole flux, $\overline{\mathbf{G}}=\frac{\text { gas mass flux }}{\text { average gas molecular weight }}$

$$
=\frac{0.95}{[(0.1)(17)+(0.9)(29)]} \quad=0.0341 \frac{\mathrm{kmol}}{\mathrm{~m}^{2} . \mathrm{s}}
$$

The liquid mole flux, $\overline{\mathbf{L}}=\frac{\text { liquid mass flux }}{\text { average liquid molecular weight }}$

$$
=\frac{0.65}{(18)}=0.0361 \frac{\mathrm{kmol}}{\mathrm{~m}^{2} . \mathrm{s}}
$$

the mole flux of the inert gas,$\overline{\mathbf{G}}_{\mathbf{s}}=\overline{\mathbf{G}}\left(\mathbf{1}-\mathbf{y}_{\mathbf{1}}\right)=(0.0341)(1-0.1)=0.0307 \frac{\mathrm{kmol}}{\mathrm{m}^{2} . \mathrm{s}}$ the mole flux of the inert liquid, $\overline{\mathbf{L}}_{\mathbf{s}}=\overline{\mathbf{L}}\left(\mathbf{1}-\mathbf{x}_{\mathbf{2}}\right)=(0.0361)(1-0)=0.0361 \frac{\mathrm{kmol}}{\mathrm{m}^{2} . \mathrm{s}}$ Therefore, for pure solvent: $\quad \overline{\mathbf{L}}_{\mathbf{s}}=\overline{\mathbf{L}}$

$$
\mathrm{HOG}=\frac{\overline{\mathrm{G}}_{\mathrm{s}}}{\text { KoG. a. } \mathrm{P}_{\mathrm{T}}}=\frac{0.0307}{(0.0008)(101.3)}=0.38 \mathrm{~m}
$$

## Since the equilibrium is linear:

$$
\phi=\frac{\mathrm{m} \overline{\mathrm{G}}_{\mathrm{s}}}{\overline{\mathrm{~L}}_{\mathrm{s}}}=\frac{(0.8)(0.0307)}{(0.0361)}=0.68
$$

$$
\mathbf{Z}=\mathbf{H O G} * \mathbf{N O G}=(0.38)(11.19)=4.25 \mathrm{~m}
$$

