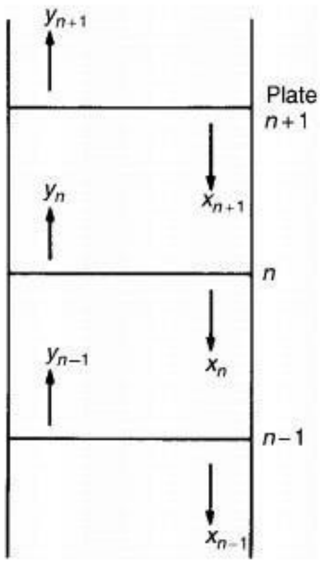
Calculation of Number of theoretical Trays (N):

A. For Linear Equilibrium Relationship $\mathbf{Y = m\,X}$:



Solute material balance over tray (n):

$$G_s\,Y_{n-1} + L_s\,X_{n+1} = G_s\,Y_n + L_s\,X_n \qquad \ldots\ldots\ldots(1)$$

The equilibrium relation is:

$$Y = m\,X \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Substitute Eq.(2) in to Eq.(1) to get:

$$G_s\,Y_{n-1} + \frac{L_s}{m}\,Y_{n+1} = G_s\,Y_n + \frac{L_s}{m}\,Y_n$$

$$G_s\,Y_{n-1} + \frac{L_s}{m}\,Y_{n+1} = \left(G_s + \frac{L_s}{m}\right)Y_n$$

$$\mathbf{Y_{n+1}} - \left(\frac{m\,G_s}{L_s} + 1\right)\mathbf{Y_n} + \frac{m\,G_s}{L_s}\,\mathbf{Y_{n-1}} = 0$$

Where:

$$\frac{m\,G_s}{L_s} = \boldsymbol{\phi}$$

$$\mathbf{Y_{n+1}} - (\mathbf{1} + \boldsymbol{\phi})\,\mathbf{Y_n} + \boldsymbol{\phi}\,\mathbf{Y_{n-1}} = 0$$

By using E-operator:

$$E\,\mathbf{Y_n} - (\mathbf{1} + \boldsymbol{\phi})\,\mathbf{Y_n} + \boldsymbol{\phi}\,E^{-1}\,\mathbf{Y_n} = 0 \qquad \ldots\ldots\ldots\ldots\ldots(3) \qquad \text{multiply by (E)}$$

$$(E^2 - (\mathbf{1} + \boldsymbol{\phi})\,E + \boldsymbol{\phi})\,\mathbf{Y_n} = 0$$

Change (E) symbol by (ρ):

$$\rho^2 - (\mathbf{1} + \boldsymbol{\phi})\,\rho + \boldsymbol{\phi} = 0$$

$$(\rho - 1)\,(\rho - \phi) = 0$$

The equation roots are: $\rho_1 = 1$

and $\rho_2 = \phi$

The general solution is:

$$Y_n = c_1 \, \rho_1^n + c_2 \, \rho_2^n$$

Substitute the equation roots in to the general solution to get:

$$\mathbf{Y_n = c_1 + c_2 \, \phi^n}$$

$$\boxed{\mathbf{n = \frac{\ln \frac{Y_n - c_1}{c_2}}{\ln \phi}}}$$

To find the total number of trays, we substitute (n) by (N) to get:

$$\boxed{\mathbf{N = \frac{\ln \frac{Y_N - c_1}{c_2}}{\ln \phi}}}$$

To find the equation constants **C_1** and **C_2** we substitute the boundary conditions:

$$\mathbf{B.C.1: \quad at \quad n = 0 \quad \rightarrow \quad Y_n = Y_0}$$
$$\mathbf{B.C.2: \quad at \quad n = 1 \quad \rightarrow \quad Y_n = Y_1 \quad \rightarrow \quad Y_1 = m\,X_1}$$

$$\mathbf{Y_n = c_1 + c_2 \, \phi^n}$$

$$\mathbf{B.C.1: \quad Y_0 = c_1 + c_2 \, \phi^0 \quad \rightarrow \quad Y_0 = c_1 + c_2} \quad \dots\dots..(1)$$
$$\mathbf{B.C.2: \quad Y_1 = c_1 + c_2 \, \phi^1 \quad \rightarrow \quad m\,X_1 = c_1 + c_2 \, \phi} \quad \dots\dots..(2)$$

From Eq.(1) and Eq.(2) we get:

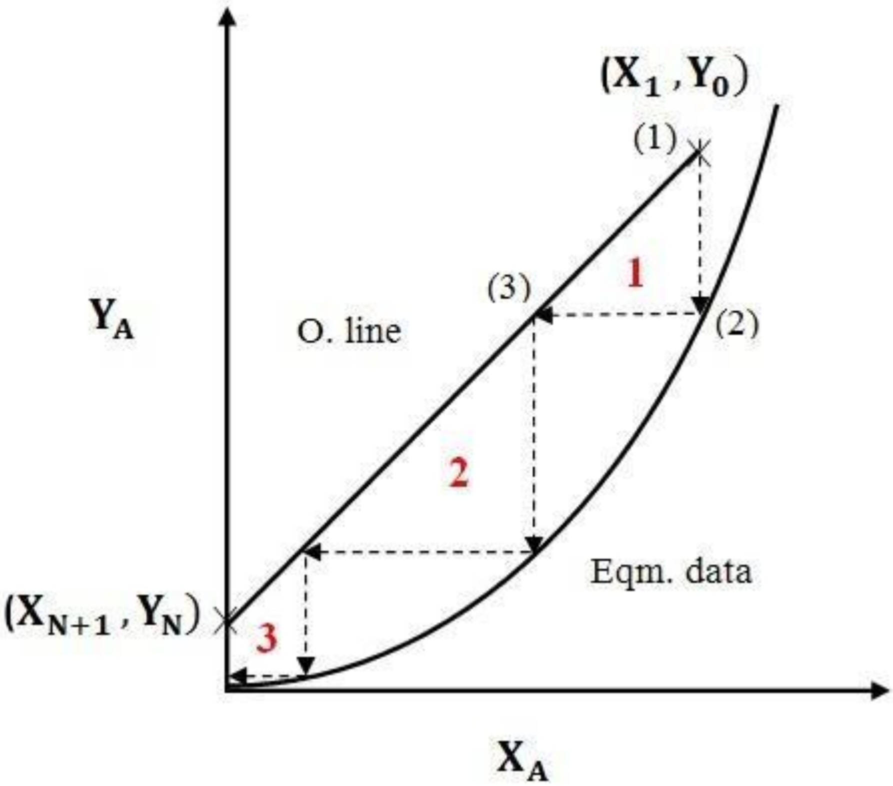
$$\boxed{c_2 = \frac{Y_0 - m\,X_1}{1 - \phi}}$$

$$\boxed{c_1 = Y_0 - c_2}$$

B. For Non-linear Equilibrium Relationship (Graphical method):

In this case the number of theoretical plates will be calculated using graphical method following steps below:

1. Complete the material balance to calculate all the unknowns (all compositions and flow rates of the inlet and the outlet streams must be known).
2. Draw the equilibrium curve (or line) either from given data or from the equilibrium equation: $Y = m\,X$.
3. Draw the operating line, from two points **(X_1, Y_0)** and **(X_{N+1}, Y_N)** or one point and slope of $\left(\frac{L_s}{G_s}\right)$ according to the condition of the process.
4. Draw a vertical line from **point 1** which represents the point **(X_1, Y_0)** {as shown in the figure} to **point 2** which will intersect the equilibrium line (Curve). Then draw a horizontal line from **point 2** to **point 3**, intersecting the operating line. The triangular formed will represent the plate number one.
5. Continue drawing the vertical lines and horizontal lines as in step 4 (shown in the fig.) until we reach to the point **(X_{N+1}, Y_N)** or pass it.
6. Count the triangles constructed, this number represents the number of theoretical plates.



(X_1, Y_0)
(1)
(2)
(3)
1
2
3
Y_A
O. line
Eqm. data
(X_{N+1}, Y_N)
X_A