

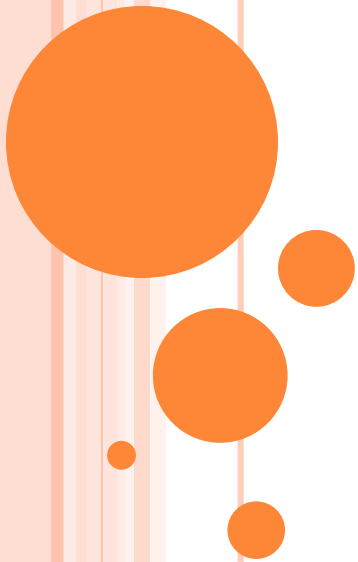
BIOPROCESS EQUIPMENT DESIGN



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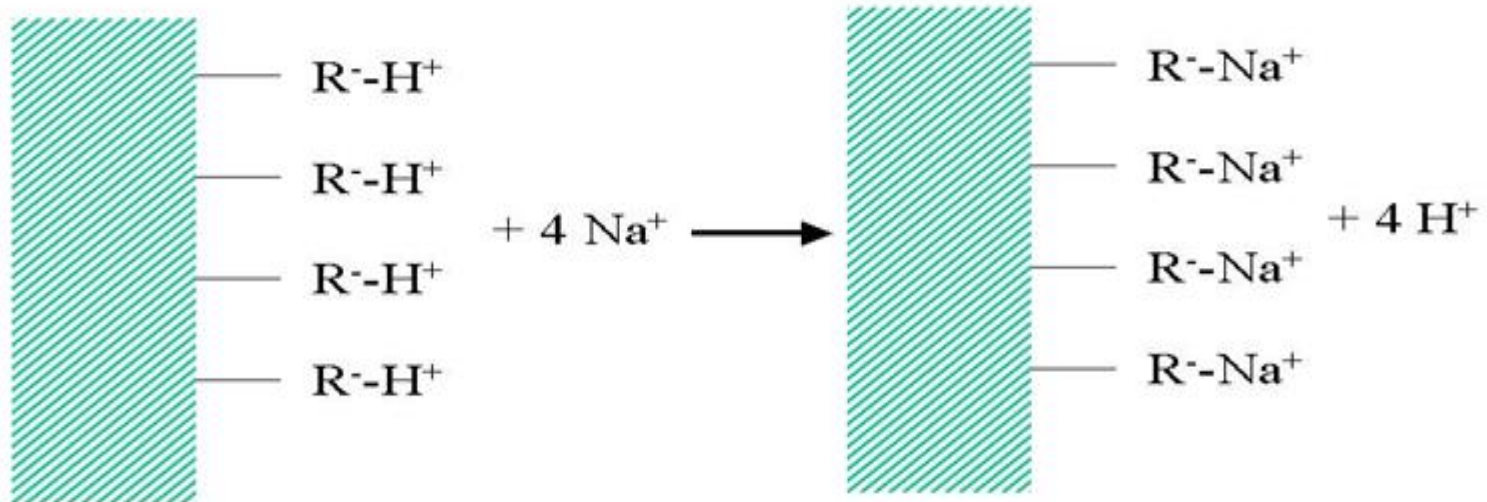
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ION EXCHANGE



Ion Exchange

Ion exchange is an adsorption phenomenon where the mechanism of adsorption is electrostatic. Electrostatic forces hold ions to charged functional groups on the surface of the ion exchange resin. The adsorbed ions replace ions that are on the resin surface on a 1:1 charge basis.

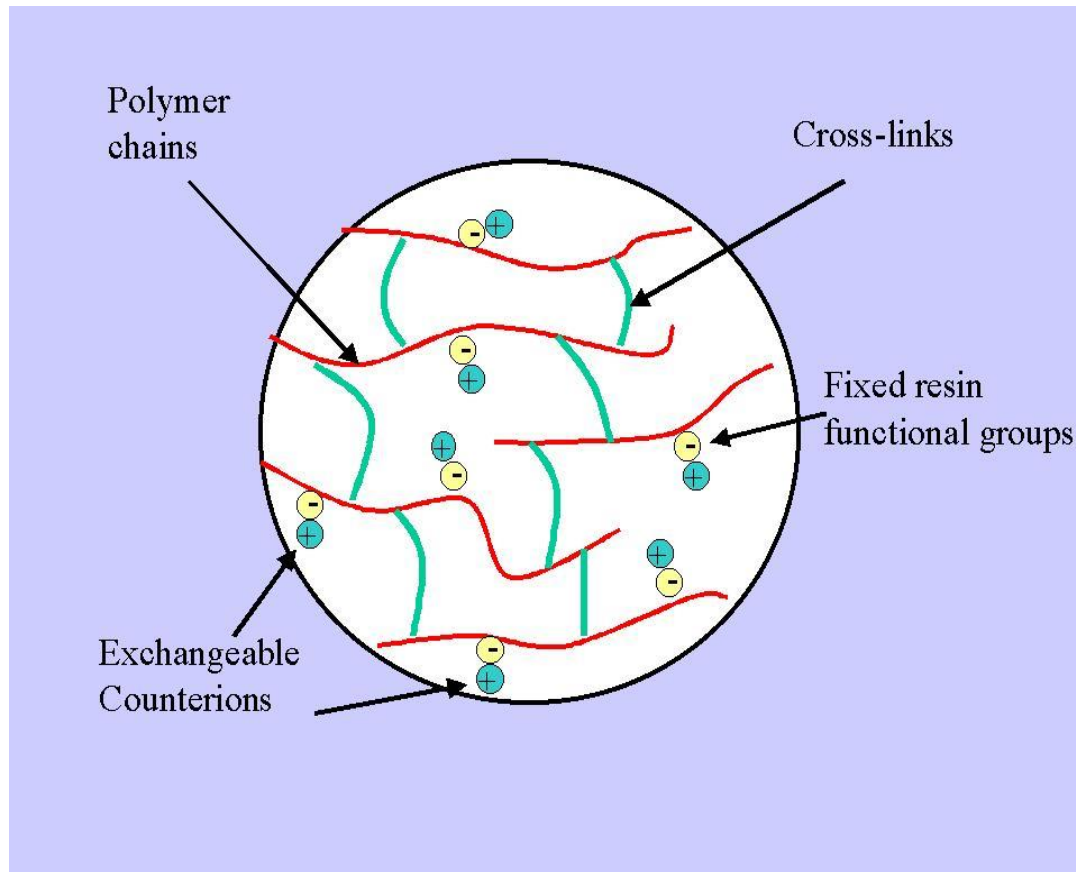


Applications of ion exchange in water & wastewater

- Ca, Mg (hardness removal) exchange with Na or H.
- Fe, Mn removal from groundwater.
- Recovery of valuable waste products Ag, Au, U
- Demineralization (exchange all cations for H all anions for OH)
- Removal of NO_3 , NH_4 , PO_4 (nutrient removal).
- Natural: Proteins, Soils, Lignin, Coal, Metal oxides, Alumino silicates (zeolites) ($\text{NaOAl}_2\text{O}_3 \cdot 4\text{SiO}_2$).
- Synthetic zeolite gels and polymeric resins (macro reticular, large pores).

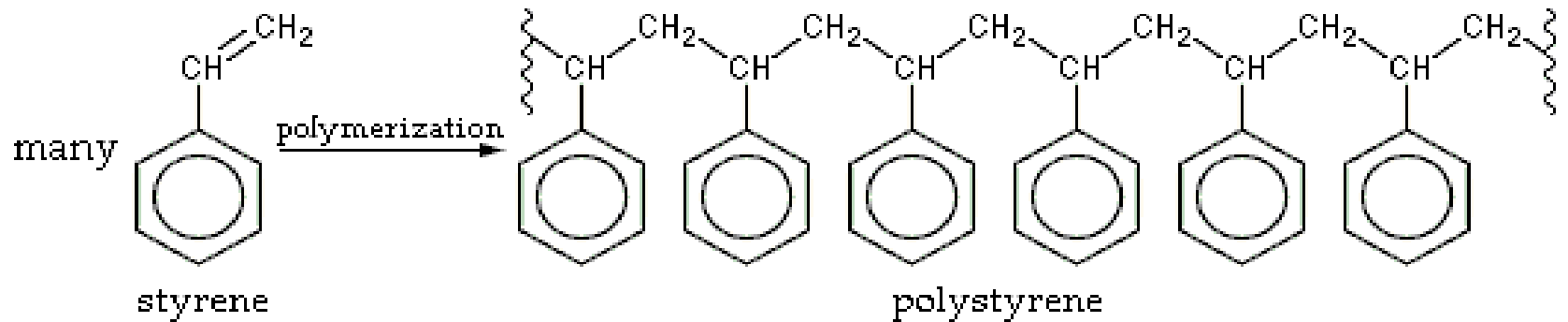


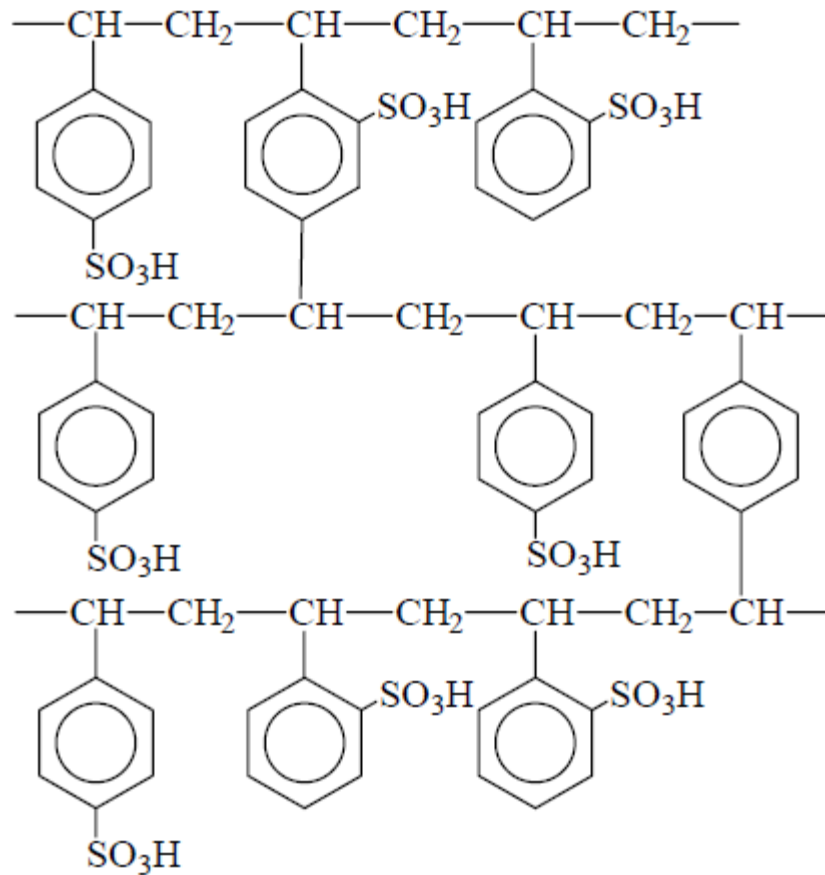
Polymeric resins are made in 3-D networks by cross-linking hydrocarbon chains. The resulting resin is insoluble, inert and relatively rigid. Ionic functional groups are attached to this framework.



Cation-Exchange Resins

These resins are generally manufactured by polymerizing neutral organic molecules such as styrene (to form polystyrene) and then a proportion of divinylbenzene is mixed with styrene, the resultant polymer becomes cross-linked and is then completely insoluble. Functional groups are then added according to the intended use. For example the resin can be sulfonated by adding sulfuric acid to get the structure shown below.



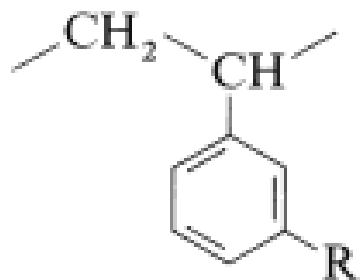


Strongly acidic sulphonated polystyrene cation exchange resin

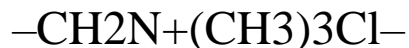


Anion-Exchange Resins

Polystyrene Materials. Cross-linked polystyrene is the basis for generating anion exchange resins carrying amine groups. These amine groups determine the degree of basicity of the exchanger. The anion exchangers listed below are arranged in order of decreasing basicity :



where R can be



e.g., Duolite A 101 (type 1 resin)



e.g., Duolite A 102 (type 2 resin)



e.g., Duolite A 378



e.g., Duolite A 365



Ion Exchange Resin Shape







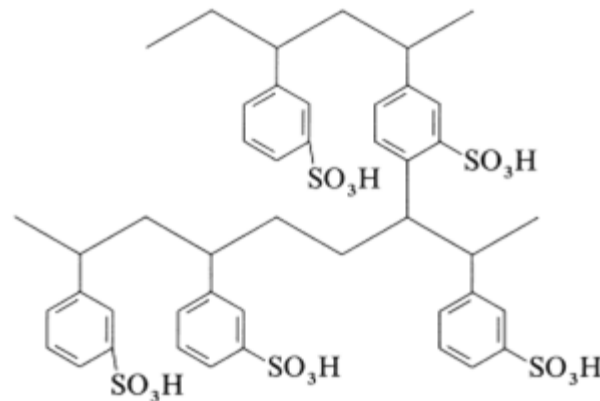


Resin classification

Resins are classified based on the type of functional group they contain and their % of cross-linkages

Cationic Exchangers:

- Strongly acidic – functional groups derived from strong acids e.g., $R-SO_3H$ (sulfonic).

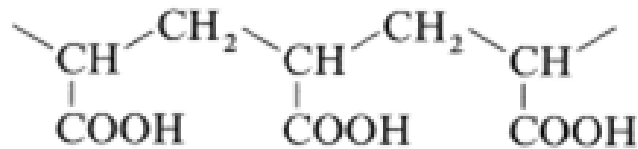


Cross-linked polystyrene 3-sulfonic acid

Examples: Amberlite IR 120, Dowex HCR, Duolite C 20, Lewatit S 100



- Weakly acidic – functional groups derived from weak acids, e.g., R-COOH (carboxylic).

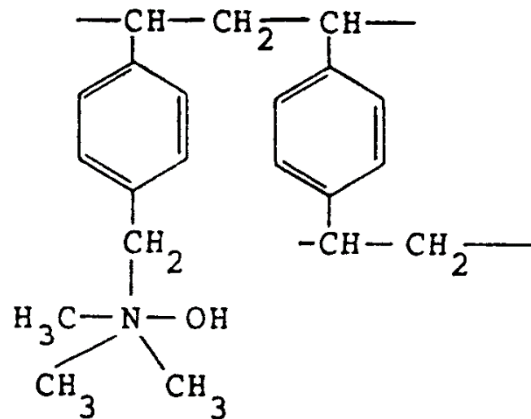


Examples: Amberlite IRC 76, Duolite C 433, Relite CC.



Anionic Exchangers

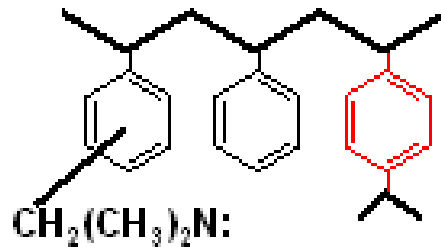
- Strongly basic – functional groups derived from quaternary ammonia compounds, R-N-OH.



Examples: DIAION SA series (Gel Type), DIAION UBA120 (Gel Type)
DIAION PA300 series (Porous Type), DIAION PA400 series (Porous Type)
DIAION HPA25 (Highly Porous Type)



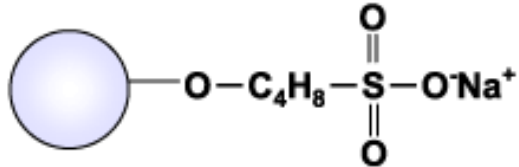
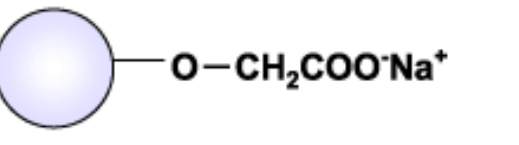
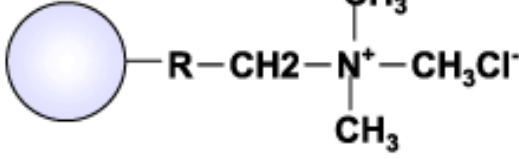
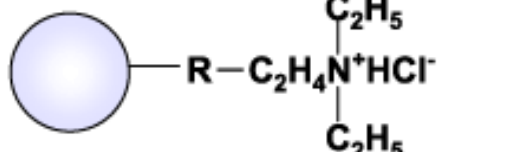
Weakly basic - functional groups derived from primary and secondary amines, $R-NH_3OH$ or CH_2NR_3OH .



Weak Base Anion Resin

Examples: DIAION WA10 (Acrylic Type), DIAION WA20 series (Polyamine Type)
DIAION WA30 (Dimethylamine Type)



<p>S - strong cation Cellufine MAX S-r Cellufine MAX S-h</p>	 $\text{O}-\text{C}_4\text{H}_8-\text{S}(\text{O})_2-\text{O}^-\text{Na}^+$
<p>CM - weak cation Cellufine MAX CM</p>	 $\text{O}-\text{CH}_2\text{COO}^-\text{Na}^+$
<p>Q - strong anion Cellufine MAX Q-r Cellufine MAX Q-h</p>	 $\text{R}-\text{CH}_2-\text{N}^+(\text{CH}_3)_2-\text{CH}_3\text{Cl}^-$
<p>DEAE - weak anion Cellufine MAX DEAE</p>	 $\text{R}-\text{C}_2\text{H}_4-\text{N}^+(\text{C}_2\text{H}_5)_2\text{HCl}^-$



Exchange Capacity

Total Capacity: The total exchange capacity of a resin, expressed in equivalents per unit weight (or per unit volume), represents the number of active sites available. The capacity is expressed in equivalents (eq) per kilogram of dry resin (the weight capacity C_p) or equivalents per liter of wet settled resin (the volume capacity C_v). Capacity values for some of the most common resins are given in Table below;



Material	Shape* of particles	Bulk wet density (drained), kg/L	Moisture content (drained), % by weight	Swelling due to exchange, %	Maximum operating temperature, † °C	Operating pH range	Exchange capacity	
							Dry, equivalent/kg	Wet, equivalent/L
Cation exchangers: strongly acidic								
Polystyrene sulfonate								
Homogeneous (gel) resin	S				120–150	0–14		
4% cross-linked		0.75–0.85	64–70	10–12			5.0–5.5	1.2–1.6
6% cross-linked		0.76–0.86	58–65	8–10			4.8–5.4	1.3–1.8
8–10% cross-linked		0.77–0.87	48–60	6–8			4.6–5.2	1.4–1.9
12% cross-linked		0.78–0.88	44–48	5			4.4–4.9	1.5–2.0
16% cross-linked		0.79–0.89	42–46	4			4.2–4.6	1.7–2.1
20% cross-linked		0.80–0.90	40–45	3			3.9–4.2	1.8–2.0
Porous structure								
12–20% cross-linked	S	0.81	50–55	4–6	120–150	0–14	4.5–5.0	1.5–1.9
Sulfonated phenolic resin	G	0.74–0.85	50–60	7	50–90	0–14	2.0–2.5	0.7–0.9
Sulfonated coal	G							
Cation exchangers: weakly acidic								
Acrylic (pK 5) or methacrylic (pK 6)								
Homogeneous (gel) resin	S	0.70–0.75	45–50	20–80	120	4–14	8.3–10	3.3–4.0
Macroporous	S	0.67–0.74	50–55	10–100	120		~8.0	2.5–3.5
Phenolic resin	G	0.70–0.80	~50	10–25	45–65	0–14	2.5	1.0–1.4
Polystyrene phosphonate	G, S	0.74	50–70	<40	120	3–14	6.6	3.0
Polystyrene iminodiacetate	S	0.75	68–75	<100	75	3–14	2.9	0.7
Polystyrene amidoxime	S	~0.75	58	10	50	1–11	2.8	0.8–0.9
Polystyrene thiol	S	~0.75	45–50		60	1–13	~5	2.0
Cellulose								
Phosphonate	F						~7.0	
Methylene carboxylate	F, P, G						~0.7	
Greensand (Fe silicate)	G	1.3	1–5	0	60	6–8	0.14	0.18
Zeolite (Al silicate)	G	0.85–0.95	40–45	0	60	6–8	1.4	0.75
Zirconium tungstate	G	1.15–1.25	~5	0	>150	2–10	1.2	1.0
Anion exchangers: strongly basic								
Polystyrene-based								
Trimethyl benzyl ammonium (type I)								
Homogeneous, 8% CL	S	0.70	46–50	~20	60–80	0–14	3.4–3.8	1.3–1.5
Porous, 11% CL	S	0.67	57–60	15–20	60–80	0–14	3.4	1.0
Dimethyl hydroxyethyl ammonium (type II)								
Homogeneous, 8% CL	S	0.71	~42	15–20	40–80	0–14	3.8–4.0	1.2
Porous, 10% CL	S	0.67	~55	12–15	40–80	0–14	3.8	1.1
Acrylic-based								
Homogeneous (gel)	S	0.72	~70	~15	40–80	0–14	~5.0	1.0–1.2
Porous	S	0.67	~60	~12	40–80	0–14	3.0–3.3	0.8–0.9
Cellulose-based								
Ethyl trimethyl ammonium	F				100	4–10	0.62	
Triethyl hydroxypropyl ammonium					100	4–10	0.57	
Anion exchangers: intermediately basic (pK 11)								
Polystyrene-based	S	0.75	~50	15–25	65	0–10	4.8	1.8
Epoxy-polyamine	S	0.72	~64	8–10	75	0–7	6.5	1.7
Anion exchangers: weakly basic (pK 9)								
Aminopolystyrene								
Homogeneous (gel)	S	0.67	~45	8–12	100	0–7	5.5	1.8
Porous	S	0.61	55–60	~25	100	0–9	4.9	1.2
Acrylic-based amine								
Homogeneous (gel)	S	0.72	~63	8–10	80	0–7	6.5	1.7
Porous	S	0.72	~68	12–15	60	0–9	5.0	1.1
Cellulose-based								
Aminoethyl	P						1.0	
Diethyl aminoethyl	P						~0.9	

*Shapes: C, cylindrical pellets; G, granules; P, powder; S, spheres.

†When two temperatures are shown, the first applies to H form for cation, or OH form for anion, exchanger; the second, to salt ion.

Water hardness

Hardness in water is caused by the presence of certain positively charged metallic ions in solution in the water. The most common of these hardness-causing ions are *calcium* and *magnesium*; others include *iron*, *strontium*, and *barium*. The two primary constituents of water that determine the hardness of water are calcium and magnesium. If the concentration of these elements in the water is known, the total hardness of the water can be calculated. To make this calculation, the equivalent weights of calcium, magnesium, and calcium carbonate must be known; the equivalent weights are given below.

Equivalent Weights	
Calcium (Ca)	20.04
Magnesium (Mg)	12.15
Calcium carbonate (CaCO ₃)	50.045



Calculating calcium hardness as CaCO₃

The hardness (in mg/L as CaCO₃) for any given metallic ions is calculated using equations below;

$$\frac{\text{calcium hardness (mg/L) as CaCO}_3}{\text{equivalent weight of CaCO}_3} = \frac{\text{calcium (mg/L)}}{\text{equivalent weight of calcium}}$$

$$\frac{\text{magnesium hardness (mg/L) as CaCO}_3}{\text{equivalent weight of CaCO}_3} = \frac{\text{magnesium (mg/L)}}{\text{equivalent weight of magnesium}}$$



Example:

Determine the total hardness as CaCO_3 of a sample of water that has calcium content of 28 mg/L and magnesium content of 9 mg/L.

Solution

Express calcium and magnesium in terms of CaCO_3 :

$$\frac{\text{calcium hardness (mg/L) as CaCO}_3}{\text{equivalent weight of CaCO}_3} = \frac{\text{calcium (mg/L)}}{\text{equivalent weight of calcium}}$$

$$\frac{x \text{ mg/L}}{50.045} = \frac{28 \text{ mg/L}}{20.04}$$

$$x = 69.9 \text{ mg/L Ca as CaCO}_3$$

$$\frac{\text{magnesium hardness (mg/L) as CaCO}_3}{\text{equivalent weight of CaCO}_3} = \frac{\text{magnesium (mg/L)}}{\text{equivalent weight of magnesium}}$$

$$\frac{x \text{ mg/L}}{50.045} = \frac{9 \text{ mg/L}}{12.15}$$

$$x = 37.1 \text{ mg/L Mg as CaCO}_3$$



Now, total hardness can be calculated using equation below;

$$\begin{aligned}\text{Total hardness (mg/L)} &= 69.9 \text{ mg/L} + 37.1 \text{ mg/L} \\ &= 107 \text{ mg/L as CaCO}_3\end{aligned}$$



Calculation of Resin Volume

From the calculated operating capacities, the volume of each type of resin can be determined as a function of the operating time chosen between two regenerations using the following equation:

$$V = \frac{Q \times t \times S}{C_v}$$

Where:

V = resin volume, m^3

Q = flow rate, m^3/h

t = operating time, h

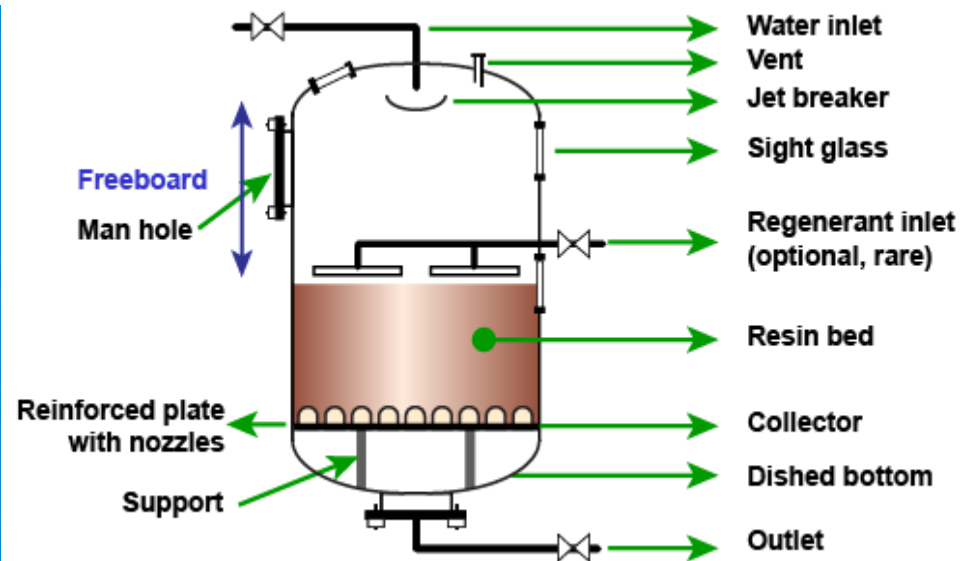
C_v = operating capacity of the resin, eq/m^3 of resin

S = hardness of water or total ions to be adsorbed by the resin, eq/m^3

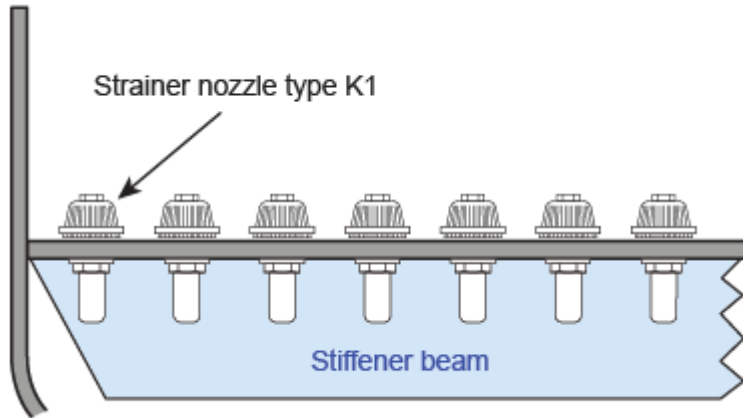


Specific Flow Rate and Cycle Duration

The specific flow rate should not be too low because a very even flow must always be obtained; irregularities in flow may cause "channeling" in the resin bed, thereby impairing complete exchange. Too high a rate produces an excessive head loss. *The specific flow rate Q/V should be between 4 and 40 bed volumes per hour V/h (m^3/h per m^3 of resin).*



Strainer nozzle for Ion Exchange

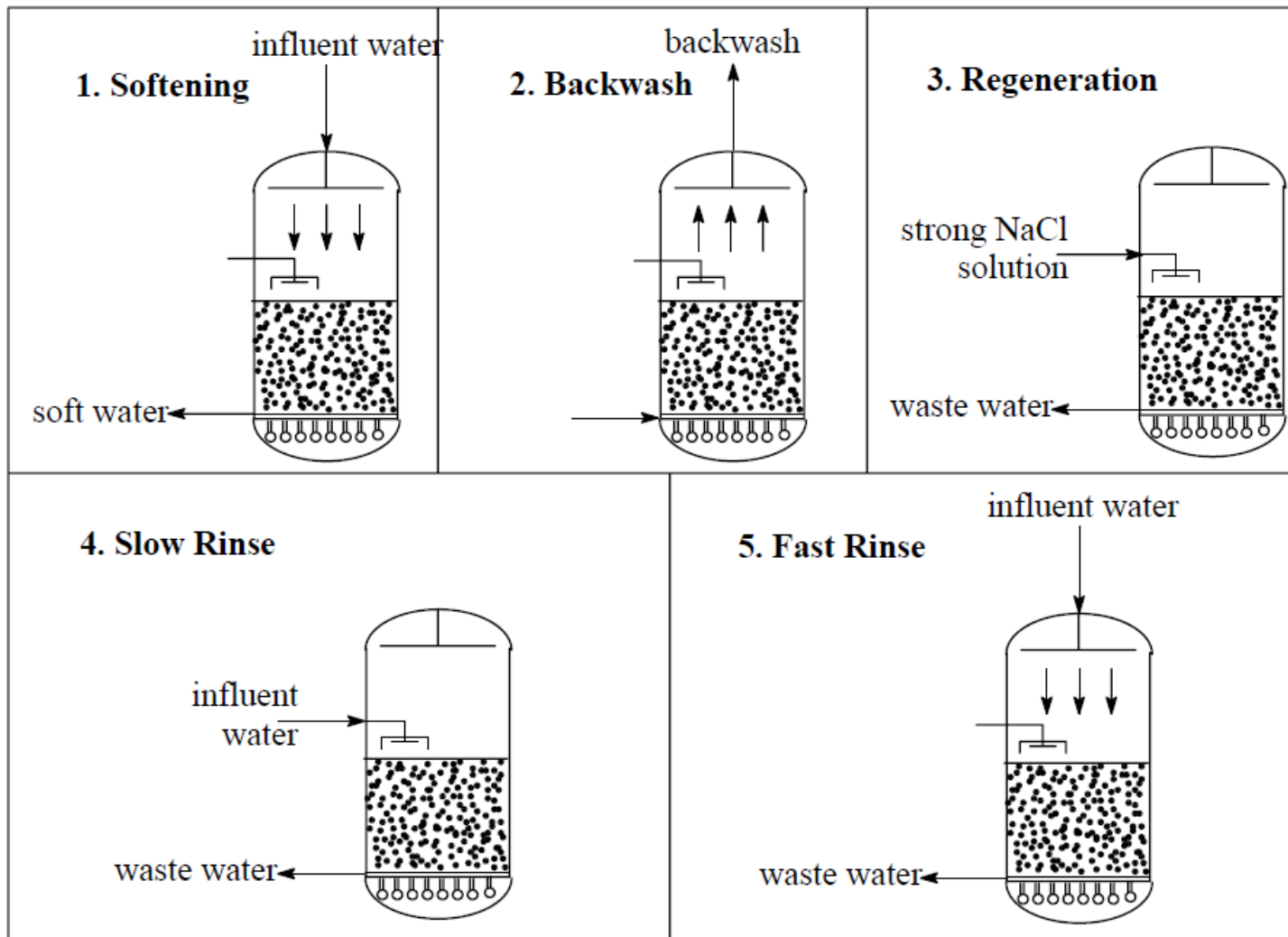


Ion-Exchange Cycle

Figures show a vertical section through ion-exchange column. The solution to be treated is introduced into the column through a distributor. It passes through the resin and emerges through a collector or collection system. In a conventional plant, the resin fills *only half the available space* so that the bed can be decompacted by an upward flow of water. This expands the resin and removes suspended matter and fragments of resin accumulated during the previous cycle. The ion-exchange cycle is divided into four stages:

- 1) Exhaustion (or softening)
- 2) Decompaction (or backwash)
- 3) Regeneration
- 4) Slow rinse
- 5) Fast rinse





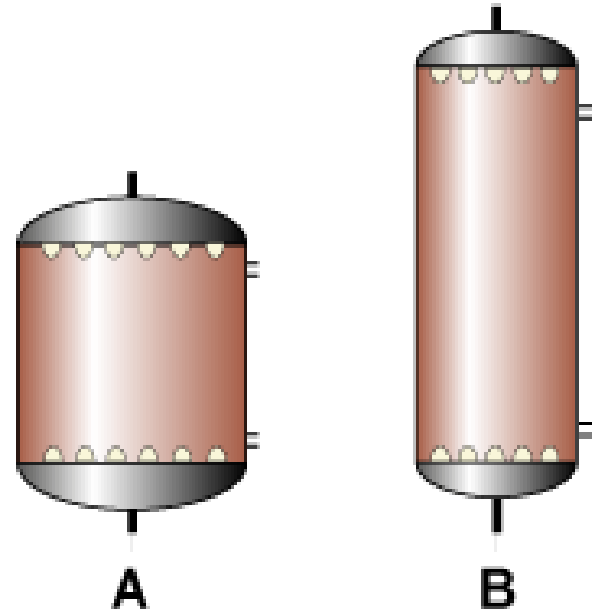
Using and cleaning a water softening ion exchange system



- 1) Exhaustion (or softening). The solution to be treated passes through the resin bed and exhausts it. As soon as the quantity of ions taken up reaches the operating capacity (i.e., the breakthrough point) and leakage reaches a predetermined limiting value, the service stage is stopped. Service runs between regenerations usually range from **12 to 48 hours**
- 2) Backwash. After the service stage, the resin is decompacted for about **15 min** by an upward current of water. This treatment also removes any particles deposited on the surface of the bed, together with any fragments of resin. The bed is then allowed to settle.
- 3) Regeneration. The regenerate solution is introduced, usually at a concentration of a few percent, and slowly percolates through the bed. This takes about **15–60 min**.
- 4) Rinsing. The regenerate is then displaced by water at a low flow rate until the resin bed contains no more than traces of regenerate. This displacement or slow rinse stage is followed by a rapid rinse stage at a higher flow rate to remove the last traces of regenerate. The concentration of the residual regenerate is measured at the end of the operation and, as soon as this falls to an appropriate limiting value, the next cycle begins. When water is being demineralized, the electrolytic conductivity of the effluent is measured.

Vessel sizing

For a given resin volume, it is generally cheaper to make a tall and narrow vessel rather than a wide and short unit: in the illustration, both vessels contain the same resin volume. Vessel B is cheaper, because the major cost components of the vessel are the dished ends and nozzle plates. There is no limit in height, except that the pressure drop at maximum flow rate should not exceed **100 to 150 kPa (1 to 1.5 bar)** at maximum flow rate with clean resins. Graphs are provided by the resin manufacturer that allow computation the expected pressure loss per foot of bed depth as a function of flow rate, at various temperatures. For simple calculation the height to vessel diameter can be selected as **$2 > H/D > 2$** .



Design Procedure for Ion Exchange

an approximate calculation by hand even without using a computer program or the engineering to understand the basic principles presented above.

Simple demineralization trains comprising a single cation exchange resin column an optional degasifier and a single anion exchange resin column. Here is the procedure for a simple demineralization plant:

1. Examine water analysis (as illustrated before)
2. Calculate cation concentration S_c [eq/L as CaCO_3],
3. Decide about the use of a degasifier: If the bicarbonate content is greater than 0.6 to 1.0 meq/L a degasifier may be justified
4. Calculate the anion concentration S_a [eq/L as CaCO_3]: it contains Cl^{-1} , SO_4^{-2} , NO_3^{-1} , SiO_2 , HCO_3^{-1} or residual CO_2 after degasser if any
5. Decide about a reasonable running time t in hours between regenerations
6. Select resin type and capacity C [eq/L], from table or manufactural manual



7. Calculate the volume of each type of resin using the following equation:

$$V = \frac{Q \times t \times S}{C_v}$$

Where:

V = resin volume, m^3

Q = flow rate, m^3/h

t = operating time, h

C_v = operating capacity of the resin, eq/m^3 of resin

S = hardness of water or total ions to be adsorbed by the resin, eq/m^3

8. At the end of this calculation, we must make sure that the specific flow rate of both resin columns is compatible with the general recommendations of the resin producer. The specific flow rate in h^{-1} (often expressed in bed volumes per hour BV/h) is equal to the flow rate in m^3/h divided by the resin volume in m^3 . The usual range is 5 to 40 h^{-1} . For a compact plant with minimum investment cost, use a specific flow rate around 30 to 35 h^{-1} .
9. If the specific flow rates calculated from the resin volumes V are too high, increase the running time t . If they are too low, reduce the running time t



10. Calculate the vessel volume (V_T) from equation below;

$$V = 0.5V_T$$

10. Calculate The vessel height and diameter from equations below;

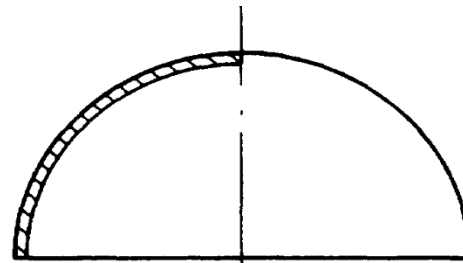
$$V_T = \frac{\pi}{4} D_T^2 H + 2 \times \frac{D_T^3}{24} \quad \text{(Elliptical dish end)}$$

$$V_T = \frac{\pi}{4} D_T^2 H + 2 \times \frac{D_T^3}{12} \quad \text{(Hemispherical dish end)}$$

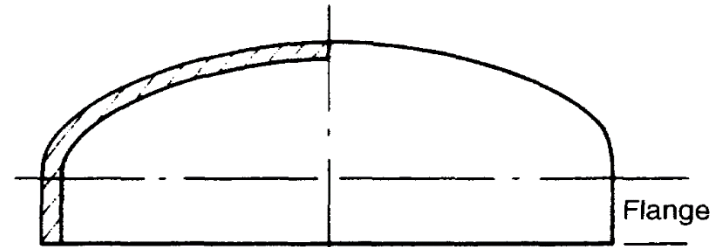
$$V_T = \frac{\pi}{4} D_T^2 H + 2 \times \frac{D_T^3}{12.36} \quad \text{(Torispherical dish end)}$$

12. Specify H/D from preferred arrangement ($2 > H/D > 2$) and dimensions depend on V_T .

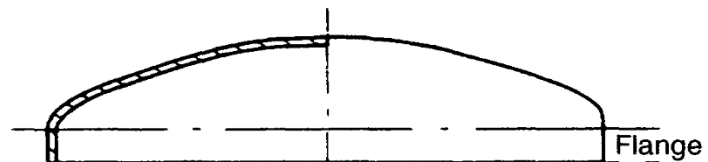




(a)



(b)



(c)

Dish ends(a) Hemispherical (b) Ellipsoidal (c) Torispherical



13. Calculate the pressure drop through the resin bed using Ergun Equation;

$$\frac{\Delta P}{L} = 150 \frac{(1 - \varepsilon)^2 \pi}{\varepsilon^3} \frac{\mu u}{d_p^2} + 1.75 \frac{(1 - \varepsilon) \pi}{\varepsilon^3} \frac{\rho u^2}{d_p}$$

$$\varepsilon = 1 - \frac{\rho_B}{\rho_P}$$

Where

ΔP = Pressure drop, Pa

L = Depth of the packed bed, m.

u = Superficial linear velocity, m/s.

ρ = Fluid density, Kg/m³

μ = Fluid viscosity, Pa.s.

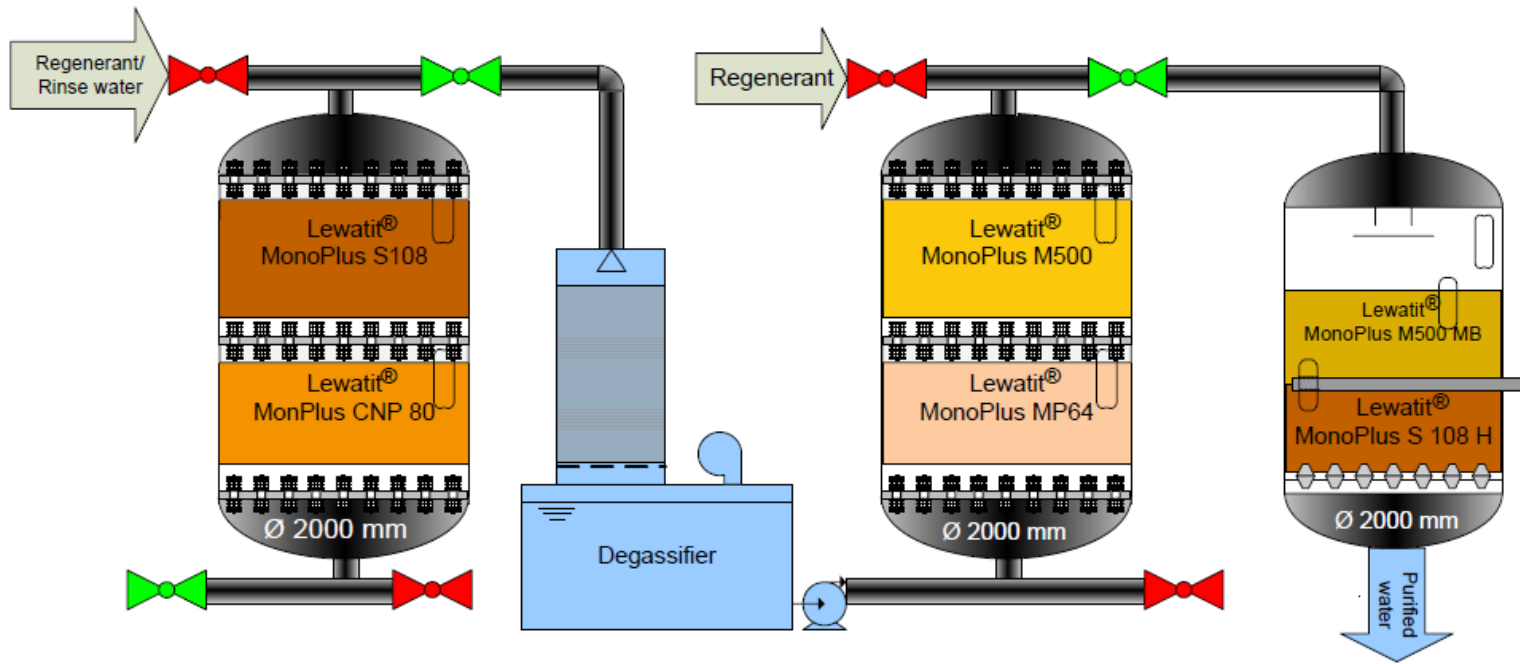
ρ_B = Resin bulk density, Kg/m³

ρ_P = Resin particle density, Kg/m³

d_p = Effective particle diameter, m.

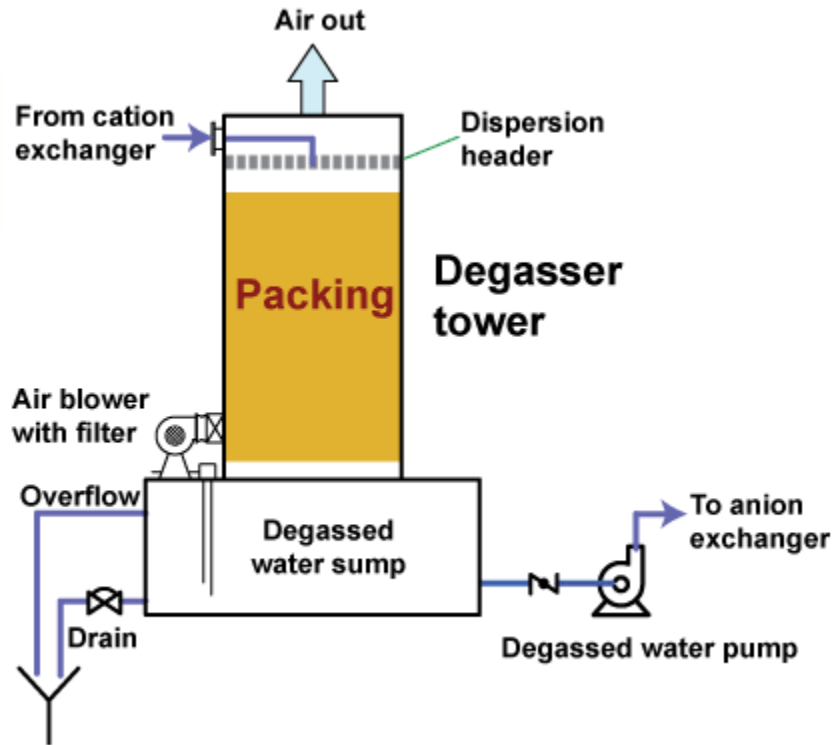
ε = Bed void fraction, dimensionless





Typical demineralized water system including polishing mixed bed





A forced draft degasifier



Design Problem

60 m³/h of industrial wastewater with following analysis (all expressed in mg/L) is to be treated by ion exchange columns Design cation and anion columns using data below;

<u>Cations</u>	<u>(mg/L)</u>	<u>Anions</u>	<u>(mg/L)</u>
Ca	64	Cl	39.05
Mg	8.4	SO ₄	28.8
Na	20.7	NO ₃	12.4
		HCO ₃	15.25

Service runs between regenerations $t = 12$ hours

Cation exchange capacity = 1.0 eq/L

Wastewater density = 1100 Kg/m³

Wastewater viscosity = 0.000862 Pa.s

Anion exchange capacity = 0.5 eq/L

Resin mean diameter = 2 mm

Density of Cation particle = 1250 Kg/m³

Density of Anion particle = 1100 Kg/m³

Bulk density of Cation particle = 800 Kg/m³

Bulk density of Anion particle = 650 Kg/m³

The height to vessel diameter is $H/D = 2$ with elliptical dish ends



Solution

From periodic tables, the equivalent weights are selected as shown below;

<u>Formula</u>	<u>Equivalent weights</u>
Ca	20
Mg	12
Na	23
Cl	35.5
SO ₄	48
NO ₃	62
HCO ₃	61
CaCO ₃	50



Calculate the cation and anion concentrations (S_c and S_a) using the following equation

$$\frac{\text{calcium hardness (mg/L) as CaCO}_3}{\text{equivalent weight of CaCO}_3} = \frac{\text{calcium (mg/L)}}{\text{equivalent weight of calcium}}$$

$$\frac{x \text{ mg/L}}{50} = \frac{64 \text{ mg/L}}{20}$$

$$\begin{aligned} x &= 160 \text{ mg/L Ca as CaCO}_3 \\ &= 3.2 \text{ meq/L Ca as CaCO}_3 \end{aligned}$$

the result as CaCO_3 are illustrated in table below;

<u><i>Cations</i></u> (<i>meq/L</i>)		<u><i>Anions</i></u> (<i>meq/L</i>)	
Ca	3.2	Cl	1.1
Mg	0.7	SO4	0.6
Na	0.9	NO3	0.2
		HCO3	0.25
<hr/>		<hr/>	
Σ	4.8	Σ	2.15



Since the concentration of $\text{HCO}_3 = 0.25 \text{ meq/L}$ as CaCO_3 , a degasifier is not required.

Calculate the volume of each type of resin using the following equation:

$$V = \frac{Q \times t \times S}{C}$$

$$V_c = \frac{60 \times 12 \times 4.8 \times 10^{-3}}{1} = 3.456 \text{ m}^3$$

$$V_a = \frac{60 \times 12 \times 2.15 \times 10^{-3}}{0.5} = 3.096 \text{ m}^3$$

Now, the Specific flow rate for cation and anion beds are checked

$$\text{SFR}_c = 60/3.456 = 16.9 \text{ h}^{-1}$$

$$\text{SFR}_a = 60/3.096 = 19.4 \text{ h}^{-1}$$

The specific flow rate values are OK, The specific flow rate Q/V should be between 4 and 40 bed volumes per hour (m^3/h per m^3 of resin).

Calculate the vessel volume (V_T) for cation and anion from equation below;

$$V = 0.5V_T$$

$$V_{Tc} = 2 \times 3.456 = 6.912 \text{ m}^3$$

$$V_{Ta} = 2 \times 3.096 = 6.192 \text{ m}^3$$

elliptical dish ends are used for two types of vessel with $H/D_T = 2$

$$V_{Tc} = \frac{\pi}{4} D_T^2 H + 2 \times \frac{D_T^3}{24}$$

$$6.912 = \frac{\pi}{4} D_T^2 \times 2 + 2 \times \frac{D_T^3}{24}$$

$$D_{Tc} = 1.6106 \text{ m} \quad H = 3.2212 \text{ m}$$



$$V_{Tc} = \frac{\pi}{4} D_T^2 H + 2 \times \frac{D_T^3}{24}$$

$$6.192 = \frac{\pi}{4} D_T^2 \times 2 + 2 \times \frac{D_T^3}{24}$$

$$D_{Ta} = 1.5526 \text{ m} \quad H = 3.1052 \text{ m}$$

Based on Resin volumes and diameters , the height of cation and anion resin are calculated as following;

$$V_c = \frac{\pi}{4} D_T^2 L_c$$

$$3.456 = \frac{\pi}{4} (1.6106)^2 L_c$$

$$L_c = 1.6963 \text{ m}$$



$$V_a = \frac{\pi}{4} D_T^2 L_a$$

$$3.096 = \frac{\pi}{4} (1.5526)^2 L_a$$

$$L_a = 1.6352 \text{ m}$$

Calculate the pressure drop through the resin beds using Ergun Equation;

$$\frac{\Delta P}{L} = 150 \frac{(1 - \varepsilon)^2 \pi}{\varepsilon^3} \frac{\mu u}{d_p^2} + 1.75 \frac{(1 - \varepsilon) \pi}{\varepsilon^3} \frac{\rho u^2}{d_p}$$

$$\varepsilon = 1 - \frac{\rho_B}{\rho_p}$$



Ion exchange vessel of cross-sectional area $= \frac{\pi}{4} D_r^2$

$$A_c = \frac{\pi}{4} (1.6106)^2 = 2.0373 \text{ m}^2$$

$$A_a = \frac{\pi}{4} (1.5526)^2 = 1.8932 \text{ m}^2$$

$$u_c = \frac{Q}{A_c} = \frac{60}{3600 \times 2.0373} = 0.0081 \text{ m/s}$$

$$u_a = \frac{Q}{A_c} = \frac{60}{3600 \times 1.8932} = 0.0088 \text{ m/s}$$

$$\varepsilon_c = 1 - \frac{800}{1250} = 0.36$$

$$\varepsilon_a = 1 - \frac{650}{1100} = 0.409$$



$$\frac{\Delta P}{L} = 150 \frac{(1-\varepsilon)^2 \pi}{\varepsilon^3} \frac{\mu u}{d_p^2} + 1.75 \frac{(1-\varepsilon)\pi}{\varepsilon^3} \frac{\rho u^2}{d_p}$$

$$\frac{\Delta P_c}{1.6963} = 150 \times \frac{(1-0.36)^2 \times \pi}{0.36^3} \times \frac{0.000862 \times 0.0081}{0.002^2} + 1.75 \times \frac{(1-0.36) \times \pi}{0.36^3} \times \frac{1100 \times 0.0081^2}{0.002}$$

$$\Delta P_c = 12.6158 \text{ kPa}$$

$$\frac{\Delta P_a}{1.6352} = 150 \times \frac{(1-0.409)^2 \times \pi}{0.409^3} \times \frac{0.000862 \times 0.0088}{0.002^2} + 1.75 \times \frac{(1-0.409) \times \pi}{0.409^3} \times \frac{1100 \times 0.0088^2}{0.002}$$

$$\Delta P_a = 10.7676 \text{ kPa}$$

