

AL MUSTAQBAL UNIVERSITY


## Lecture: (1)

Principles of information theory

Subject: Coding Techniques

## First Stage

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# AI- Mustaqbal University <br> College of Sciences <br> Department of Cybersecurity 

Probability can be defined as the chance of an event occurring.
Probability is a way of assigning every "event" a value between " 0 " and
"1"
$\boldsymbol{P}$ - denotes a probability.
$A, B$, and $C$ - denote specific events.
$\boldsymbol{P}(\boldsymbol{A})$ - denotes the probability of event A occurring.

## Basic Concepts

- Probability Experiment is a chance process that leads to well defined results called outcomes.
- An outcome is the result of a single trial of a probability experiment.
- An event consists of a set of outcomes of a probability experiment.
- Sample Space is the set of all possible outcomes of a probability experiment. Sample Space Consists of all possible simple events.

| EXPERIMENT | SAMPLE SPACE |
| :---: | :---: |
| Toss one coin | $\mathbf{H , ~ T}$ |
| Roll a die | $\mathbf{1 , 2 , 3 , 4 , 5 , 6}$ |
| Answer a true-false question | True, False |
| Toss two coins | HH, HT, TH, TT |

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The probability of any event $E$ is $\square$
This probability is denoted by
$P(E)=\frac{\mathrm{n}(E)}{\mathrm{n}(S)}$

Example 1: A coin is tossed 3 times. Find the following:

1. List the sample space
2. Find the probability of tossing heads exactly twice
3. Find the probability of tossing tails at least two.

## Solution:

1. The sample space for tossing the coin 3 times has eight outcomes, S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT $\}$.
$\mathrm{N}(\mathrm{S})=8(\mathrm{n}$ is the number of all possible outcomes in sample space)
2. The event A represents the outcomes of exactly two heads H .

Event $(\mathrm{A})=\{$ HHT, HTH, THH $\}$
$\mathrm{N}(\mathrm{A})=3$
$\mathrm{P}(\mathrm{A})=n(A) / n(S)$
$=3 / 8$
$=0.375$
3. The event B represents the outcomes of at least two tails T .

$$
\text { Event }(B)=\{\text { THT, TTH, HTT, TTT }\}
$$

$$
\begin{aligned}
\mathrm{N}(\mathrm{~A}) & =4 \\
\mathrm{P}(\mathrm{~A}) & =n(A) / n(S) \\
& =4 / 8 \\
& =0.5
\end{aligned}
$$

Example 2: A box contains 3 red balls, 2 blue balls, and 5 white balls. A ball is selected from the box. Find the probability of selecting red balls.

## Solution

$$
\begin{aligned}
\mathrm{N}(\mathrm{~S}) & =3(\text { red })+2(\text { blue })+5(\text { white }) \\
& =10 \\
\mathrm{P}(\mathrm{red}) & =n(r e d) / n(S) \\
& =3 / 10 \\
& =0.33
\end{aligned}
$$

Example 3: If a family has three children, find the following:

1. List the sample space
2. Find the probability of a family having at least 1 girl.

3 . Find the probability of a family having exactly two boys.

## Solution:

Use B for boy and G for girl.

1. $\mathrm{S}=\{\mathrm{BBB}, \mathrm{BBG}, \mathrm{BGB}, \mathrm{GBB}, \mathrm{GGG}, \mathrm{GGB}, \mathrm{GBG}, \mathrm{BGG}\}$
2. Event (at least one girl) $=\{\mathrm{BBG}, \mathrm{BGB}, \mathrm{GBB}, \mathrm{GGG}, \mathrm{GGB}, \mathrm{GBG}$,

BGG $\}$
$\mathrm{N}(\mathrm{G})=7$

$$
\begin{aligned}
\mathrm{P}(\mathrm{G}) & =n(G) / n(S) \\
& =7 / 8 \\
& =0.875
\end{aligned}
$$

3.Event (two boys) $=\{$ BBG, BGB, GBB $\}$
$\mathrm{N}(\mathrm{B})=3$
$\mathrm{P}(\mathrm{B})=n(B) / n(S)$

$$
\begin{aligned}
& =3 / 8 \\
& =0.375
\end{aligned}
$$

Example4: Two dice are rolled. Find the probability of the sum of two dice is equal to 5 .

Solution: The sample space of rolling two dice as shown:

| Die 1 | Die 2 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
|  | $(1,1)$ | $(1,2)$ | $(1,3)$ | $(1,4)$ | $(1,5)$ | $(1,6)$ |
| 2 | $(2,1)$ | $(2,2)$ | $(2,3)$ | $(2,4)$ | $(2,5)$ | $(2,6)$ |
| 3 | $(3,1)$ | $(3,2)$ | $(3,3)$ | $(3,4)$ | $(3,5)$ | $(3,6)$ |
| 4 | $(4,1)$ | $(4,2)$ | $(4,3)$ | $(4,4)$ | $(4,5)$ | $(4,6)$ |
| 5 | $(5,1)$ | $(5,2)$ | $(5,3)$ | $(5,4)$ | $(5,5)$ | $(5,6)$ |
| 6 | $(6,1)$ | $(6,2)$ | $(6,3)$ | $(6,4)$ | $(6,5)$ | $(6,6)$ |

$\mathrm{n}(\mathrm{S})=36$
The event A represents the outcomes of the sum of two dice is equal to 5 .
$\operatorname{Event}(\mathrm{A})=\{(1,4),(2,3),(3,2),(4,1)\}$.
$\mathrm{n}(\mathrm{A})=4$
$\mathrm{P}(\mathrm{A})=n(A) / n(S)$
$=4 / 36$

$$
=1 / 9
$$

The Addition Rules for Probability
$\mathrm{A}=\{1,2,3,4,5\}, \mathrm{n}(\mathrm{A})=5$
$B=\{4,5,6,7\}, n(B)=4$
$\mathrm{AUB}=\{1,2,3,4,5,6,7\}$
$\mathrm{n}(\mathrm{AUB})=7$
$\mathrm{n}(\mathrm{AUB})=\mathrm{n}(\mathrm{A})+\mathrm{n}(\mathrm{B})=5+4=9$
$A \cap B=\{4,5\}$
$\mathrm{n}(\mathrm{A} \cap \mathrm{B})=2$
$\mathbf{n}(\mathbf{A U B}) \neq \mathbf{n}(\mathbf{A})+\mathbf{n}(\mathbf{B})$
then $n(A U B)=n(A)+n(B)-n(A \cap B)$

$$
=\mathrm{p}(\mathrm{~A})+\mathrm{p}(\mathrm{~B})-\mathrm{p}(\mathrm{~A} \cap \mathrm{~B})
$$

If $(A \cap B)=0$ then $A$ and $B$ are mutually exclusive
Mutually exclusive events: Two events are mutually exclusive events if they cannot occur at the same time.
Rule1: When two events $\boldsymbol{A}$ and $B$ are mutually exclusive, the
probability that $A$ or $B$ will occur is:
$\boldsymbol{P}(\boldsymbol{A}$ or $\boldsymbol{B})=\mathbf{P}(\mathbf{A})+\mathbf{P}(\mathbf{B})$

Example5: A day of the week is selected at random. Find the probability that it is a weekend.

Solution: $P$ (Friday or Saturday)
$=P($ Friday $)+\mathrm{P}($ Saturday $)=1 / 7+1 / 7=2 / 7$.

Rule 2:When two events $A$ and $B$ are Not mutually exclusive ,the probability that $A$ or $B$ occur is:
$P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$

Example 6: A box contains balls numbered from 1 to 10 , what is the probability of getting (odd or prime) ball?

Solution: $\mathrm{S}=\{1,2,3,4,5,6,7,8,9,10\}, \mathrm{n}(\mathrm{S})=10$
Event A: odd ball $=\{1,3,5,7,9\} n(A)=5$
Event B: prime ball $=\{2,3,5,7\} n(B)=4$
$\mathrm{A} \cap \mathrm{B}=\{3,5,7\} \mathrm{n}(\mathrm{A} \cap \mathrm{B})=3$

$$
\begin{aligned}
\mathrm{P}(\mathrm{AUB}) & =\frac{n(\mathrm{AUB})}{n(S)}=\frac{n(A)+n(B)-n(\mathrm{~A} \cap \mathrm{~B})}{n(S)} \\
& =\frac{n(\mathrm{~A})}{n(S)}+\frac{n(B)}{n(S)}-\frac{n(\mathrm{~A} \cap \mathrm{~B})}{n(S)} \\
& =\frac{5}{10}+\frac{4}{10}-\frac{3}{10} \\
& =\frac{9}{10}-\frac{3}{10}=\frac{6}{10}
\end{aligned}
$$

Example7: In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected, find the probability that the subject is a nurse or a male.

## Solution:

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| Staff | Females | Males | Totals |
| :--- | :--- | :--- | :--- |
| Nurses | 7 | 1 | 8 |
| Physicians | 3 | 2 | 5 |
| Total | 10 | 3 | 13 |

$\mathrm{N}(\mathrm{S})=13$
Event A: nurse

$$
\mathrm{n}(\mathrm{~A})=8
$$

Event B: male

$$
n(B)=3
$$

$\mathrm{A} \cap \mathrm{B}=$ nurse OR male
$\mathrm{n}(\mathrm{A} \cap \mathrm{B})=1$
$P($ nurse or male $)=P($ nurse $)+P($ male $)-P($ male nurse $)$

$$
=\frac{n(\mathrm{~A})}{n(S)}+\frac{n(B)}{n(S)}-\frac{n(\mathrm{~A} \cap \mathrm{~B})}{n(S)}
$$

$$
=\frac{8}{13}+\frac{3}{13}-\frac{1}{13}
$$

$$
=\frac{11}{13}-\frac{1}{13}
$$

$$
=\frac{10}{13}
$$

What is the information theory?
The purpose of a communication system is to carry information-bearing baseband signals from one place to another over a communication channel. with high efficiency and reliability.

Information theory provides a quantitative measure of the information
contained in message signal and allows us to determine the capacity of a communication system to transfer this information from source to destination. Through the use of coding, a major topics of information theory، redundancy can be reduced from message signal so that channels can be used with improved efficiency. In addition systematic redundancy can be introduced to the transmitted signal so that channels can be used with improved reliability.

## Examples:

Baseband signals: Audio signals and video signals
Channel: Optical fiber and free space
-Information" in communication theory refers to the amount of uncertainty in a system that a message will get rid of.

Information Theory deals with mathematical modeling and analysis of a communication system rather than with physical sources and physical channels.

Even if information theory is considered a branch of communication theory, it actually spans a wide number of disciplines including computer science probability, statistics, economics, etc. The most basic questions treated by information theory are: how can 'information' measured? How can information be transmitted?

- Data compression addresses the problem of reducing the amount of data required to represent a digital file, so that it can be stored or transmitted so efficiently.

The principle of data compression is that, it compress data by removing redundancy from the original data in the source file.

- On the other hand, information theory tells us that the amount of information conveyed by an event relates to its probability of occurrence. An event that is less likely to occur is said to contain more information than an event that is more likely to occur.

