جامـــــعة المـــستـقبـل
AL MUSTAQBAL UNIVERSITY

كلية العلوم
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Lecture: (2)

Mathematical Measure of Information

Subject: Coding Techniques

First Stage

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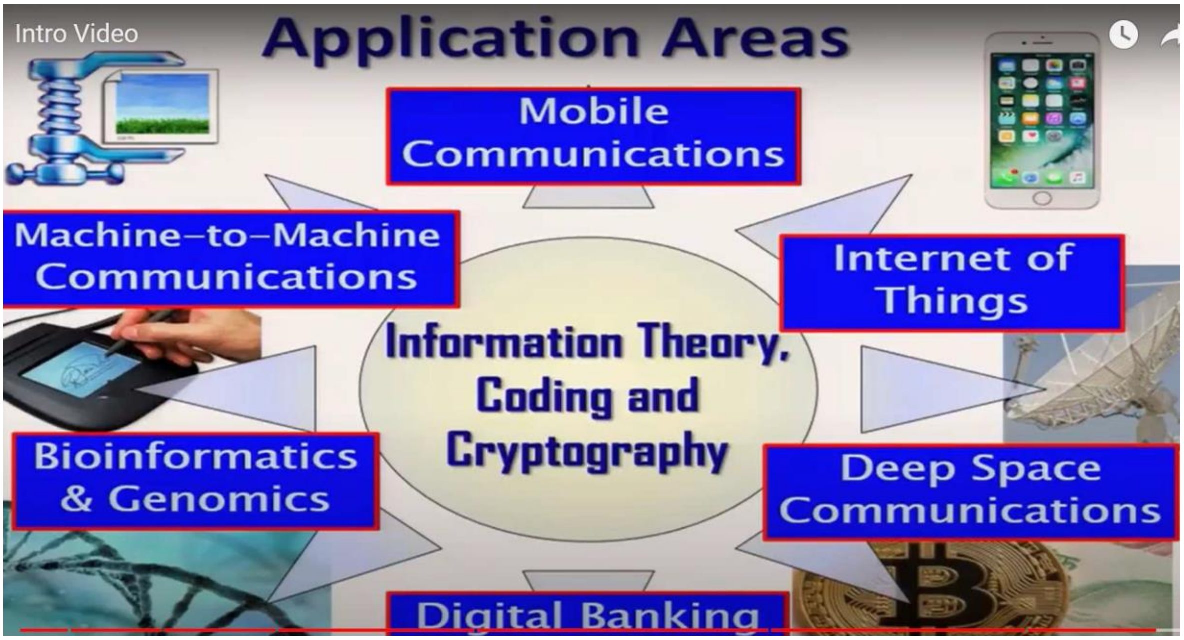
The Role of Coding and Information Theory

Coding Theory and Information Theory are pivotal in the digital age, each playing a unique role in the transmission, processing, and storage of information. While Coding Theory deals with designing codes for error detection and correction, Information Theory concerns itself with the quantification, storage, and communication of information. Together, they provide the foundation for understanding and improving digital communication systems.

- Data compression addresses the problem of reducing the amount of data required to represent a digital file, so that it can be stored or transmitted so efficiently.
  The principle of data compression is that, it compress data by removing redundancy from the original data in the source file.

- On the other hand, information theory tells us that the amount of information conveyed by an event relates to its probability of occurrence. An event that is less likely to occur is said to contain more information than an event that is more likely to occur.



The problem of representing the source alphabet symbols (usually the binary system consisting of the two symbols 0 & 1) is the main topic of **coding theory**.

An optimum coding scheme will use more bits for the symbols that less likely to occur, and a fewer bits for the symbols that frequently occur. coding theory leads to information theory and ***information theory*** provides the performance limits on what can be done by suitable encoding of the information. Thus the two theories are intimately related. Both ***coding and information*** theory give a central role to errors ***(noise)*** and are therefore of special interest since in real-life noise is everywhere.

The conventional system is modeled by:

1- An information source.

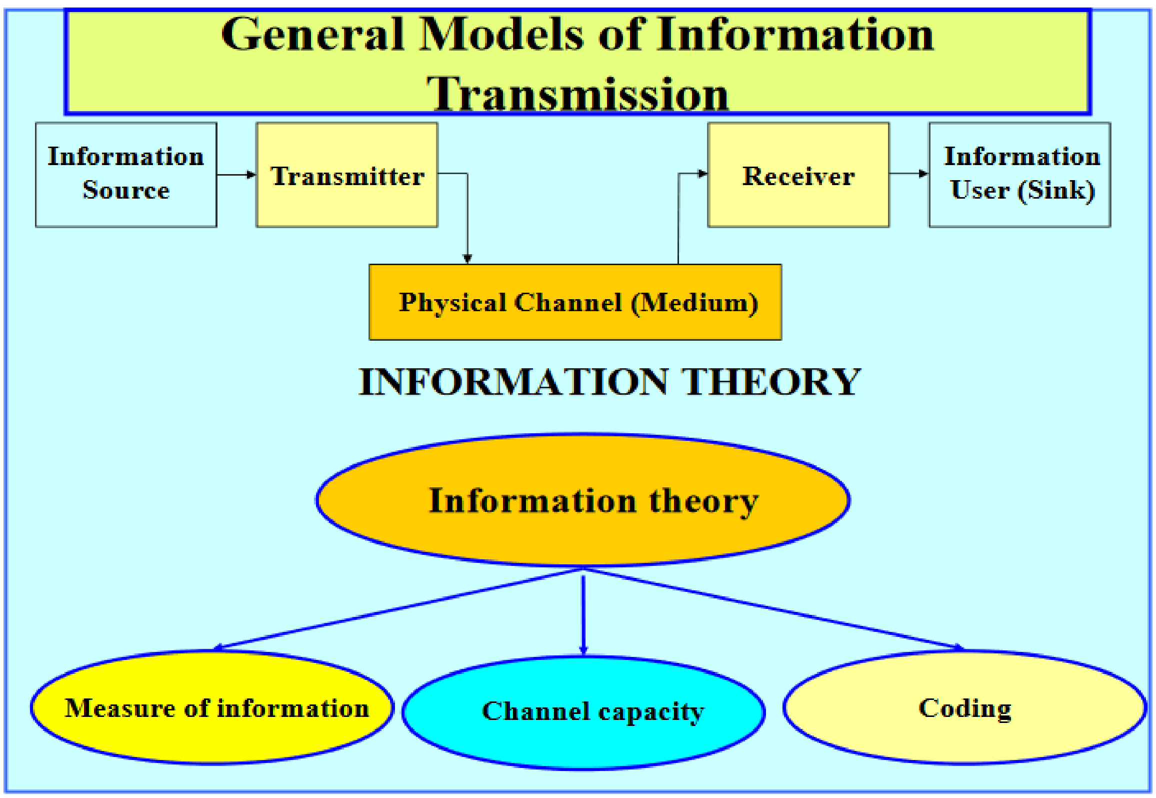
2- An encoding of this source.

3- A channel over or through which the information is sent.

4- A noise (error) source that is added to the signal in the channel.

5- A decoding and recovery of the original information from the received signal with noise.

6- A destination for the information.



Information theory tells us that the amount of information conveyed by an event relates to its probability of occurrence. An event that is less likely to occur is said to contain more information than an event that is more likely to occur. The amount of information of an event and its probability are thus opposite.

Information Theory is a mathematical subject dealing with

three basic concepts:

1- The measure of information.

2- The capacity of communication channel to transfer information.

3- Coding as a means of utilizing channels at full capacity.

These three concepts are tied together in what can be called the fundamental theorem of information theory as follows:

Given an information source and a communication channel, there exists a coding technique such that the information can be transmitted over the channel at any rate less than the channel capacity and with arbitrarily frequency of error despite the presence of noise.

Mathematical Measure of Information

Self - information

Consider a discrete random variable X with possible value $X=x_i$, $i=1,2,3,\ldots n$ the self - information of the event $X=x_i$ is

1 bit is the amount of information needed to choose between two equally likely alternatives.

So:

$$I(x) = -\log_2 p(x) = \log_2 \frac{1}{p(x)} (bit)$$

$$\log_2(x) = \frac{\ln(x)}{\ln(2)} = \frac{\log_{10}(x)}{\log_{10}(2)}$$

We note that a high probability event conveys less information than the low probability events. For an event with P(x) =1 , I(x)=0 . Since a low probability implies a higher degree of uncertainty (and vice-versa), a random variable with high degree of uncertainty contains more information. i.e., self - information is non – negative

$$P \uparrow \quad I \downarrow \qquad\qquad P \downarrow \quad I \uparrow$$

with I(x)=0 only for the certain event.

Properties of information:

Following properties can be written for information.

i) If there is more uncertainty about the message, information carried is also more.

ii) If receiver knows the message being transmitted, the amount of information carried is zero.

iii) If I_1 is the information carried by message m_1, and I_2 is the information carried by m_2, then amount of information carried combinely due to m_1 and m_2 is $I_1 + I_2$.

iv) If there are $M = 2^N$ equally likely messages, then amount of information carried by each message will be N bits.

Q. Comments on the information content of the following message

1. Tomorrow the sun will rise from the east.
2. It will snow fall in Amaravati this winter
3. The phone will ring in the next one hour.

Ans. 1. The first statement does not carry any information since it is sure that sun always rises from east. The probability of occurrence of first event is high or sure. Hence it carries less or negligible information

$$I_k = \text{Log}1/p_k = \text{Log}1/1 = 0$$

2. In the winter season snow fall in amaravati is very rare. Hence probability of occurrence of this event is very rare, so it carries large amount of information.

3. In the third statement predicts about phone ring in the time span of one hour. It does not mention exact time but span of one hour is mentioned. Hence it carries moderate information.

Example1: Consider a random experiment with 16 equally likely outcomes. The information associated with each outcome is

$$I(x_j) = -\log_2(1/16) = \log_2(16) = 4bits$$

Where j ranges from 1 to 16. The information is greater than one bit since the probability of each outcome is less than half.

Example2: Consider a binary source which tosses a fair coin and outputs a 1 if a head (H) appears and a 0 if a trail (T) appears. For this source, P(1) = P(0) = 0.5 . The information content of each output from the source is:

$$I(1)=I(0)= \log_2 (1/0.5) = \log_2 2 = 1 \text{ bits of information.}$$

Example3: When throwing a fair die, the probability of 'four' is 1/6. When it is proclaimed that 'four' has been thrown, the amount of self-information is :

$$I(\text{'four'}) = \log_2 (1/(1/6)) = \log_2 (6) = 2.585 \text{ bits.}$$

Example4: Q. A source emit one of 4 possible symbol X0 to X3 during each signaling interval. The symbol occur with probability as given in table:

Symbol	Probability
X0	P0=0.4
X1	P1=0.3
X2	P2=0.2
X3	P3=0.1

Find the amount of information gained by observing the source emitting each of these symbol?

Ans:

I_0=1.322bits, I_1=1.737bits, I_2=2.322bits, I_3=3.322bits

Q/ Calculate the amount of information if p_k=1/4?

Example5: In binary PCM if '0' occur with probability 1/4 and '1' occur with probability 3/4 then calculate amount of information conveyed by each bits?

Ans:

I_1=2 bits, I_2=0.415bits