Electrical Engineering Fundamentals

First class

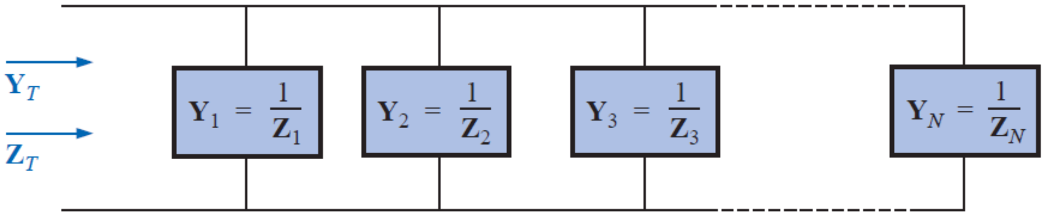
AC

Lecture 8, 9 & 10

Dr. Saad Mutashar Abbas

2024-2025

PARALLEL ac CIRCUITS



$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N$$

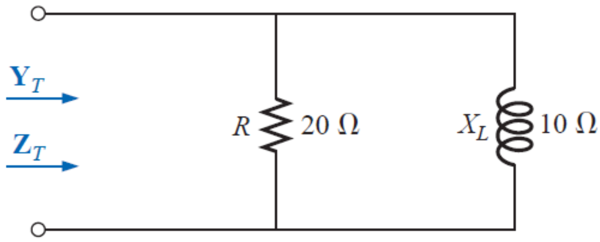
$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}$$

For two impedances in parallel

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

EXAMPLE: For the network

a. Find the admittance of each parallel branch.
b. Determine the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.



Solutions:

a. $\mathbf{Y}_R = G\angle 0° = \dfrac{1}{R}\angle 0° = \dfrac{1}{20\ \Omega}\angle 0°$

$= \mathbf{0.05\ S\ \angle 0° = 0.05\ S + \mathit{j}\ 0}$

$\mathbf{Y}_L = B_L\angle -90° = \dfrac{1}{X_L}\angle -90° = \dfrac{1}{10\ \Omega}\angle -90°$

$= \mathbf{0.1\ S\ \angle -90° = 0 - \mathit{j}\ 0.1\ S}$

b. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ \text{S} + j\ 0) + (0 - j\ 0.1\ \text{S})$

$= \mathbf{0.05\ S - \mathit{j}\ 0.1\ S} = G - jB_L$

c. $\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \dfrac{1}{0.05\ \text{S} - j\ 0.1\ \text{S}} = \dfrac{1}{0.112\ \text{S}\ \angle -63.43°}$

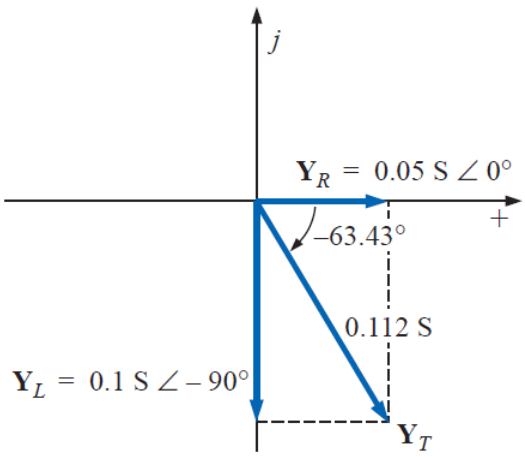
$= \mathbf{8.93\ \Omega\ \angle 63.43°}$

Or

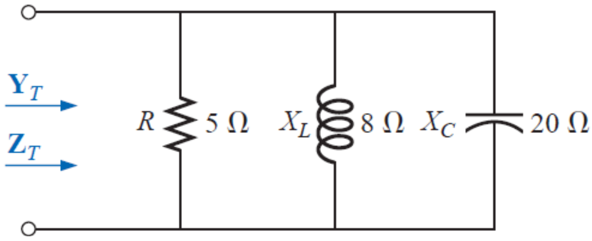
$\mathbf{Z}_T = \dfrac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \dfrac{(20\ \Omega\ \angle 0°)(10\ \Omega\ \angle 90°)}{20\ \Omega + j\ 10\ \Omega}$

$= \dfrac{200\ \Omega\ \angle 90°}{22.36\ \angle 26.57°} = \mathbf{8.93\ \Omega\ \angle 63.43°}$

d.



EXAMPLE: Repeat the above Example for the parallel network.



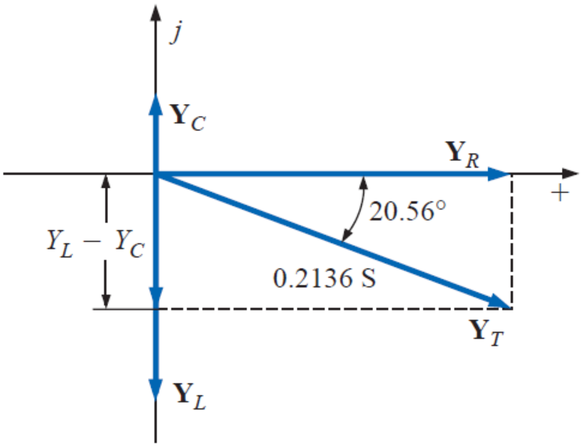
Solutions:

a. $$\mathbf{Y}_R = G\angle 0^\circ = \frac{1}{R}\angle 0^\circ = \frac{1}{5\ \Omega}\angle 0^\circ$$
$$= \mathbf{0.2\ S\ \angle 0^\circ = 0.2\ S + \mathit{j}\ 0}$$
$$\mathbf{Y}_L = B_L\angle -90^\circ = \frac{1}{X_L}\angle -90^\circ = \frac{1}{8\ \Omega}\angle -90^\circ$$
$$= \mathbf{0.125\ S\ \angle -90^\circ = 0 - \mathit{j}\ 0.125\ S}$$
$$\mathbf{Y}_C = B_C\angle 90^\circ = \frac{1}{X_C}\angle 90^\circ = \frac{1}{20\ \Omega}\angle 90^\circ$$
$$= \mathbf{0.050\ S\ \angle +90^\circ = 0 + \mathit{j}\ 0.050\ S}$$

b. $$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$$
$$= (0.2\ S + j\ 0) + (0 - j\ 0.125\ S) + (0 + j\ 0.050\ S)$$
$$= 0.2\ S - j\ 0.075\ S = \mathbf{0.2136\ S\ \angle -20.56^\circ}$$

c. $$\mathbf{Z}_T = \frac{1}{0.2136\ S\ \angle -20.56^\circ} = \mathbf{4.68\ \Omega\ \angle 20.56^\circ}$$

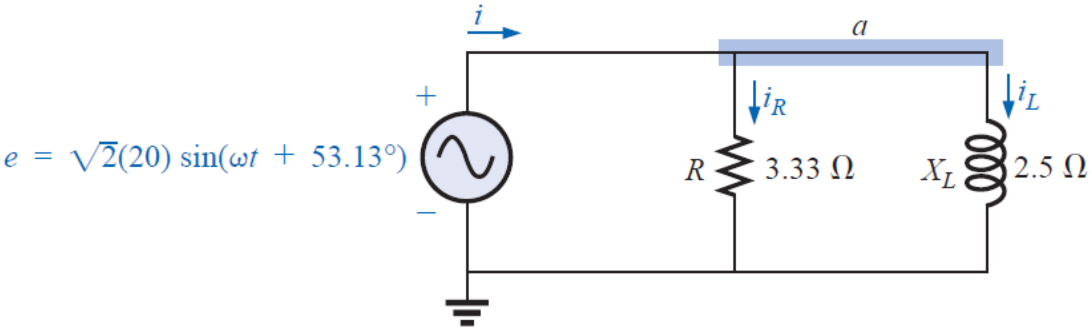
d.



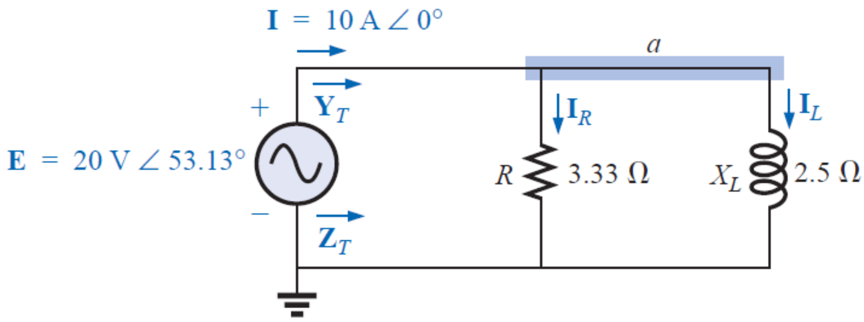
PARALLEL ac NETWORKS

1) R-L

EXAMPLE: find the total impedance and the current in each branch for the network.



Solutions:



$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L$$

$$= G\angle 0° + B_L\angle -90° = \frac{1}{3.33\ \Omega}\angle 0° + \frac{1}{2.5\ \Omega}\angle -90°$$

$$= 0.3\text{ S}\angle 0° + 0.4\text{ S}\angle -90° = 0.3\text{ S} - j\,0.4\text{ S}$$

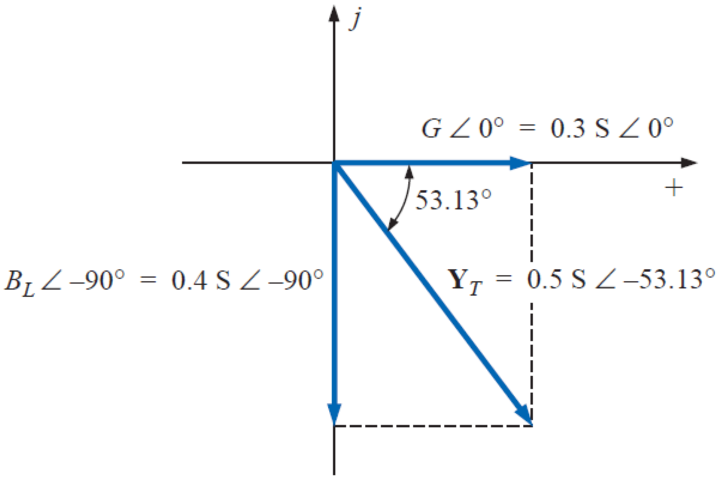
$$= \mathbf{0.5\ S\angle -53.13°}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\text{ S}\angle -53.13°} = \mathbf{2\ \Omega\angle 53.13°}$$

Or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3.33\ \Omega\angle 0°)(2.5\ \Omega\angle 90°)}{3.33\ \Omega\angle 0° + 2.5\ \Omega\angle 90°}$$

$$= \frac{8.325\angle 90°}{4.164\angle 36.87°} = \mathbf{2\ \Omega\angle 53.13°}$$



$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (20\text{ V} \angle 53.13°)(0.5\text{ S} \angle -53.13°) = \mathbf{10\ A \angle 0°}$$

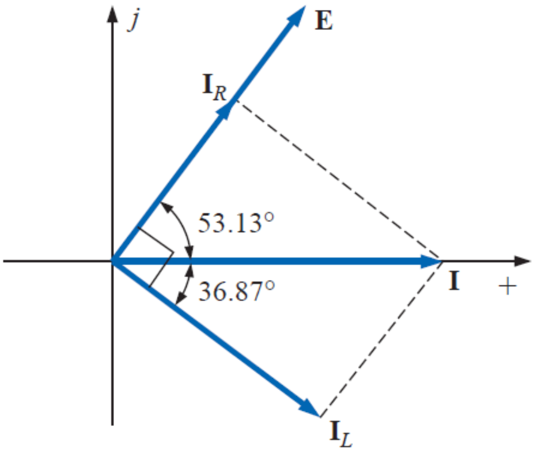
$$\mathbf{I}_R = \frac{E \angle \theta}{R \angle 0°} = (E \angle \theta)(G \angle 0°)$$

$$= (20\text{ V} \angle 53.13°)(0.3\text{ S} \angle 0°) = \mathbf{6\ A \angle 53.13°}$$

$$\mathbf{I}_L = \frac{E \angle \theta}{X_L \angle 90°} = (E \angle \theta)(B_L \angle -90°)$$

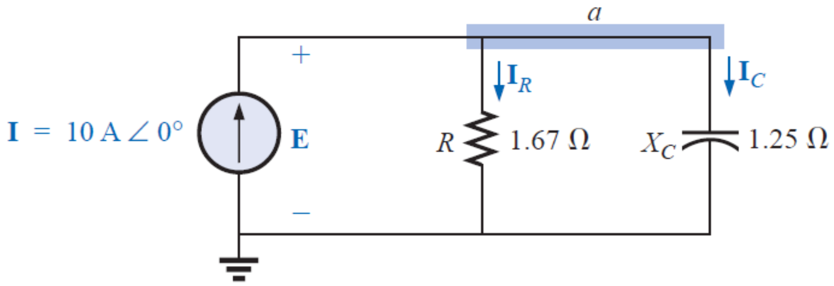
$$= (20\text{ V} \angle 53.13°)(0.4\text{ S} \angle -90°)$$

$$= \mathbf{8\ A \angle -36.87°}$$



2) R-C

EXAMPLE: find the total impedance and the current in each branch for the network.



Solutions:

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_C = G\angle 0^\circ + B_C\angle 90^\circ = \frac{1}{1.67\ \Omega}\angle 0^\circ + \frac{1}{1.25\ \Omega}\angle 90^\circ$$

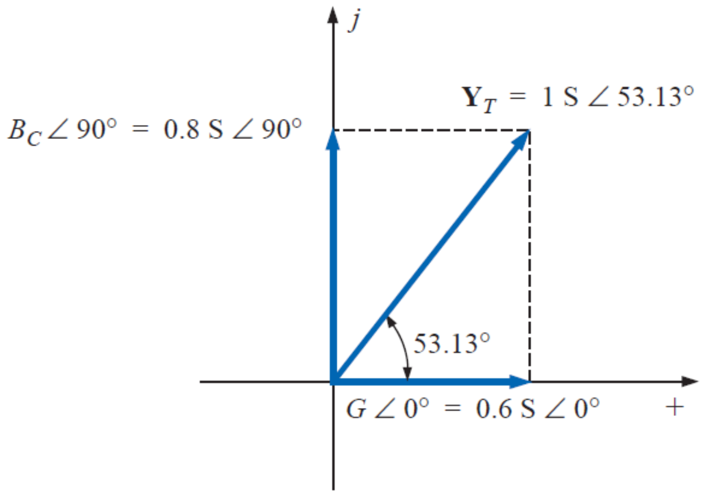
$$= 0.6\ \text{S}\angle 0^\circ + 0.8\ \text{S}\angle 90^\circ = 0.6\ \text{S} + j\,0.8\ \text{S} = \mathbf{1.0\ S\angle 53.13^\circ}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{1.0\ \text{S}\angle 53.13^\circ} = \mathbf{1\ \Omega\angle -53.13^\circ}$$

Or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_C} = \frac{(1.67\ \Omega\angle 0^\circ)(1.25\ \Omega\angle -90^\circ)}{1.67\ \Omega\angle 0^\circ + 1.25\ \Omega\angle -90^\circ}$$

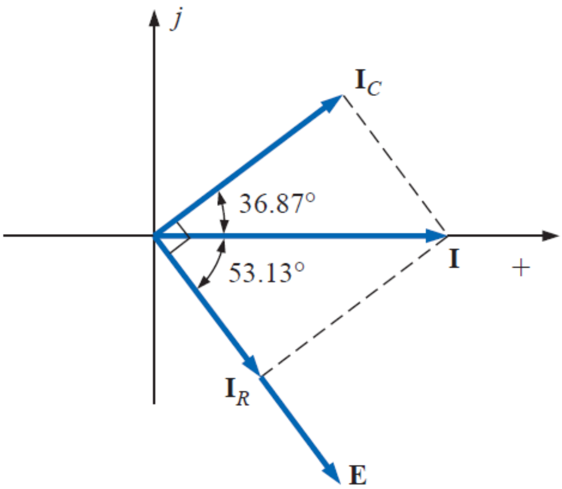
$$= \frac{2.09\angle -90^\circ}{2.09\angle -36.81^\circ} = \mathbf{1\ \Omega\angle -53.19^\circ}$$



$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \frac{\mathbf{I}}{\mathbf{Y}_T} = \frac{10\ \text{A}\angle 0^\circ}{1\ \text{S}\angle 53.13^\circ} = \mathbf{10\ V\angle -53.13^\circ}$$

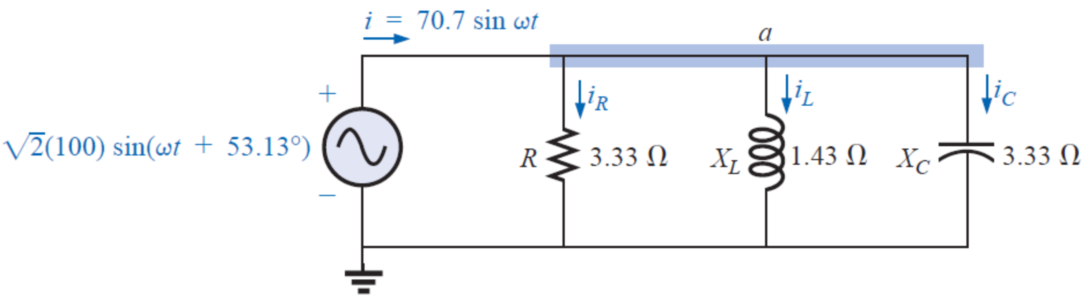
$$\mathbf{I}_R = (E\angle\theta)(G\angle 0^\circ)$$
$$= (10\ \text{V}\angle -53.13^\circ)(0.6\ \text{S}\angle 0^\circ) = \mathbf{6\ A\angle -53.13^\circ}$$

$$\mathbf{I}_C = (E\angle\theta)(B_C\angle 90^\circ)$$
$$= (10\ \text{V}\angle -53.13^\circ)(0.8\ \text{S}\angle 90^\circ) = \mathbf{8\ A\angle 36.87^\circ}$$



3) R-L-C

EXAMPLE: find the total impedance and the current in each branch for the network.



Solutions:

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G\angle 0^\circ + B_L\angle -90^\circ + B_C\angle 90^\circ$$

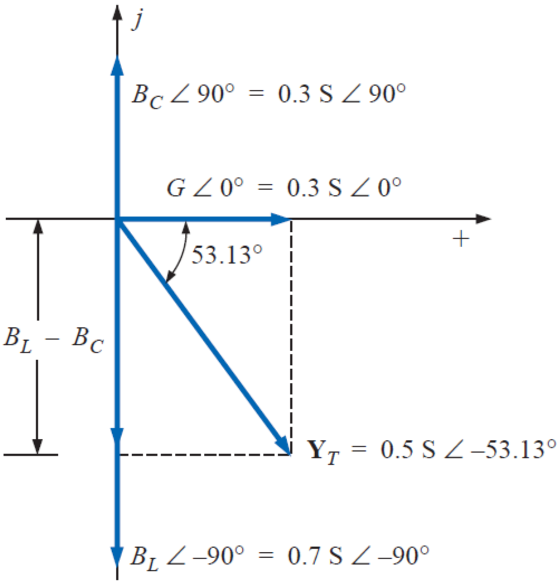
$$= \frac{1}{3.33\ \Omega}\angle 0^\circ + \frac{1}{1.43\ \Omega}\angle -90^\circ + \frac{1}{3.33\ \Omega}\angle 90^\circ$$

$$= 0.3\text{ S}\angle 0^\circ + 0.7\text{ S}\angle -90^\circ + 0.3\text{ S}\angle 90^\circ$$

$$= 0.3\text{ S} - j\,0.7\text{ S} + j\,0.3\text{ S}$$

$$= 0.3\text{ S} - j\,0.4\text{ S} = \mathbf{0.5\ S\angle -53.13^\circ}$$

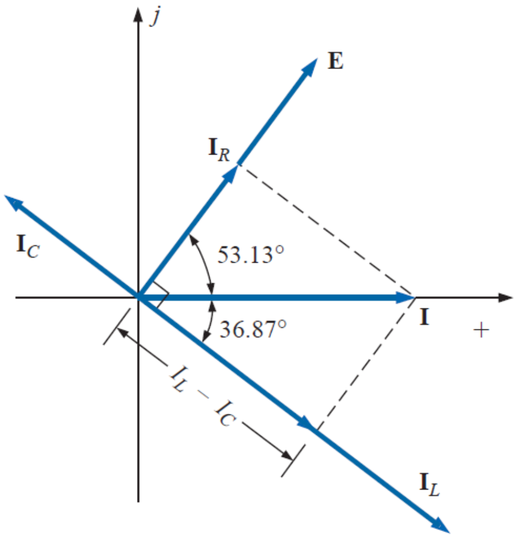
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\text{ S}\angle -53.13^\circ} = \mathbf{2\ \Omega\angle 53.13^\circ}$$



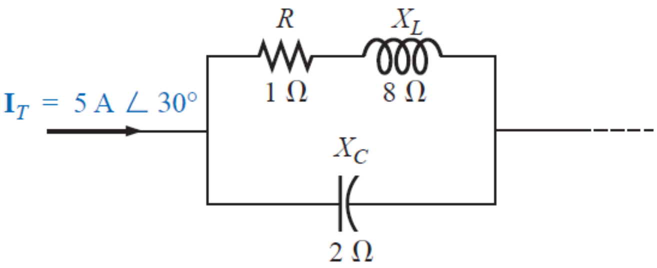
$$\mathbf{I}_R = (E \angle \theta)(G \angle 0°)$$
$$= (100\ \text{V} \angle 53.13°)(0.3\ \text{S} \angle 0°) = \mathbf{30\ A \angle 53.13°}$$

$$\mathbf{I}_L = (E \angle \theta)(B_L \angle -90°)$$
$$= (100\ \text{V} \angle 53.13°)(0.7\ \text{S} \angle -90°) = \mathbf{70\ A \angle -36.87°}$$

$$\mathbf{I}_C = (E \angle \theta)(B_C \angle 90°)$$
$$= (100\ \text{V} \angle 53.13°)(0.3\ \text{S} \angle +90°) = \mathbf{30\ A \angle 143.13°}$$



EXAMPLE: Using the current divider rule, find the current through each parallel branch.



Solutions:

$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C \mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega \angle -90^\circ)(5\ \text{A} \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A} \angle -60^\circ}{1 + j\,6}$$

$$= \frac{10\ \text{A} \angle -60^\circ}{6.083 \angle 80.54^\circ} \cong \mathbf{1.644\ A \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L} \mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\,8\ \Omega)(5\ \text{A} \angle 30^\circ)}{6.08\ \Omega \angle 80.54^\circ}$$

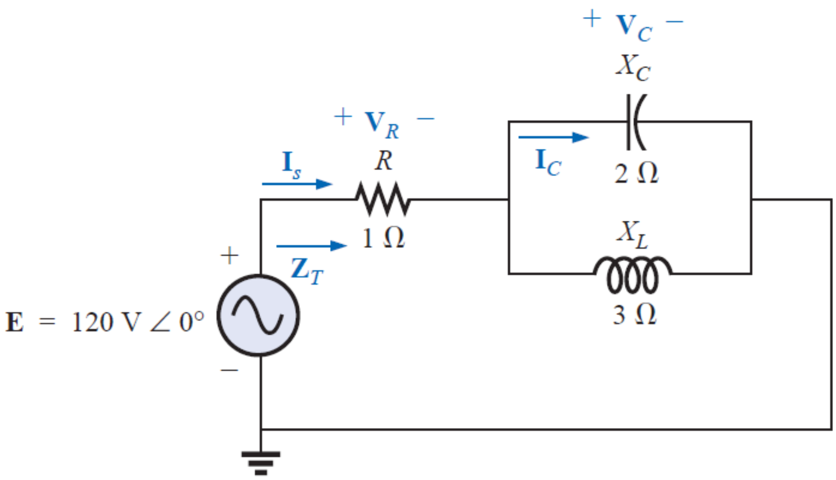
$$= \frac{(8.06 \angle 82.87^\circ)(5\ \text{A} \angle 30^\circ)}{6.08 \angle 80.54^\circ} = \frac{40.30\ \text{A} \angle 112.87^\circ}{6.083 \angle 80.54^\circ}$$

$$= \mathbf{6.625\ A \angle 32.33^\circ}$$

Series-Parallel ac Networks

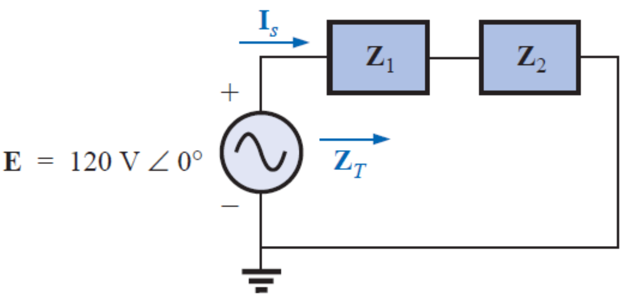
EXAMPLE: For the network

a. Calculate $\mathbf{Z}_T$.
b. Determine $\mathbf{I}_s$.
c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_C$.
d. Find $\mathbf{I}_C$.



Solutions:

a)



$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$\mathbf{Z}_1 = R \angle 0° = 1\ \Omega \angle 0°$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-j\,X_C + j\,X_L} = \frac{(2\ \Omega \angle -90°)(3\ \Omega \angle 90°)}{-j\,2\ \Omega + j\,3\ \Omega}$$

$$= \frac{6\ \Omega \angle 0°}{j\,1} = \frac{6\ \Omega \angle 0°}{1 \angle 90°} = 6\ \Omega \angle -90°$$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\,6\ \Omega = \mathbf{6.08\ \Omega \angle -80.54°}$$

b)

$$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\ \text{V} \angle 0°}{6.08\ \Omega \angle -80.54°} = \mathbf{19.74\ A \angle 80.54°}$$

c)

$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\ \text{A} \angle 80.54°)(1\ \Omega \angle 0°) = \mathbf{19.74\ V \angle 80.54°}$$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\ \text{A} \angle 80.54°)(6\ \Omega \angle -90°)$$

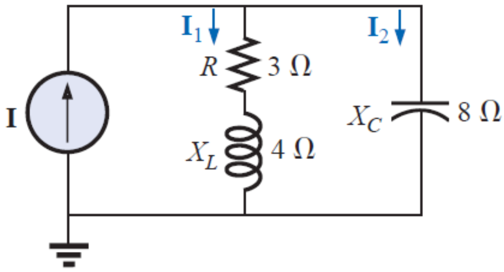
$$= \mathbf{118.44\ V \angle -9.46°}$$

d)

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\ \text{V} \angle -9.46°}{2\ \Omega \angle -90°} = \mathbf{59.22\ A \angle 80.54°}$$

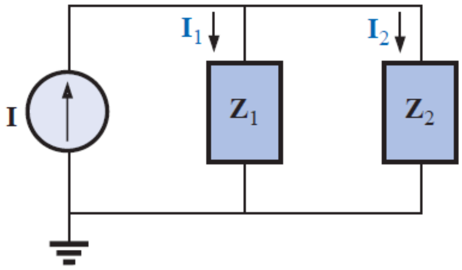
EXAMPLE: For the network

a. If **I** is 50 A $\angle 30^\circ$, calculate $\mathbf{I}_1$ using the current divider rule.
b. Repeat part (a) for $\mathbf{I}_2$.
c. Verify Kirchhoff's current law at one node.



Solutions:

a)



$$\mathbf{Z}_1 = R + j X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega \angle 53.13^\circ$$

$$\mathbf{Z}_2 = -j X_C = -j\,8\ \Omega = 8\ \Omega \angle -90^\circ$$

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\ \Omega \angle -90^\circ)(50\ \text{A} \angle 30^\circ)}{(-j\,8\ \Omega) + (3\ \Omega + j\,4\ \Omega)} = \frac{400 \angle -60^\circ}{3 - j\,4}$$

$$= \frac{400 \angle -60^\circ}{5 \angle -53.13^\circ} = \mathbf{80\ A \angle -6.87^\circ}$$

b)

$$\mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(5\ \Omega \angle 53.13^\circ)(50\ \text{A} \angle 30^\circ)}{5\ \Omega \angle -53.13^\circ} = \frac{250 \angle 83.13^\circ}{5 \angle -53.13^\circ}$$

$$= \mathbf{50\ A \angle 136.26^\circ}$$

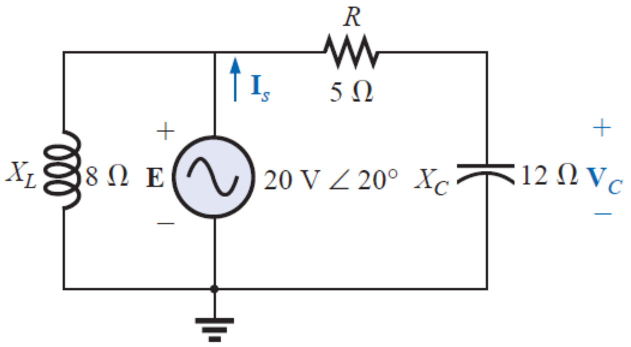
c)

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$
$$50\text{ A }\angle 30^\circ = 80\text{ A }\angle -6.87^\circ + 50\text{ A }\angle 136.26^\circ$$
$$= (79.43 - j\,9.57) + (-36.12 + j\,34.57)$$
$$= 43.31 + j\,25.0$$
$$50\text{ A }\angle 30^\circ = 50\text{ A }\angle 30^\circ \quad \text{(checks)}$$

EXAMPLE: For the network

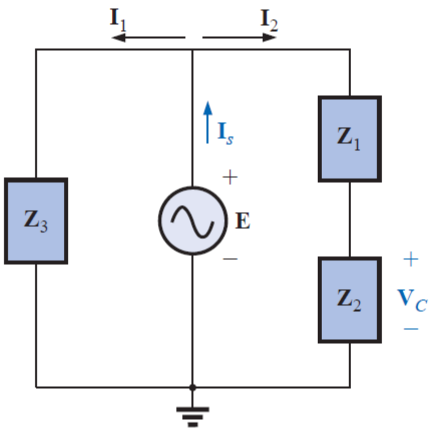
a. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.

b. Calculate the current $\mathbf{I}_s$.



Solutions:

a)



$$\mathbf{V}_C = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(12\ \Omega \angle -90^\circ)(20\text{ V }\angle 20^\circ)}{5\ \Omega - j\,12\ \Omega} = \frac{240\text{ V }\angle -70^\circ}{13 \angle -67.38^\circ}$$
$$= \mathbf{18.46\ V \angle -2.62^\circ}$$

b)

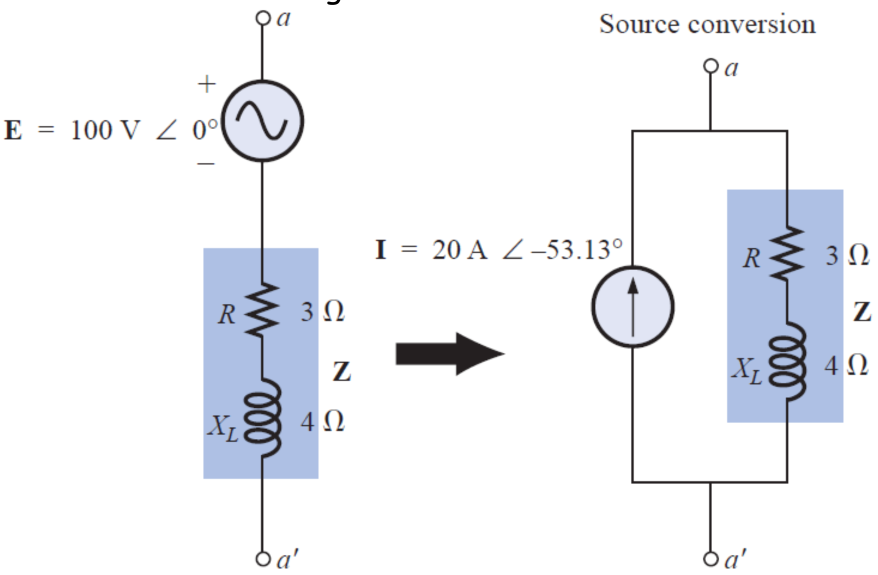
$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{20\text{ V }\angle 20^\circ}{8\ \Omega \angle 90^\circ} = 2.5\text{ A }\angle -70^\circ$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\text{ V }\angle 20^\circ}{13\ \Omega \angle -67.38^\circ} = 1.54\text{ A }\angle 87.38^\circ$$

$$\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2$$
$$= 2.5\text{ A }\angle -70^\circ + 1.54\text{ A }\angle 87.38^\circ$$
$$= (0.86 - j\,2.35) + (0.07 + j\,1.54)$$
$$\mathbf{I}_s = 0.93 - j\,0.81 = \mathbf{1.23\ A \angle -41.05^\circ}$$

Methods of Analysis (ac)

EXAMPLE: Convert the voltage source to a current source.

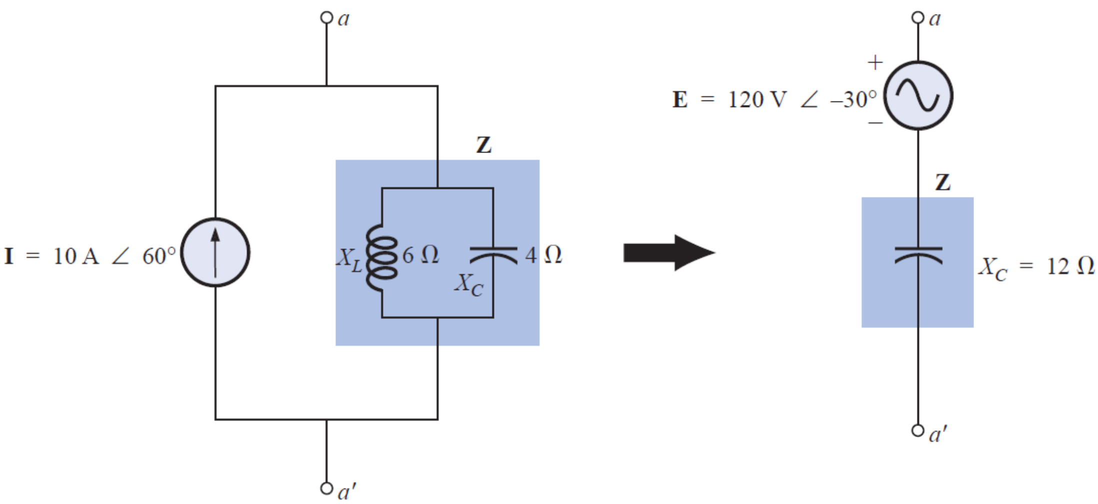


Solutions:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{100\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ}$$

$$= \mathbf{20\ A\ \angle -53.13^\circ}$$

EXAMPLE: Convert the current source to a voltage source.



Solutions:

$$\mathbf{Z} = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-j\,X_C + j\,X_L}$$

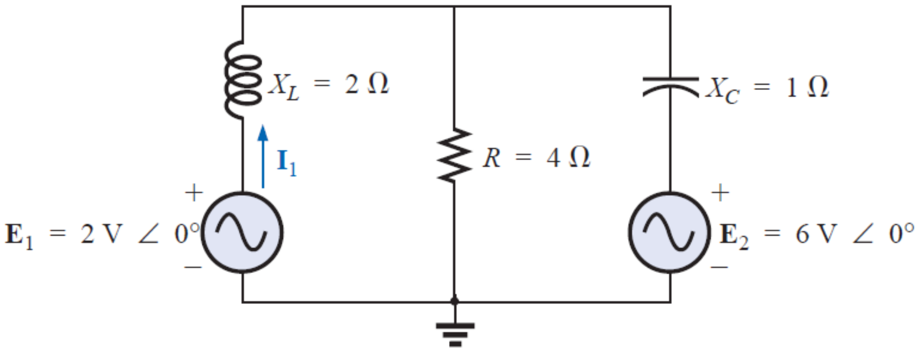
$$= \frac{(4\;\Omega \angle -90°)(6\;\Omega \angle 90°)}{-j\,4\;\Omega + j\,6\;\Omega} = \frac{24\;\Omega \angle 0°}{2 \angle 90°}$$

$$= \mathbf{12\;\Omega \angle -90°} \qquad \text{[Fig. 17.7(b)]}$$

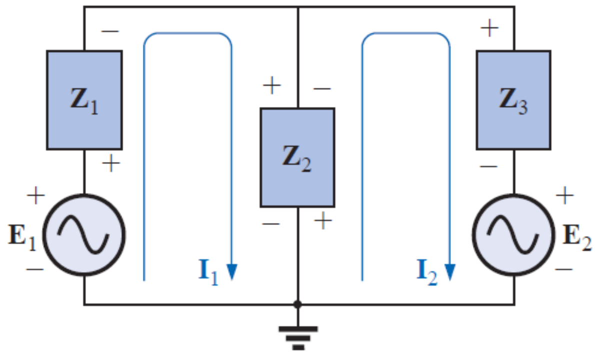
$$\mathbf{E} = \mathbf{IZ} = (10\text{ A} \angle 60°)(12\;\Omega \angle -90°)$$

$$= \mathbf{120\text{ V} \angle -30°} \qquad \text{[Fig. 17.7(b)]}$$

EXAMPLE: Using the general approach to mesh analysis, find the current **I**1.



Solutions:



$\mathbf{Z}_1 = +j\,X_L = +j\,2\;\Omega$ $\quad$ $\mathbf{E}_1 = 2\text{ V} \angle 0°$

$\mathbf{Z}_2 = R = 4\;\Omega$ $\quad$ $\mathbf{E}_2 = 6\text{ V} \angle 0°$

$\mathbf{Z}_3 = -j\,X_C = -j\,1\;\Omega$

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2$$

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\qquad &= \mathbf{E}_1 \\ -\mathbf{I}_1\mathbf{Z}_2 \qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2 \end{aligned}$$

$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$

$$= \frac{\mathbf{E}_1(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{E}_2(\mathbf{Z}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - (\mathbf{Z}_2)^2}$$

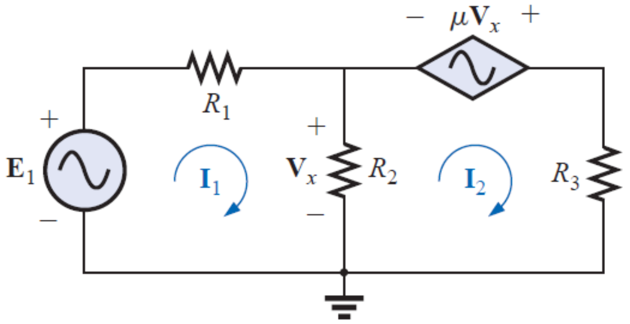
$$= \frac{(\mathbf{E}_1 - \mathbf{E}_2)\mathbf{Z}_2 + \mathbf{E}_1\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

$$\mathbf{I}_1 = \frac{(2\text{ V} - 6\text{ V})(4\ \Omega) + (2\text{ V})(-j\,1\ \Omega)}{(+j\,2\ \Omega)(4\ \Omega) + (+j\,2\ \Omega)(-j\,2\ \Omega) + (4\ \Omega)(-j\,2\ \Omega)}$$

$$= \frac{-16 - j\,2}{j\,8 - j^2 2 - j\,4} = \frac{-16 - j\,2}{2 + j\,4} = \frac{16.12\text{ A}\angle -172.87^\circ}{4.47\angle 63.43^\circ}$$

$$= \mathbf{3.61\ A\angle -236.30^\circ} \quad \text{or} \quad \mathbf{3.61\ A\angle 123.70^\circ}$$

EXAMPLE: Write the mesh currents for the network having a dependent voltage source.



Solutions:

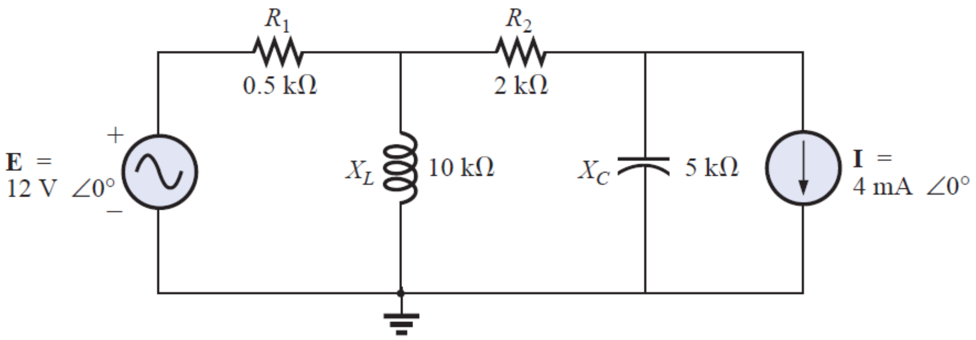
$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu\mathbf{V}_x - \mathbf{I}_2 R_3 = 0 \qquad \mathbf{V}_x = (\mathbf{I}_1 - \mathbf{I}_2)R_2$$

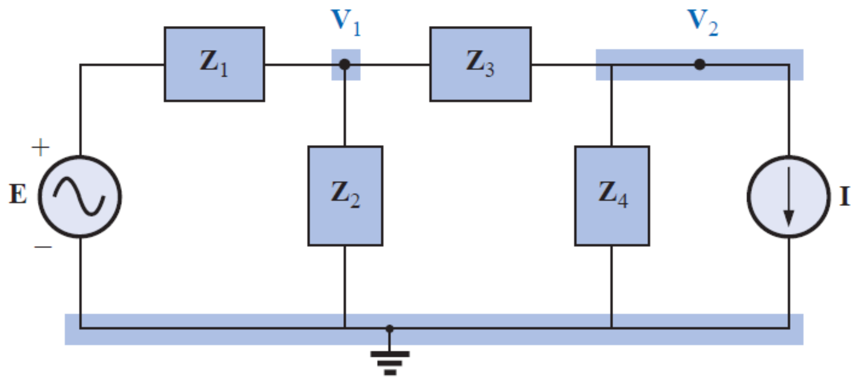
$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I} - \mathbf{I}_2) = 0$$

$$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu R_2(\mathbf{I}_1 - \mathbf{I}_2) - \mathbf{I}_2 R_3 = 0$$

EXAMPLE: Write the nodal equations for the network having a dependent current source.



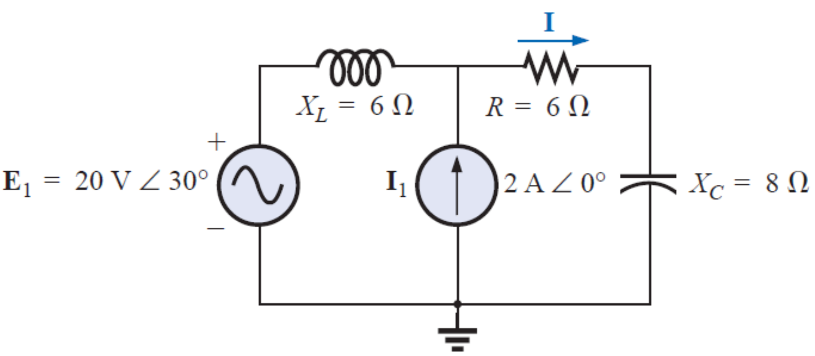
Solutions:



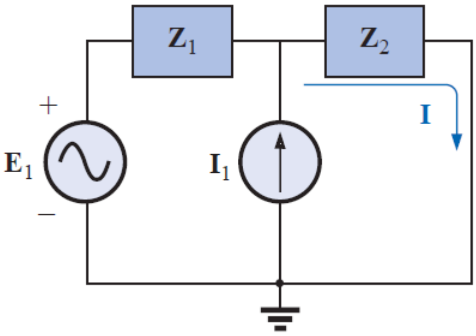
$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right]=\frac{\mathbf{E}_1}{\mathbf{Z}_1}$$

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}\right]-\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right]=-\mathbf{I}$$

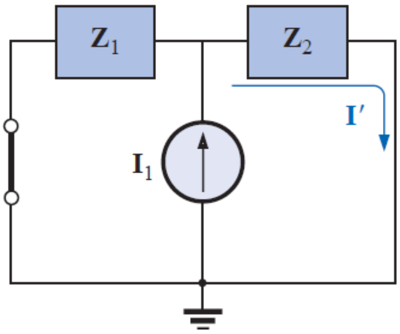
EXAMPLE: Using superposition, find the current **I** through the 6-Ω resistor.



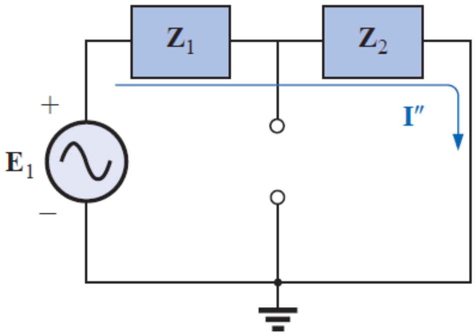
Solutions:



$\mathbf{Z}_1 = j\,6\,\Omega \qquad \mathbf{Z}_2 = 6 - j\,8\,\Omega$



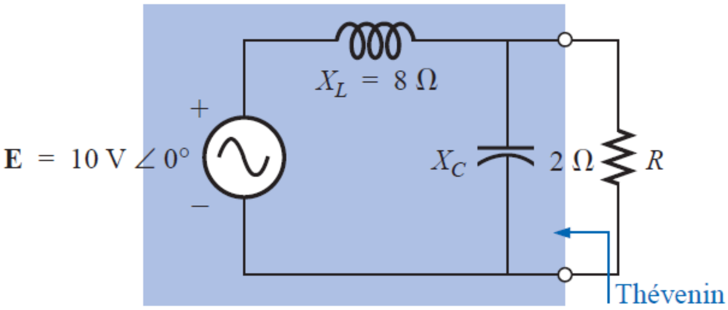
$$\mathbf{I}' = \frac{\mathbf{Z}_1\mathbf{I}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,6\,\Omega)(2\text{ A})}{j\,6\,\Omega + 6\,\Omega - j\,8\,\Omega} = \frac{j\,12\text{ A}}{6 - j\,2}$$
$$= \frac{12\text{ A}\,\angle 90^\circ}{6.32\,\angle -18.43^\circ}$$
$$\mathbf{I}' = 1.9\text{ A}\,\angle 108.43^\circ$$



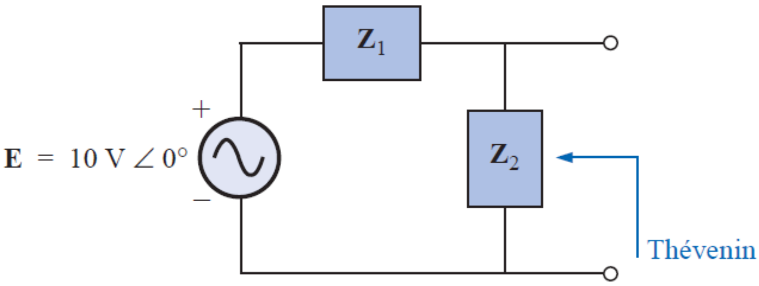
$$\mathbf{I}'' = \frac{\mathbf{E}_1}{\mathbf{Z}_T} = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\text{ V}\,\angle 30^\circ}{6.32\,\Omega\,\angle -18.43^\circ}$$
$$= 3.16\text{ A}\,\angle 48.43^\circ$$

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}''$$
$$= 1.9\text{ A}\,\angle 108.43^\circ + 3.16\text{ A}\,\angle 48.43^\circ$$
$$= (-0.60\text{ A} + j\,1.80\text{ A}) + (2.10\text{ A} + j\,2.36\text{ A})$$
$$= 1.50\text{ A} + j\,4.16\text{ A}$$
$$\mathbf{I} = \mathbf{4.42\text{ A}\,\angle 70.2^\circ}$$

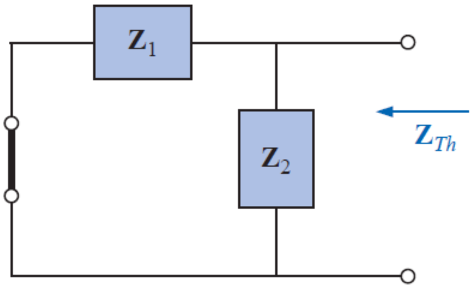
EXAMPLE: Find the Thévenin equivalent circuit for the network external to resistor *R*.



Solutions:

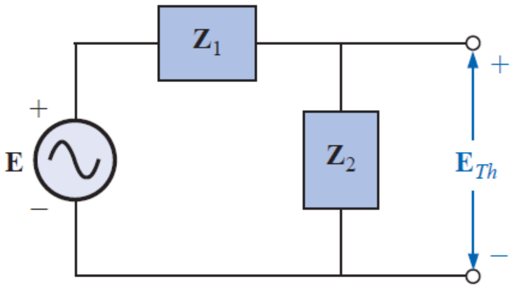


$$\mathbf{Z}_1 = j X_L = j\,8\ \Omega \qquad \mathbf{Z}_2 = -j X_C = -j\,2\ \Omega$$



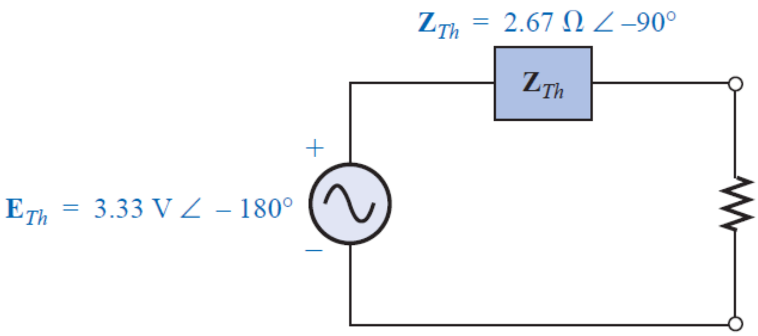
$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\ \Omega)(-j\,2\ \Omega)}{j\,8\ \Omega - j\,2\ \Omega} = \frac{-j^2 16\ \Omega}{j\,6} = \frac{16\ \Omega}{6\ \angle 90^\circ}$$

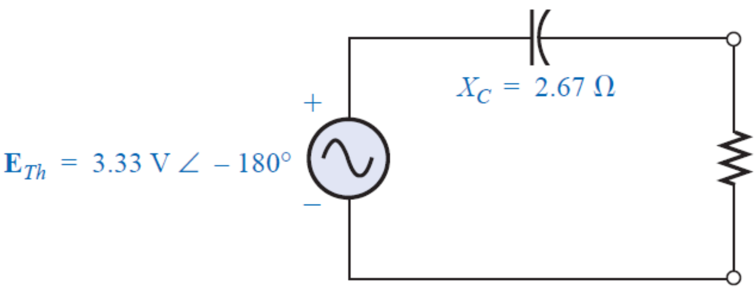
$$= \mathbf{2.67\ \Omega\ \angle -90^\circ}$$



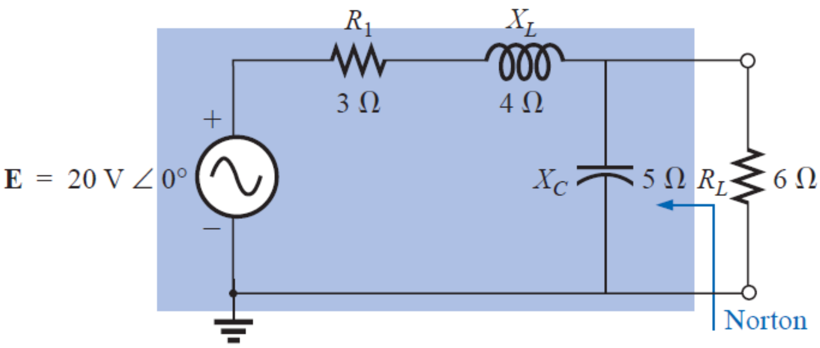
$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(voltage divider rule)}$$

$$= \frac{(-j\,2\,\Omega)(10\text{ V})}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j\,20\text{ V}}{j\,6} = \mathbf{3.33\ V\ \angle -180^\circ}$$

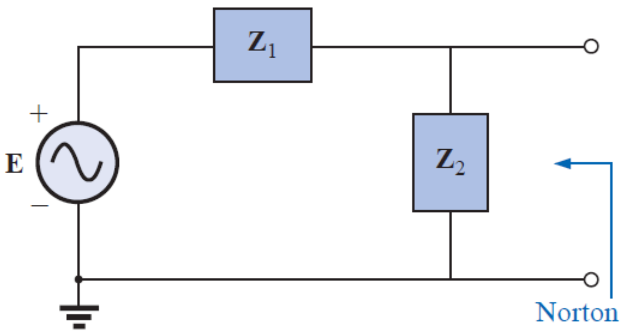




EXAMPLE: Determine the Norton equivalent circuit for the network external to the 6-Ω resistor.

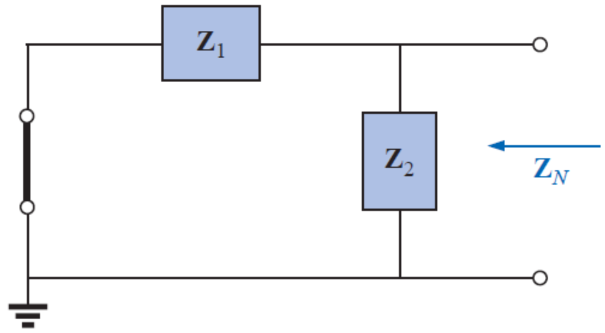


Solutions:

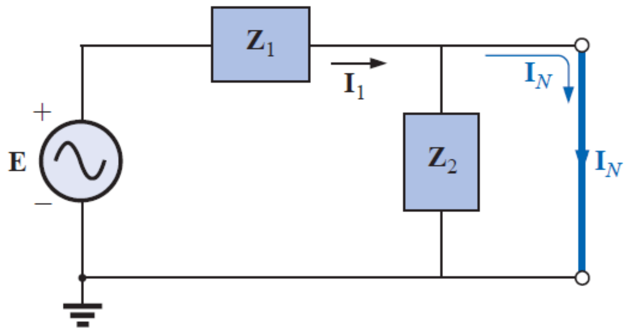


$$\mathbf{Z}_1 = R_1 + j\,X_L = 3\,\Omega + j\,4\,\Omega = 5\,\Omega\,\angle 53.13^\circ$$

$$\mathbf{Z}_2 = -j\,X_C = -j\,5\,\Omega$$



$$Z_N = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(5\ \Omega \angle 53.13^\circ)(5\ \Omega \angle -90^\circ)}{3\ \Omega + j\,4\ \Omega - j\,5\ \Omega} = \frac{25\ \Omega \angle -36.87^\circ}{3 - j\,1}$$
$$= \frac{25\ \Omega \angle -36.87^\circ}{3.16 \angle -18.43^\circ} = 7.91\ \Omega \angle -18.44^\circ = \mathbf{7.50\ \Omega - j\,2.50\ \Omega}$$



$$I_N = I_1 = \frac{E}{Z_1} = \frac{20\ V \angle 0^\circ}{5\ \Omega \angle 53.13^\circ} = \mathbf{4\ A \angle -53.13^\circ}$$