Electrical Engineering Fundamentals

First class

AC

Lecture 11

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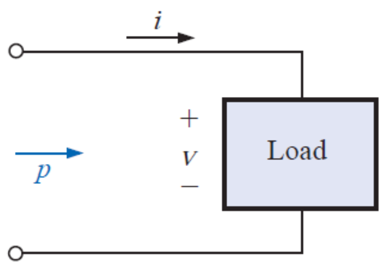
Power (ac)

AVERAGE POWER AND POWER FACTOR

The average power, or **real power** as it is sometimes called, is the power delivered to and dissipated by the load. It corresponds to the power calculations performed for dc networks.



$$p = vi$$

Since v and i are sinusoidal quantities, let us establish a general case where:

$$v = V_m \sin(\omega t + \theta)$$

$$i = I_m \sin \omega t$$

* The angle (θ) is the phase angle between v and i.

Substituting the above equations for v and i into the power equation will result in

$$p = V_m I_m \sin \omega t \sin(\omega t + \theta)$$

If we now apply a number of trigonometric identities, the following form for the power equation will result:

$$p = VI \cos \theta(1 - \cos 2\omega t) + VI \sin \theta(\sin 2\omega t)$$

* where V and I are the **rms** values.

$$V = \frac{Vm}{\sqrt{2}} \quad and \quad I = \frac{Im}{\sqrt{2}}$$

So that the power is

$$p = \underbrace{VI\cos\theta}_{\text{Average}} - \underbrace{VI\cos\theta}_{\text{Peak}}\underbrace{\cos 2\omega t}_{2x} + \underbrace{VI\sin\theta}_{\text{Peak}}\underbrace{\sin 2\omega t}_{2x}$$

The average power equal

$$\boxed{P = VI\cos\theta}$$

* The magnitude of average power delivered is independent of whether v leads i or i leads v.

- for resistor $\theta = 0$ then $P = VI\cos 0 = VI$
- for Inductor v leads i by 90°, then $P = VI\cos 90 = 0$
- for Capacitor i leads v by 90°, then $P = VI\cos 90 = 0$

Power factor

The power factor is the factor that has significant control over the delivered power level.

$$\boxed{\text{Power factor} = F_p = \cos\theta}$$

- Capacitive networks have leading power factors,
- Inductive networks have lagging power factors.

APPARENT POWER

It is a power rating of significant usefulness in the description and analysis of sinusoidal ac networks and in the maximum rating of a number of electrical components and systems.

$$\boxed{S = VI} \qquad \text{(volt-amperes, VA)}$$

$$S = I^2 Z \quad \text{(VA)}$$

$$S = \frac{V^2}{Z} \quad \text{(VA)}$$

Therefore

$$P = S\cos\theta \quad \text{(W)}$$

$$F_p = \cos\theta = \frac{P}{S}$$

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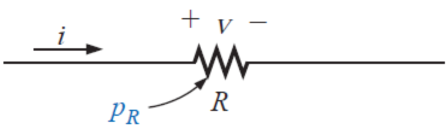
1) RESISTIVE CIRCUIT

For a purely resistive circuit, *v* and *i* are in phase, and $\theta = 0°$,



$$p_R = VI\cos(0°)(1 - \cos 2\omega t) + VI\sin(0°)\sin 2\omega t$$
$$= VI(1 - \cos 2\omega t) + 0$$

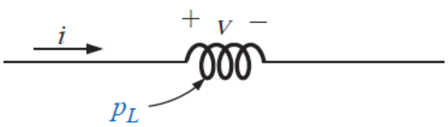
$$p_R = VI - VI\cos 2\omega t$$

The **average (real) power** is

$$P = VI = \frac{V_m I_m}{2} = I^2 R = \frac{V^2}{R} \quad \text{(watts, W)}$$

2) INDUCTIVE CIRCUIT

For a purely inductive circuit, *v* leads *i* by 90°, $\theta = 90°$.



$$p_L = VI\cos(90°)(1 - \cos 2\omega t) + VI\sin(90°)(\sin 2\omega t)$$
$$= 0 + VI\sin 2\omega t$$

$$\boxed{p_L = VI\sin 2\omega t}$$

The **reactive power** is

$$\boxed{Q = VI\sin\theta} \quad \text{(volt-ampere reactive, VAR)}$$

$$\boxed{Q_L = I^2 X_L} \quad \text{(VAR)}$$

$$\boxed{Q_L = \frac{V^2}{X_L}} \quad \text{(VAR)}$$

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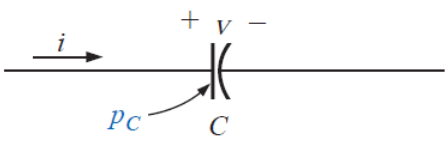
3) CAPACITIVE CIRCUIT

For a purely capacitive circuit, *i* leads *v* by 90°, θ =-90°.



$$p_C = VI\cos(-90°)(1 - \cos 2\omega t) + VI\sin(-90°)(\sin 2\omega t)$$
$$= 0 - VI\sin 2\omega t$$

$$\boxed{p_C = -VI\sin 2\omega t}$$

The **reactive power** is

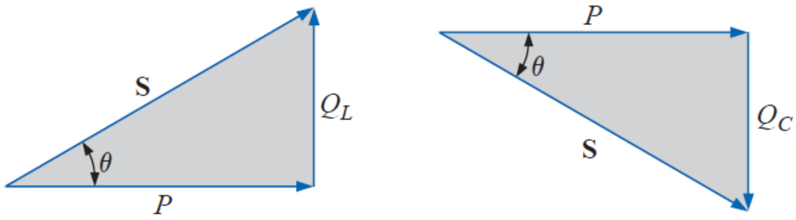
$$\boxed{Q = VI\sin\theta} \quad \text{(volt-ampere reactive, VAR)}$$

$$\boxed{Q_C = I^2 X_C}$$

$$Q_C = \frac{V^2}{X_C}$$

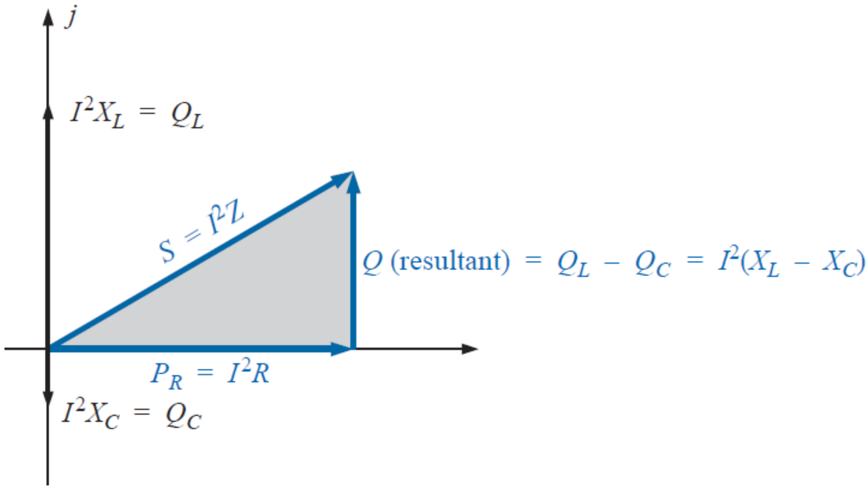
THE POWER TRIANGLE

The three quantities **average power (P)**, **apparent power (S)**, and **reactive power (Q)** can be related in the vector domain by



$$S = P + jQ$$
$$Q = Q_L - Q_C$$

$$S^2 = P^2 + Q^2$$



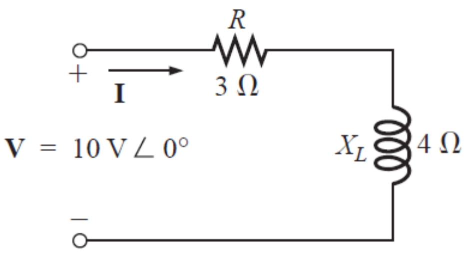
$$S = \sqrt{P^2 + Q^2}$$

$$F_p = \cos\theta = \frac{P}{S}$$

1. Find the real power and reactive power for each branch of the circuit.

2. The total real power of the system (PT) is then the sum of the average power delivered to each branch.
3. The total reactive power (QT) is the difference between the reactive power of the inductive loads and that of the capacitive loads.
4. The total apparent power is $S_T = \sqrt{P_T + Q_T}$.
5. The total power factor is P_T / S_T .

EXAMPLE: Find the total number of watts, volt-amperes reactive, and volt-amperes for the network.



Solutions:

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10\text{ V}\angle 0°}{3\ \Omega + j\,4\ \Omega} = \frac{10\text{ V}\angle 0°}{5\ \Omega\angle 53.13°} = 2\text{ A}\angle -53.13°$$

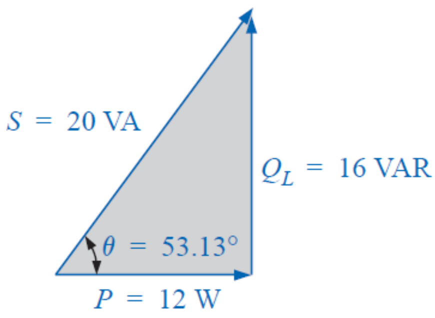
$$P = I^2R = (2\text{ A})^2(3\ \Omega) = 12\text{ W}$$

$$Q_L = I^2X_L = (2\text{ A})^2(4\ \Omega) = 16\text{ VAR }(L)$$

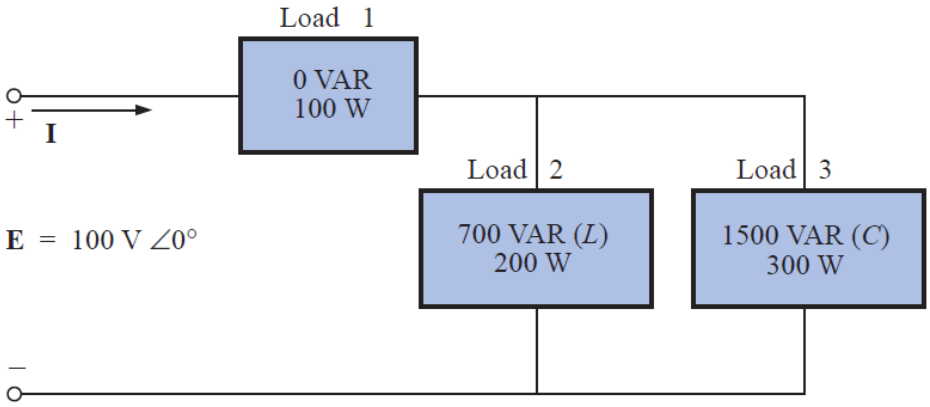
$$\mathbf{S} = P + j\,Q_L = 12\text{ W} + j\,16\text{ VAR }(L) = 20\text{ VA}\angle 53.13°$$

Or

$$\mathbf{S} = \mathbf{VI}^* = (10\text{ V}\angle 0°)(2\text{ A}\angle +53.13°) = 20\text{ VA}\angle 53.13°$$



EXAMPLE: Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor *Fp* of the network. Draw the power triangle and find the current in phasor form.



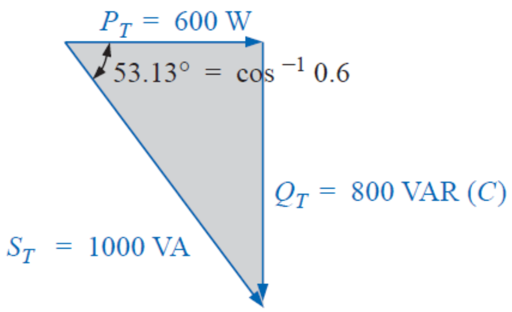
Solutions:

Load	W	VAR	VA
1	100	0	100
2	200	700 (L)	$\sqrt{(200)^2 + (700)^2} = 728.0$
3	300	1500 (C)	$\sqrt{(300)^2 + (1500)^2} = 1529.71$
	$P_T = \mathbf{600}$ Total power dissipated	$Q_T = \mathbf{800\ (C)}$ Resultant reactive power of network	$S_T = \sqrt{(600)^2 + (800)^2} = \mathbf{1000}$ (Note that $S_T \neq$ sum of each branch: $1000 \neq 100 + 728 + 1529.71$)

$$F_p = \frac{P_T}{S_T} = \frac{600\text{ W}}{1000\text{ VA}} = \mathbf{0.6\ leading\ (C)}$$

$$I = 1000\text{ VA}/100\text{ V} = 10\text{ A}$$

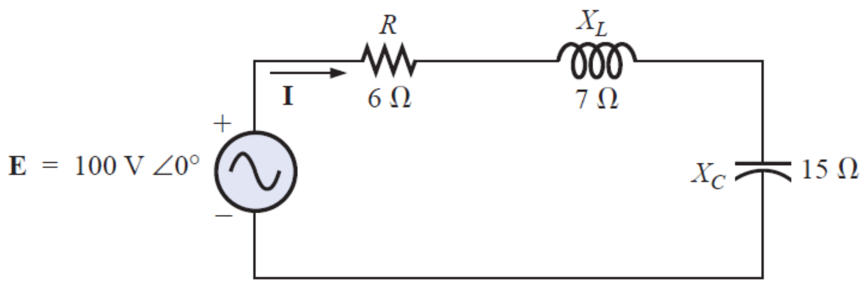
$$\mathbf{I = 10\ A\ \angle +53.13^\circ}$$



Peoblem:

a) Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor *Fp* for the network.

b) Sketch the power triangle.



EXAMPLE: An electrical device is rated 5 kVA, 100 V at a 0.6 power-factor lag. What is the impedance of the device in rectangular coordinates?

Solutions:

$S = EI = 5000 \text{ VA}$

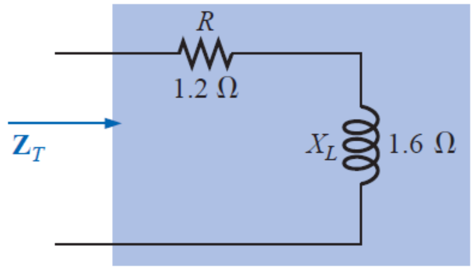
$$I = \frac{5000 \text{ VA}}{100 \text{ V}} = 50 \text{ A}$$

$\theta = \cos^{-1} 0.6 = 53.13°$

for $\mathbf{E} = 100 \text{ V} \angle 0°$,

$\mathbf{I} = 50 \text{ A} \angle -53.13°$

$$\mathbf{Z}_T = \frac{\mathbf{E}}{\mathbf{I}} = \frac{100 \text{ V} \angle 0°}{50 \text{ A} \angle -53.13°} = 2\ \Omega \angle 53.13° = \mathbf{1.2\ \Omega + \mathit{j}\, 1.6\ \Omega}$$

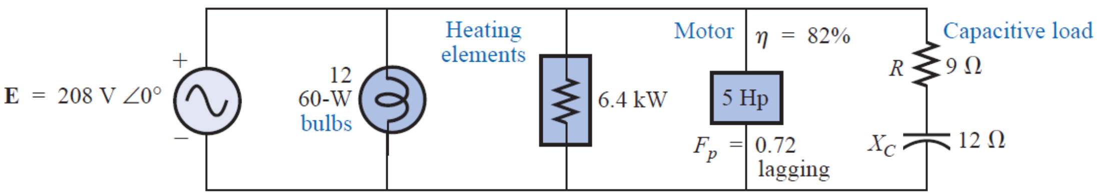


EXAMPLE: For the system

a. Find the average power, apparent power, reactive power, and *Fp* for each branch.

b. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of the system. Sketch the power triangle.

c. Find the source current *I*.



Solutions:

a)

Bulbs:

Total dissipation of applied power

$$P_1 = 12(60\text{ W}) = \mathbf{720\ W}$$
$$Q_1 = \mathbf{0\ VAR}$$
$$S_1 = P_1 = \mathbf{720\ VA}$$
$$F_{p_1} = \mathbf{1}$$

Heating elements:

Total dissipation of applied power

$$P_2 = \mathbf{6.4\ kW}$$
$$Q_2 = \mathbf{0\ VAR}$$
$$S_2 = P_2 = \mathbf{6.4\ kVA}$$
$$F_{p_2} = \mathbf{1}$$

Motor:

$$\eta = \frac{P_o}{P_i} \longrightarrow P_i = \frac{P_o}{\eta} = \frac{5(746\text{ W})}{0.82} = \mathbf{4548.78\ W} = P_3$$

$$F_p = \mathbf{0.72\ lagging}$$

$$P_3 = S_3 \cos\theta \longrightarrow S_3 = \frac{P_3}{\cos\theta} = \frac{4548.78\text{ W}}{0.72} = \mathbf{6317.75\ VA}$$

Also, $\theta = \cos^{-1} 0.72 = 43.95°$, so that

$$Q_3 = S_3 \sin\theta = (6317.75\text{ VA})(\sin 43.95°)$$
$$= (6317.75\text{ VA})(0.694) = \mathbf{4384.71\ VAR\ (\mathit{L})}$$

Capacitive load:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{208\text{ V}\angle 0°}{9\ \Omega - j\,12\ \Omega} = \frac{208\text{ V}\angle 0°}{15\ \Omega\angle -53.13°} = 13.87\text{ A}\angle 53.13°$$

$$P_4 = I^2R = (13.87\text{ A})^2 \cdot 9\ \Omega = \mathbf{1731.39\ W}$$

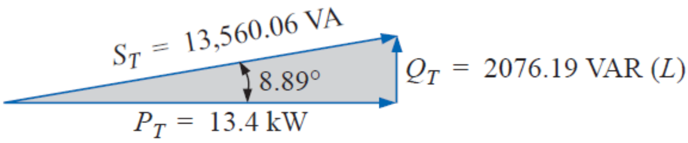
$$Q_4 = I^2X_C = (13.87\text{ A})^2 \cdot 12\ \Omega = \mathbf{2308.52\ VAR\ (\mathit{C})}$$

$$S_4 = \sqrt{P_4^2 + Q_4^2} = \sqrt{(1731.39\text{ W})^2 + (2308.52\text{ VAR})^2}$$
$$= \mathbf{2885.65\ VA}$$

$$F_p = \frac{P_4}{S_4} = \frac{1731.39\text{ W}}{2885.65\text{ VA}} = \mathbf{0.6\ leading}$$

b)

$$P_T = P_1 + P_2 + P_3 + P_4$$
$$= 720\text{ W} + 6400\text{ W} + 4548.78\text{ W} + 1731.39\text{ W}$$
$$= \mathbf{13{,}400.17\ W}$$
$$Q_T = \pm Q_1 \pm Q_2 \pm Q_3 \pm Q_4$$
$$= 0 + 0 + 4384.71\text{ VAR }(L) - 2308.52\text{ VAR }(C)$$
$$= \mathbf{2076.19\ VAR\ (L)}$$
$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13{,}400.17\text{ W})^2 + (2076.19\text{ VAR})^2}$$
$$= 13{,}560.06\text{ VA}$$
$$F_p = \frac{P_T}{S_T} = \frac{13.4\text{ kW}}{13{,}560.06\text{ VA}} = \mathbf{0.988\ lagging}$$
$$\theta = \cos^{-1} 0.988 = 8.89^\circ$$



c)

$$S_T = EI \longrightarrow I = \frac{S_T}{E} = \frac{13{,}559.89\text{ VA}}{208\text{ V}} = 65.19\text{ A}$$

Lagging power factor: **E** leads **I** by 8.89°, and

$$\mathbf{I = 65.19\ A\ \angle -8.89^\circ}$$