Electrical Engineering Fundamentals

First class

AC

Lecture 1 & 2

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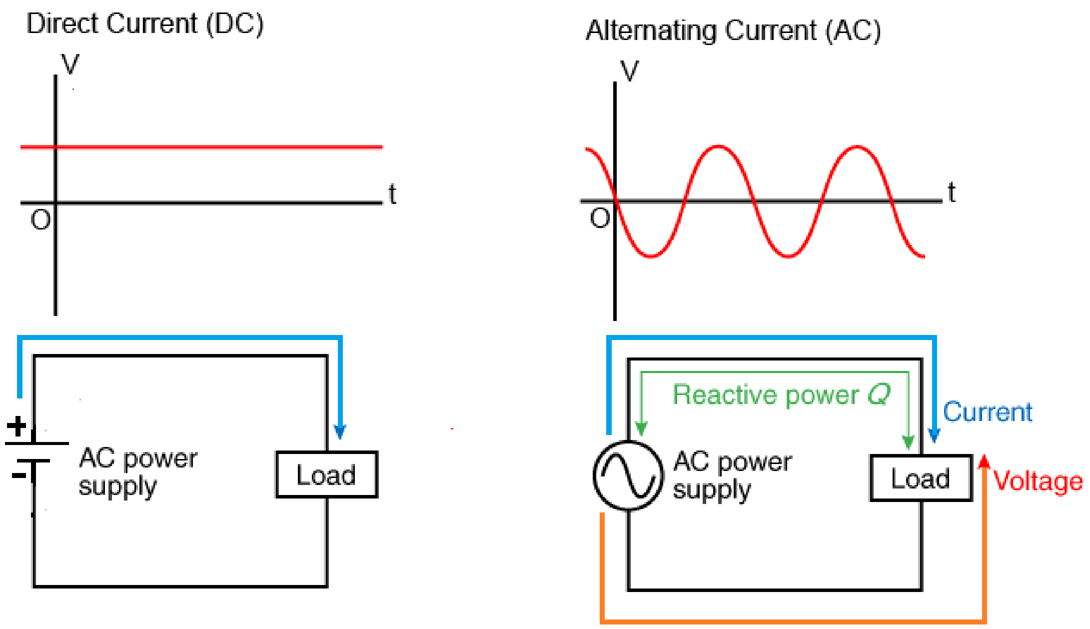
Difference Between ac and dc

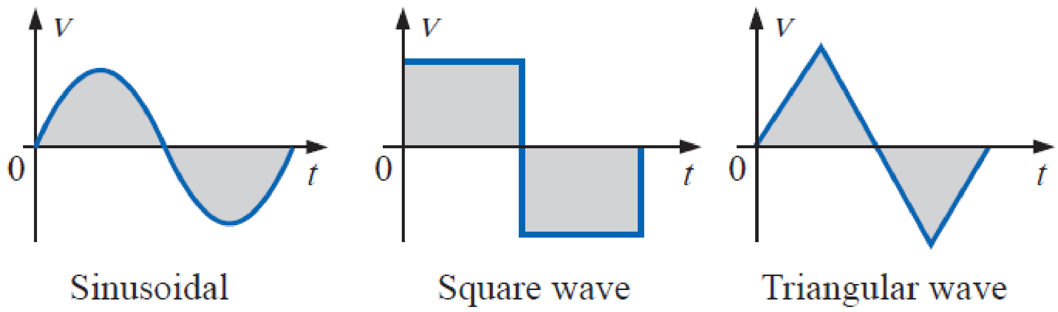
Electric current flows in two ways as an alternating current (AC) or direct current (DC). The main difference between AC and DC lies in the direction in which the electrons flow. In DC, the electrons flow steadily in a single direction, while electrons keep switching directions, going forward and then backwards in AC.



Alternating Current (AC)	Direct Current (DC)
AC is easy to be transferred over longer distances – even between two cities – without much energy loss.	DC cannot be transferred over a very long distance. It loses electric power.
The rotating magnets cause the change in direction of electric flow.	The steady magnetism makes DC flow in a single direction.
The frequency of AC is dependent upon the country. But, generally, the frequency is 50 Hz or 60 Hz.	DC has no frequency or zero frequency.
In AC the flow of current changes its direction forward and backward periodically.	It flows in a single direction steadily.
Electrons in AC keep changing their directions – backward and forward.	Electrons only move in one direction – forward.

Sinusoidal Alternating Waveforms

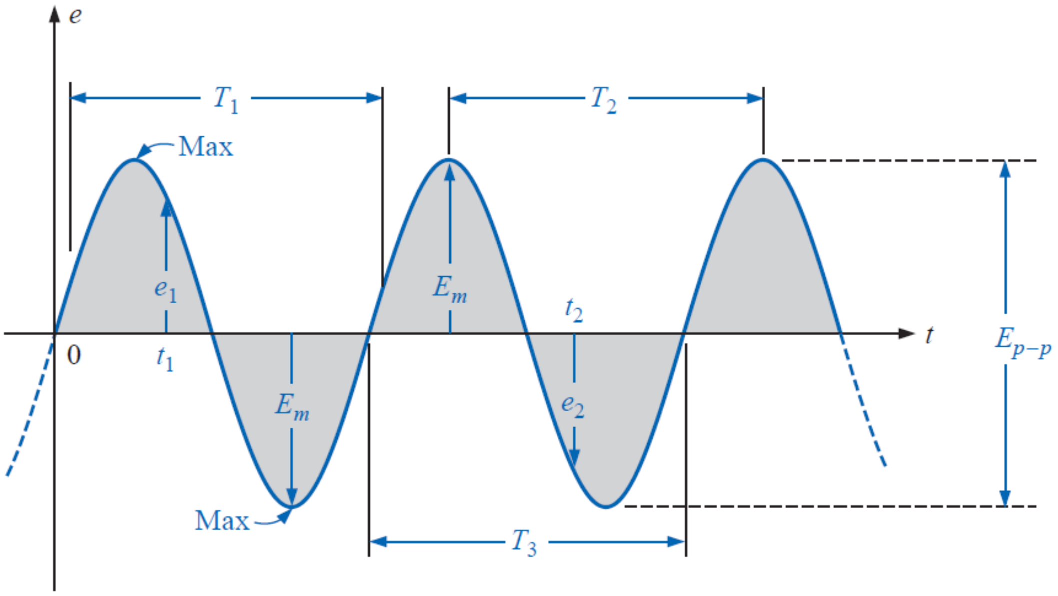
The terminology **(ac)** voltage or **(ac)** current refers to alternating voltage or current. The term alternating indicates only that waveforms alternate between two prescribed levels in a set time sequence. To be absolutely correct the term sinusoidal, square, triangular must be also applied.



The pattern of particular interest is the **sinusoidal ac** waveform voltage.

SINUSOIDAL ac VOLTAGE DEFINITIONS

The vertical scaling is in volts or amperes and the horizontal scaling is always in units of time.



Waveform: The path traced by a quantity, such as the voltage plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters ($e1$, $e2$).

Peak amplitude: The maximum value of a waveform as measured from its *average,* or *mean,* value, denoted by uppercase letters (such as *Em* for sources of voltage and *Vm* for the voltage drop across a load).

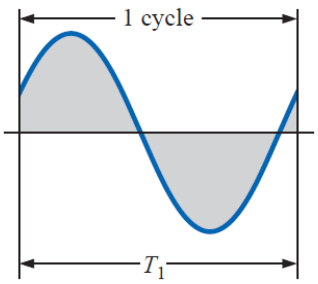
Peak value: The maximum instantaneous value of a function as measured from the zero-volt level.

Peak-to-peak value: Denoted by *Ep-p* or *Vp-p,* the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

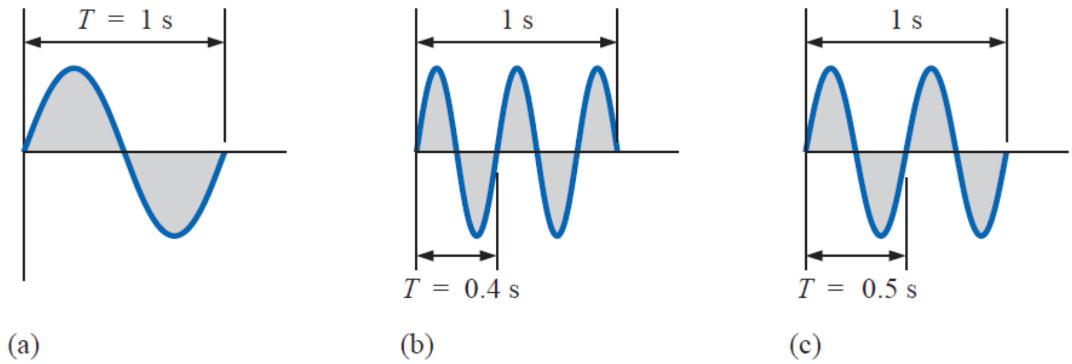
Periodic waveform: A waveform that continually repeats itself after the same time interval.

Period (*T*): The time interval between successive repetitions of a periodic waveform (the period $T1 = T2 = T3$),

Cycle: The portion of a waveform contained in *one period* of time.



Frequency (*f*): The number of cycles that occur in 1 s.



(a) is 1 cycle per second, and for (b), 2 1⁄2 cycles per second. If a waveform of similar shape had a period of 0.5 s (c), the frequency would be 2 cycles per second.

$$\boxed{1 \text{ hertz (Hz)} = 1 \text{ cycle per second (c/s)}}$$

$$\boxed{f = \frac{1}{T}} \qquad \begin{aligned} f &= \text{Hz} \\ T &= \text{seconds (s)} \end{aligned}$$

$$\boxed{T = \frac{1}{f}}$$

EXAMPLE: Find the period of a periodic waveform with a frequency of

a. 60 Hz.

b. 1000 Hz.

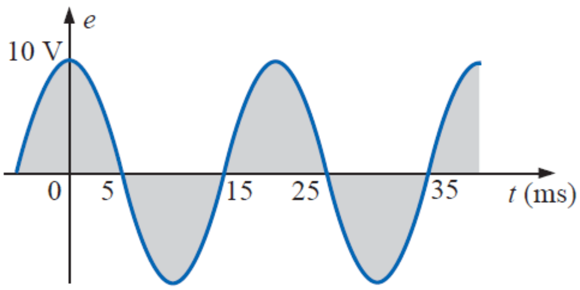
Solutions:

a. $T = \frac{1}{f} = \frac{1}{60 \text{ Hz}} \cong 0.01667 \text{ s or } \mathbf{16.67 \text{ ms}}$

(a recurring value since 60 Hz is so prevalent)

b. $T = \frac{1}{f} = \frac{1}{1000 \text{ Hz}} = 10^{-3} \text{ s} = \mathbf{1 \text{ ms}}$

EXAMPLE: Determine the frequency of the waveform



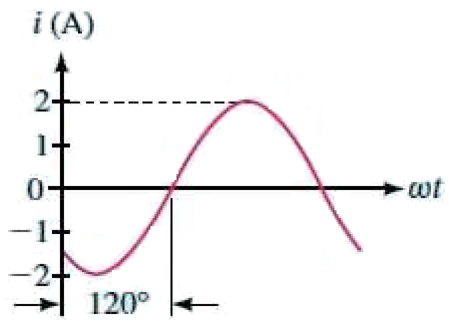
Solutions:

From the figure, $T = (25 \text{ ms} - 5 \text{ ms}) = 20 \text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3} \text{ s}} = \mathbf{50 \text{ Hz}}$$

Problem 1

Determine the equation for the waveform shown below, given f =60 Hz.



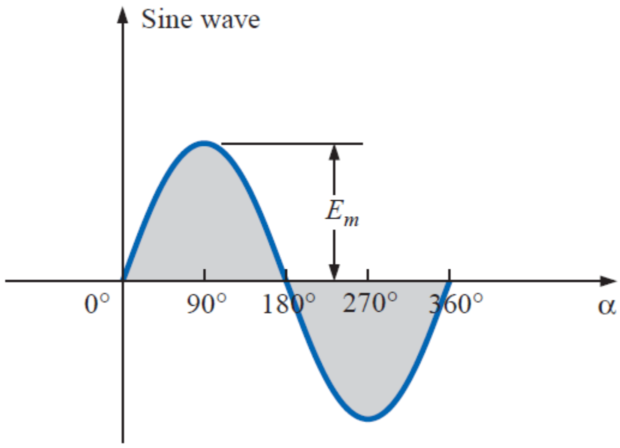
Problem 2

The current in an a.c. circuit at any time t seconds is given by:
$i = 120\sin(100\pi t + 0.36)$ amperes. Find:

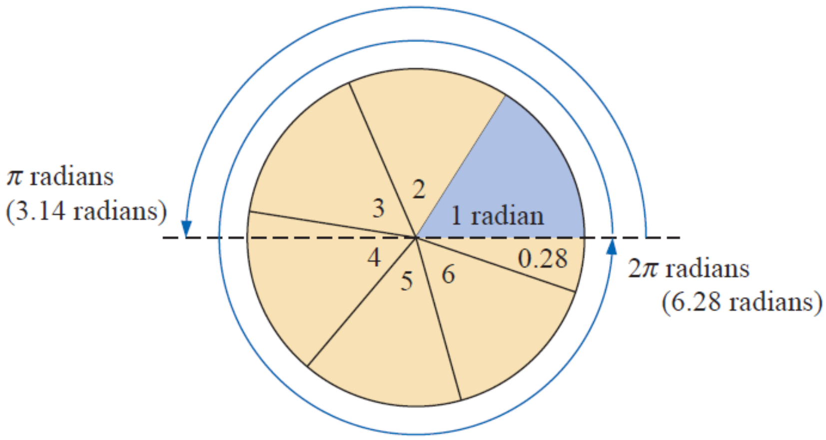
(a) The peak value, the periodic time, the frequency and phase angle relative to $120\sin 100\pi t$
(b) The value of the current when t =0
(c) The value of the current when t =8ms
(d) The time when the current first reaches 60A

THE SINE WAVE

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.
In other words, if the voltage across (or current through) a resistor, coil, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics.



$$\boxed{2\pi \text{ rad} = 360^\circ}$$



$$\boxed{\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})}$$

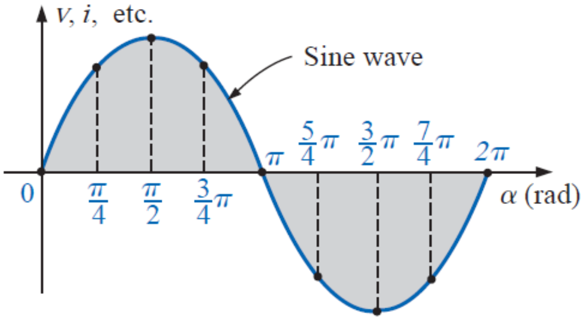
$$\boxed{\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})}$$

$$\mathbf{90°:} \quad \text{Radians} = \frac{\pi}{180°}(90°) = \frac{\boldsymbol{\pi}}{\mathbf{2}}\ \mathbf{rad}$$

$$\mathbf{30°:} \quad \text{Radians} = \frac{\pi}{180°}(30°) = \frac{\boldsymbol{\pi}}{\mathbf{6}}\ \mathbf{rad}$$

$$\frac{\boldsymbol{\pi}}{\mathbf{3}}\ \mathbf{rad:} \quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60°}$$

$$\frac{\mathbf{3}\boldsymbol{\pi}}{\mathbf{2}}\ \mathbf{rad:} \quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270°}$$



Angular velocity: The velocity, with which the radius vector rotates about the center, can be determined from the following equation:

$$\boxed{\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}}$$

$$\boxed{\omega = \frac{2\pi}{T}} \qquad \text{(rad/s)}$$

$$\boxed{\omega = 2\pi f} \qquad \text{(rad/s)}$$

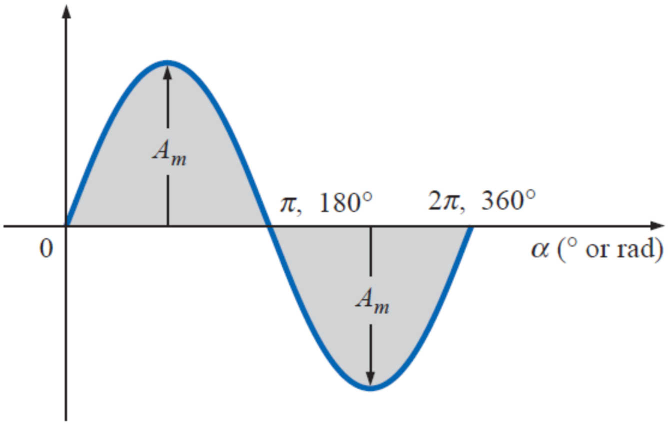
GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is:

$$\boxed{A_m \sin \alpha}$$

where ***Am*** is the peak value of the waveform and α is the unit of measure for the horizontal axis.

$$\alpha = \omega t$$



$$\boxed{A_m \sin \omega t}$$

For electrical quantities such as current and voltage, the general format is:

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

EXAMPLE: Given $e = 5 \sin \alpha$, determine e at $\alpha = 40°$ and $\alpha = 0.8\,\pi$.

Solution:

For $\alpha = 40°$,

$$e = 5 \sin 40° = 5(0.6428) = \mathbf{3.214\ V}$$

For $\alpha = 0.8\pi$,

$$\alpha\ (°) = \frac{180°}{\pi}(0.8\pi) = 144°$$

and $$e = 5 \sin 144° = 5(0.5878) = \mathbf{2.939\ V}$$

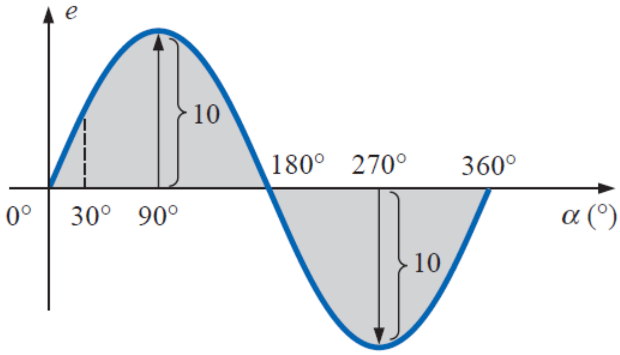
EXAMPLE: Sketch $e = 10 \sin 314\, t$ with the abscissa

a. angle (α) in degrees.

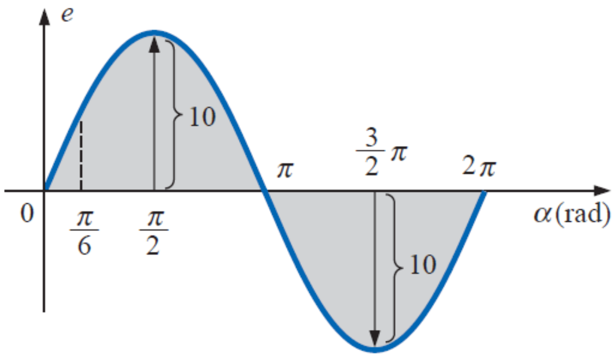
b. angle (α) in radians.

c. time (t) in seconds.

Solution:



(a)



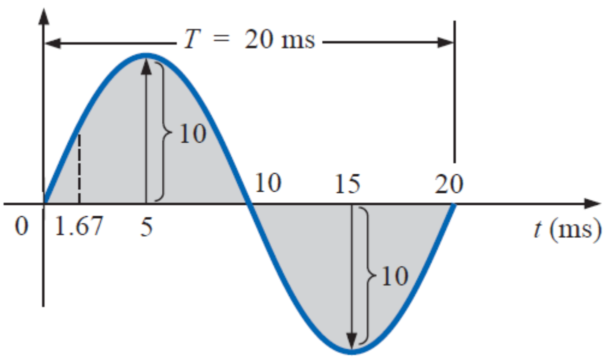
(b)

$$\text{c. } 360°: \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{314} = 20 \text{ ms}$$

$$180°: \quad \frac{T}{2} = \frac{20 \text{ ms}}{2} = 10 \text{ ms}$$

$$90°: \quad \frac{T}{4} = \frac{20 \text{ ms}}{4} = 5 \text{ ms}$$

$$30°: \quad \frac{T}{12} = \frac{20 \text{ ms}}{12} = 1.67 \text{ ms}$$



EXAMPLE: Given $i = 6 * 10^{-3} \sin 1000\,t$, determine i at $t = 2$ ms.

Solution:

$$\alpha = \omega t = 1000t = (1000 \text{ rad/s})(2 \times 10^{-3} \text{ s}) = 2 \text{ rad}$$

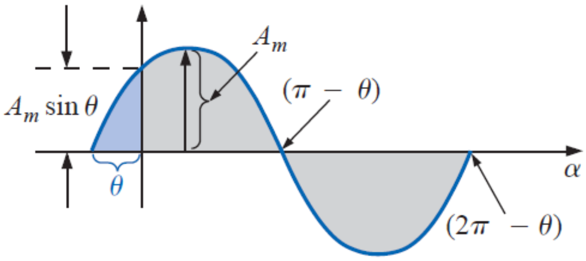
$$\alpha\ (^\circ) = \frac{180^\circ}{\pi \text{ rad}}(2 \text{ rad}) = 114.59^\circ$$

$$i = (6 \times 10^{-3})(\sin 114.59^\circ) = (6 \text{ mA})(0.9093) = \mathbf{5.46\ mA}$$

PHASE RELATIONS

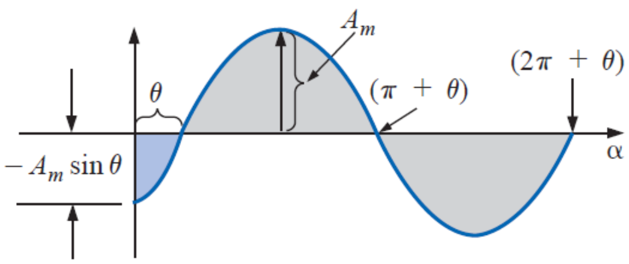
* If the waveform passes through the horizontal axis with a *positive going* (increasing with time) slope *before* 0°.

$$\boxed{A_m \sin(\omega t + \theta)}$$



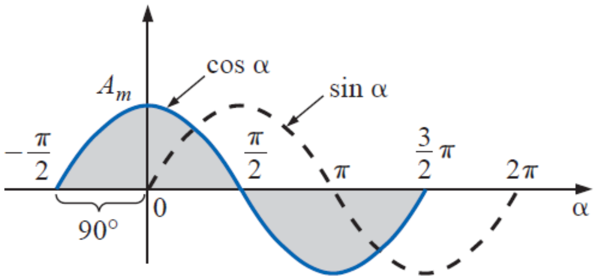
* If the waveform passes through the horizontal axis with a positive-going slope *after* 0°,

$$\boxed{A_m \sin(\omega t - \theta)}$$



* If the waveform crosses the horizontal axis with a positive-going slope 90°(π/2) sooner, it is called a *cosine wave*.

$$\boxed{\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t}$$



EXAMPLE 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30^\circ)$
   $i = 5 \sin(\omega t + 70^\circ)$

b. $i = 15 \sin(\omega t + 60^\circ)$
   $v = 10 \sin(\omega t - 20^\circ)$

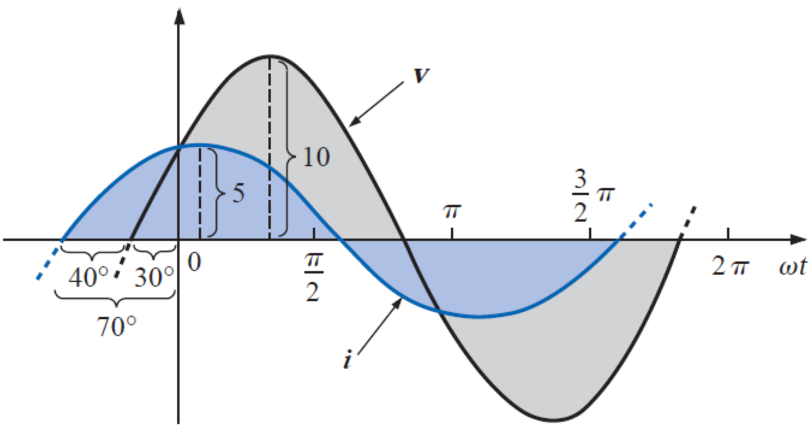
c. $i = 2 \cos(\omega t + 10^\circ)$
   $v = 3 \sin(\omega t - 10^\circ)$

d. $i = -\sin(\omega t + 30^\circ)$
   $v = 2 \sin(\omega t + 10^\circ)$

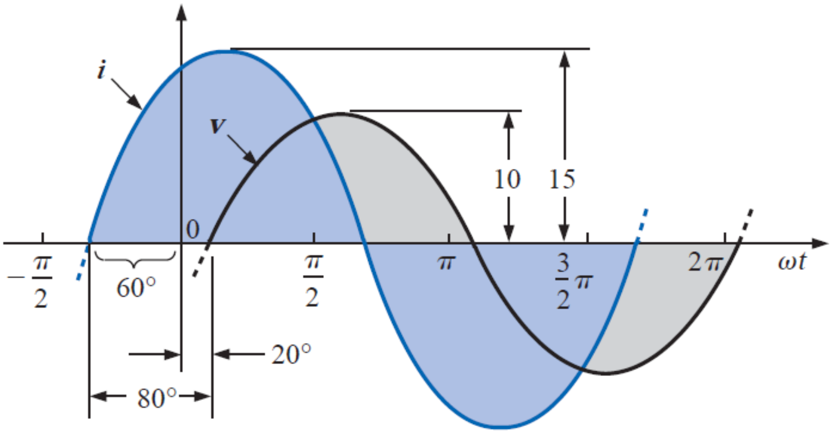
e. $i = -2 \cos(\omega t - 60^\circ)$
   $v = 3 \sin(\omega t - 150^\circ)$

Solutions:

a) ***i* leads *v* by 40°, or *v* lags *i* by 40°.**



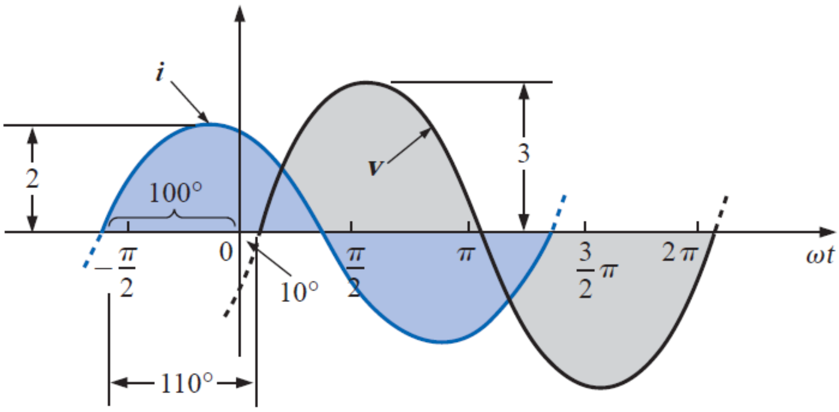
b) ***i* leads *v* by 80°, or *v* lags *i* by 80°.**



c)

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$

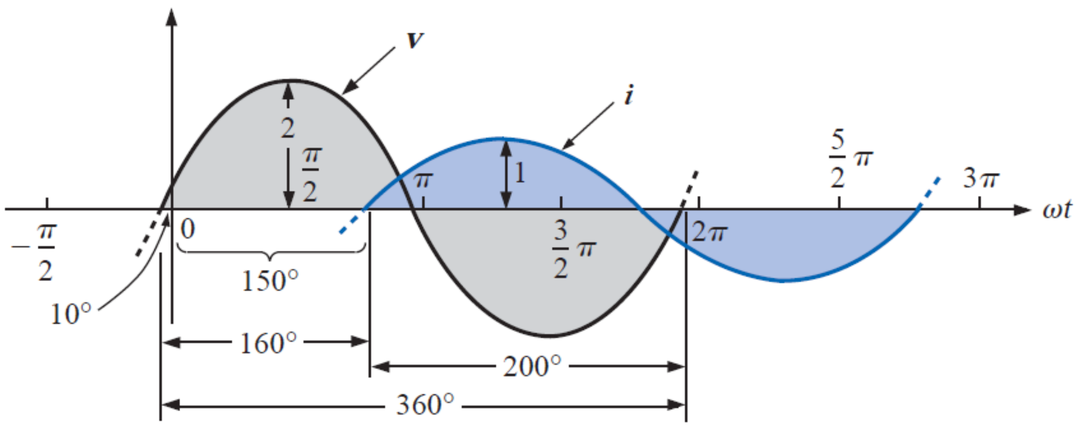
***i* leads *v* by 110°, or *v* lags *i* by 110°.**



d)

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° - 180°)$$
$$= \sin(\omega t - 150°)$$

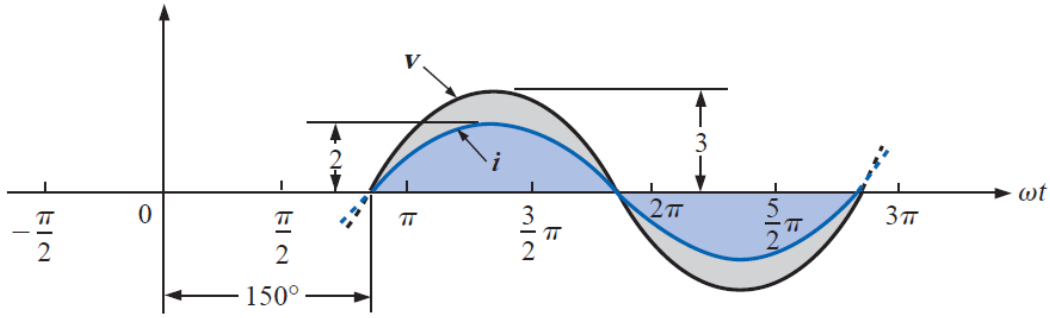
***v* leads *i* by 160°, or *i* lags *v* by 160°.**



e)

By choice

$$i = -2\cos(\omega t - 60°) = 2\cos(\omega t - 60° - 180°)$$
$$= 2\cos(\omega t - 240°)$$



However, $\cos\alpha = \sin(\alpha + 90°)$

so that $2\cos(\omega t - 240°) = 2\sin(\omega t - 240° + 90°)$
$$= 2\sin(\omega t - 150°)$$

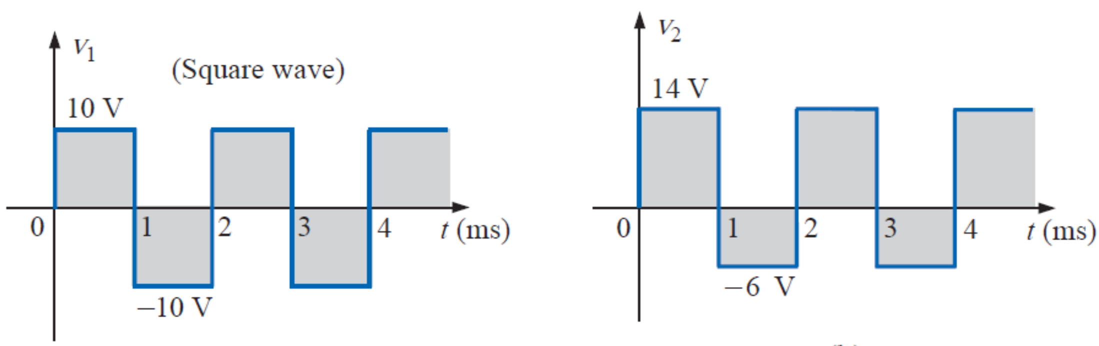
***v* and *i* are in phase.**

AVERAGE VALUE (mean)

The average value of alternating waveform is the equivalent (DC) value over a complete cycle. In general the average value of a waveform is given as:

$$G \text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

EXAMPLE: Determine the average value of the waveforms



(a) (b)

Solutions:

a) By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts.

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

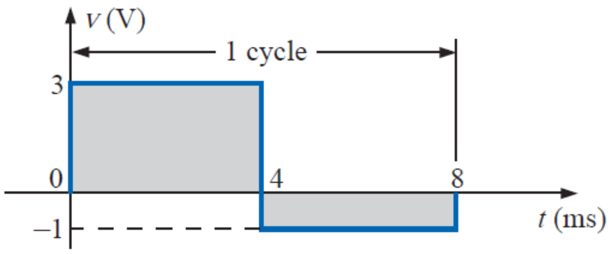
b)

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}}$$

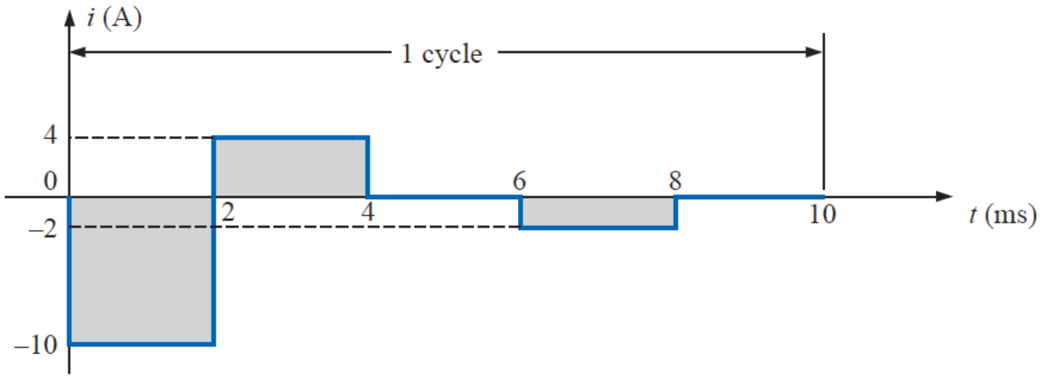
$$= \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

EXAMPLE: Find the average values of the following waveforms over one full cycle:

a)



b)



Solutions:

$$\text{a. } G = \frac{+(3\text{ V})(4\text{ ms}) - (1\text{ V})(4\text{ ms})}{8\text{ ms}} = \frac{12\text{ V} - 4\text{ V}}{8} = \mathbf{1\text{ V}}$$

$$\text{b. } G = \frac{-(10\text{ V})(2\text{ ms}) + (4\text{ V})(2\text{ ms}) - (2\text{ V})(2\text{ ms})}{10\text{ ms}}$$

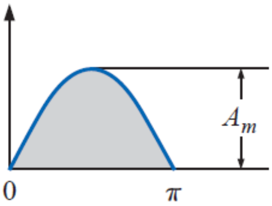
$$= \frac{-20\text{ V} + 8\text{ V} - 4\text{ V}}{10} = -\frac{16\text{ V}}{10} = \mathbf{-1.6\text{ V}}$$

The area of **sine wave** (for one half) can be calculated by the following:

$$\text{Area} = \int_0^{\pi} A_m \sin \alpha \, d\alpha$$

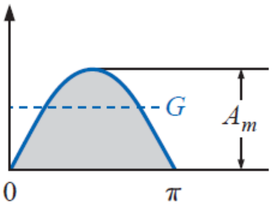
$$\text{Area} = A_m[-\cos \alpha]_0^{\pi}$$
$$= -A_m(\cos \pi - \cos 0°)$$
$$= -A_m[-1 - (+1)] = -A_m(-2)$$

$$\boxed{\text{Area} = 2A_m}$$

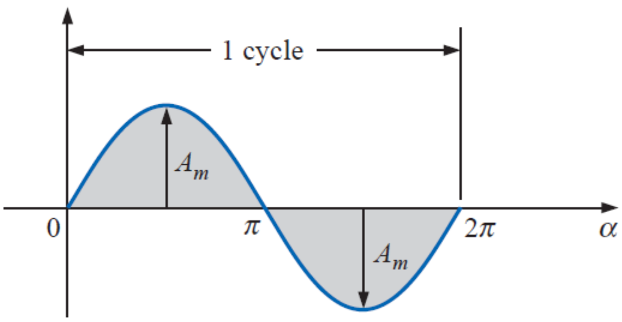


$$G = \frac{2A_m}{\pi}$$

$$\boxed{G = 0.637A_m}$$



EXAMPLE: Determine the average value of the sinusoidal waveform

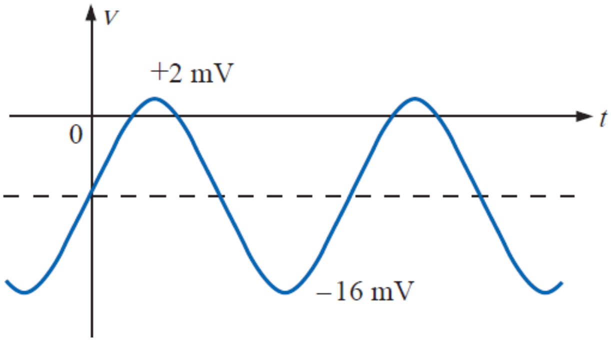


Solutions:

The average value of a pure sinusoidal waveform over one full cycle is zero.

$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

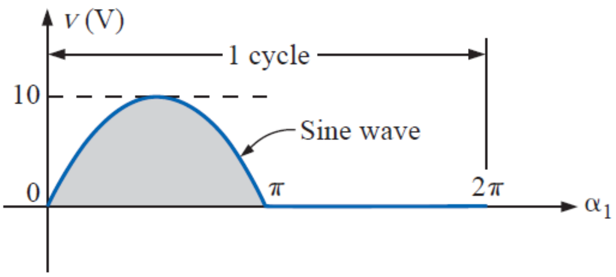
EXAMPLE: Determine the average value of the waveform



Solutions:

The peak-to-peak value of the sinusoidal function is 16 mV + 2 mV = 18 mV. The peak amplitude of the sinusoidal waveform is, therefore, 18 mV/2 = 9 mV. Counting down 9 mV from 2 mV (or 9 mV up from -16 mV) results in an average or dc level of -7 mV.

EXAMPLE: Determine the average value of the waveform



Solutions:

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\ \text{V})}{2\pi} \cong \mathbf{3.18\ V}$$
