Electrical Engineering Fundamentals

First class

AC

Lecture 2 & 3

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EFFECTIVE (rms) VALUES

The effective value (or the rms value) of an alternating waveform is given by the steady (dc) current which when flowing through a given circuit, for a given time produces the same heat produced by the alternating current when flowing the same circuit for the same time.

Effective value of the sinusoidal is:

$$I_{eff} = 0.707 I_m$$

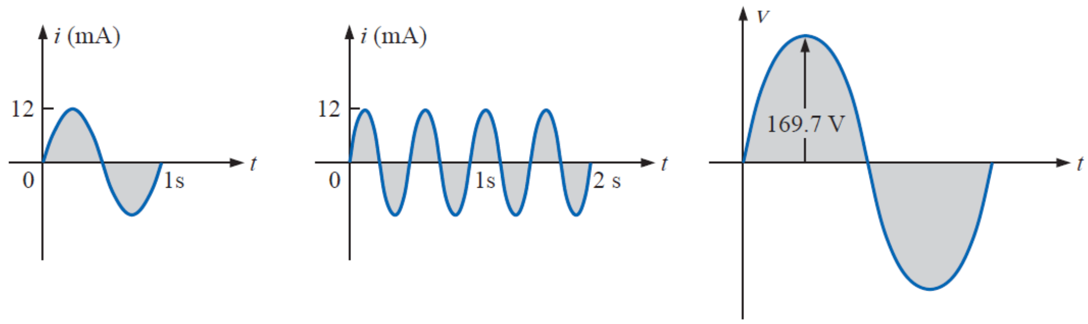
$$E_{eff} = 0.707 E_m$$

The effective value of any quantity plotted as a function of time can be found by using the following equation:

$$I_{eff} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}}$$

$$I_{eff} = \sqrt{\frac{\text{area } (i^2(t))}{T}}$$

EXAMPLE: Find the rms values of the sinusoidal waveform

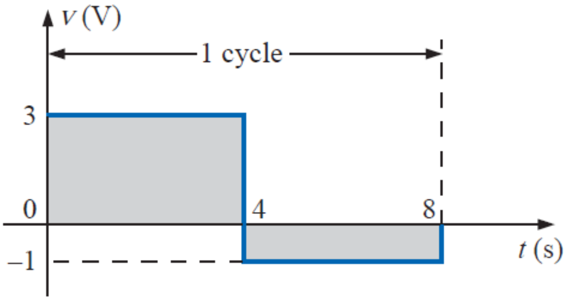


(a) (b) (c)

Solution:

For part (a), I_{rms} = 0.707(12 * 10^{-3} A) = **8.484 mA.** For part (b), again I_{rms} = **8.484 mA.** Note that frequency did not change the effective value in (b) above compared to (a). For part (c), V_{rms} = 0.707(169.73 V) ≅ **120 V**.

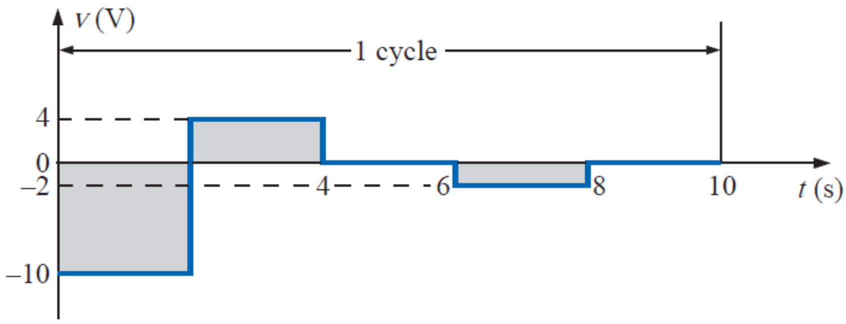
EXAMPLE: Find the effective or rms value of the waveform



Solution:

$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.236\ V}$$

EXAMPLE: Calculate the rms value of the voltage

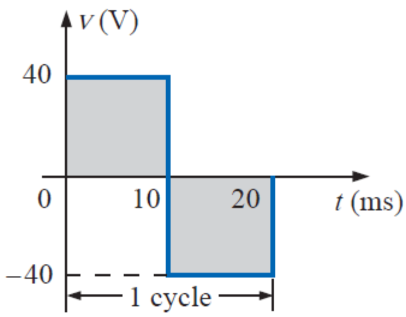


Solution:

$$V_{rms} = \sqrt{\frac{(100)(2) + (16)(2) + (4)(2)}{10}} = \sqrt{\frac{240}{10}}$$

$$= \mathbf{4.899\ V}$$

EXAMPLE: Determine the average and rms values of the square wave.



Solution:

$$V_{rms} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{32{,}000 \times 10^{-3}}{20 \times 10^{-3}}} = \sqrt{1600}$$

$$V_{rms} = \mathbf{40\ V}$$

Problem 3

Determine the effective values of
a) i = 50sin(wt + 20) mA
b) v = 10cos 2wt V

Problem 4

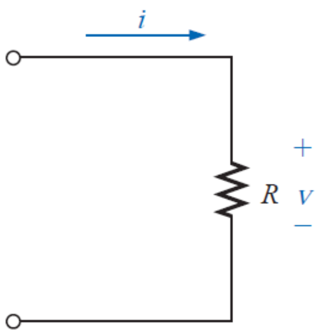
The 120-V dc source delivers 3.6 W to the load. Determine the peak value of the applied voltage (Vm) and the current (Im) if the ac source is to deliver the same power to the load.

RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURREN

R, L, and C circuit elements each have quite different electrical properties. Resistance, for example, opposes current, while inductance opposes changes in current, and capacitance opposes changes in voltage.

1) **Resistor**

For a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.



For $v = V_m \sin \omega t$,

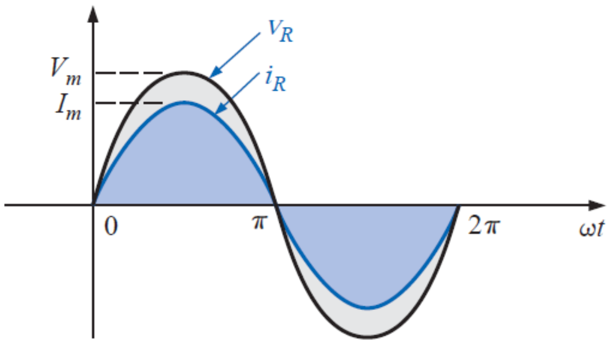
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

Or

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

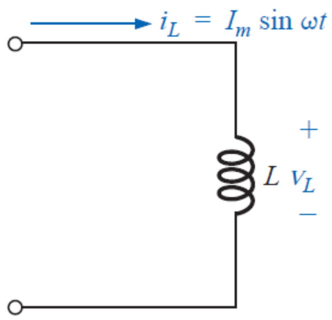
$$V_m = I_m R$$



2) **Inductor**

For an inductor, vL leads iL by 90°, or iL lags vL by 90°.

$$v_L = L \frac{di_L}{dt}$$



$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

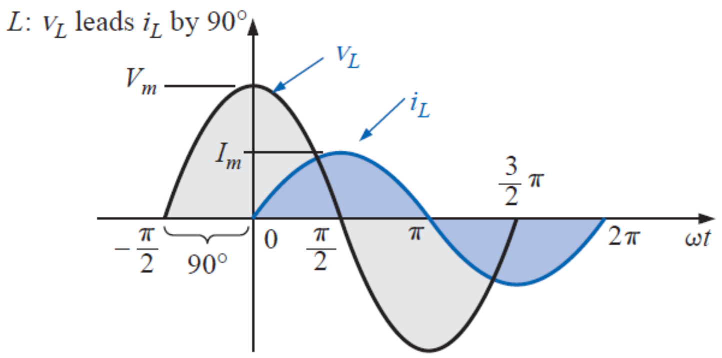
$$v_L = L\frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$$

$$v_L = V_m \sin(\omega t + 90°)$$

$$V_m = \omega L I_m$$

$$i_L = I_m \sin(\omega t \pm \theta)$$

$$v_L = \omega L I_m \sin(\omega t \pm \theta + 90°)$$



The quantity ωL, called the **reactance** (from the word *reaction*) of an inductor, is symbolically represented by *XL* and is measured in ohms; that is,

$$X_L = \omega L \quad \text{(ohms, } \Omega\text{)}$$

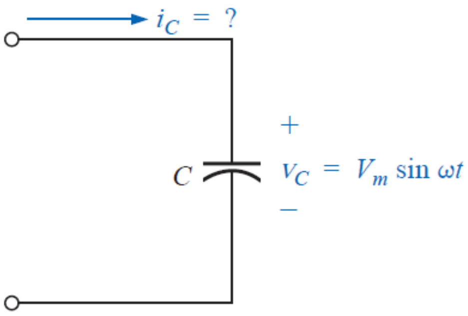
$$X_L = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega\text{)}$$

$$X_L = \omega L = 2\pi f L = 2\pi L f$$

3) **Capacitor**

For a particular capacitance, the greater the rate of change of voltage across the capacitor, the greater the capacitive current.

$$i_C = C\frac{dv_C}{dt}$$



$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$

$$i_C = C\frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

$$i_C = I_m \sin(\omega t + 90^\circ)$$

$$I_m = \omega C V_m$$

For a capacitor, iC leads vC by 90°, or vC lags iC by 90°.

$$v_C = V_m \sin(\omega t \pm \theta)$$

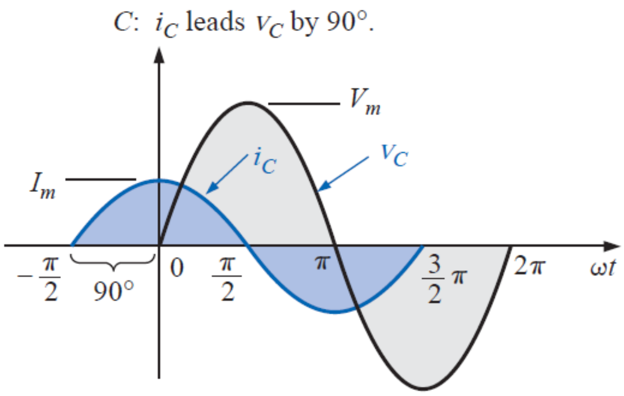
$$i_C = \omega C V_m \sin(\omega t \pm \theta + 90^\circ)$$

The quantity $1/\omega C$, called the **reactance** of a capacitor, is symbolically represented by XC and is measured in ohms; that is,

$$X_C = \frac{1}{\omega C} \quad \text{(ohms, } \Omega\text{)}$$

$$X_C = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega\text{)}$$

$$X_C = \frac{1}{2\pi f C}$$



EXAMPLE: The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for v and i.

a) v = 100 sin 377t

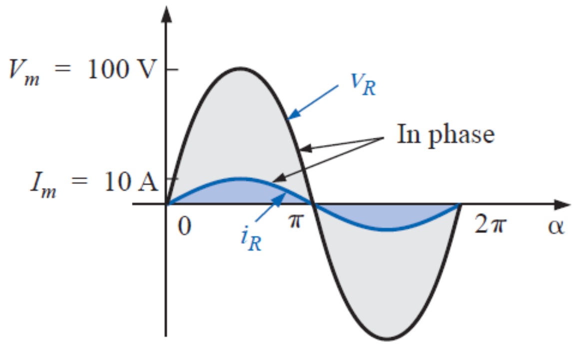
b) v = 25 sin(377t + 60°)

Solutions:

a)

$$I_m = \frac{V_m}{R} = \frac{100\text{ V}}{10\ \Omega} = 10\text{ A}$$

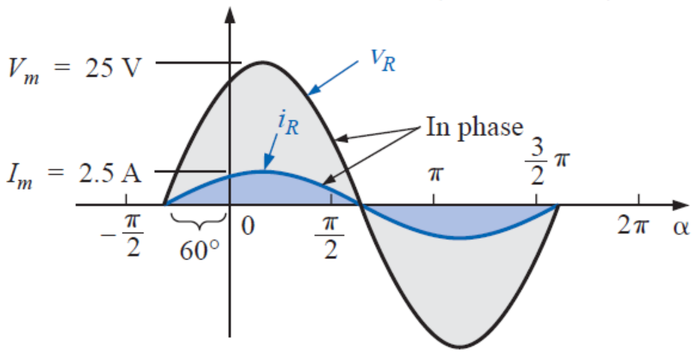
(v and i are in phase) $\quad i = \mathbf{10\ sin\ 377}\boldsymbol{t}$



b)

$$I_m = \frac{V_m}{R} = \frac{25\text{ V}}{10\ \Omega} = 2.5\text{ A}$$

(v and i are in phase) $\quad i = \mathbf{2.5\ sin(377}\boldsymbol{t}\mathbf{ + 60°)}$



EXAMPLE: The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a) i = 10 sin 377t

b) i = 7 sin(377t _ 70°)

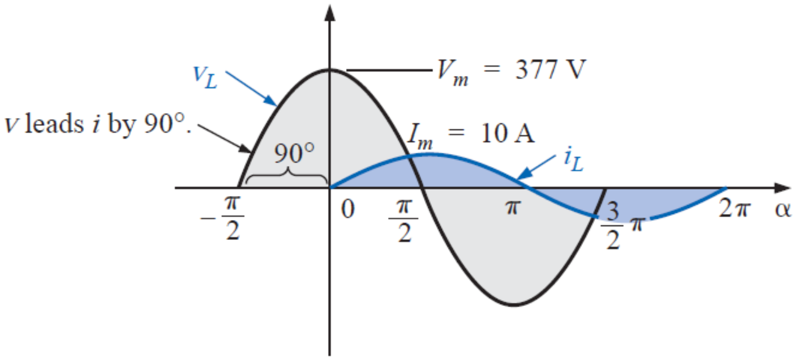
Solutions:

a)

$X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7\ \Omega$

$V_m = I_m X_L = (10 \text{ A})(37.7\ \Omega) = 377 \text{ V}$

v leads i by 90° $\quad v = \mathbf{377 \sin(377t + 90°)}$

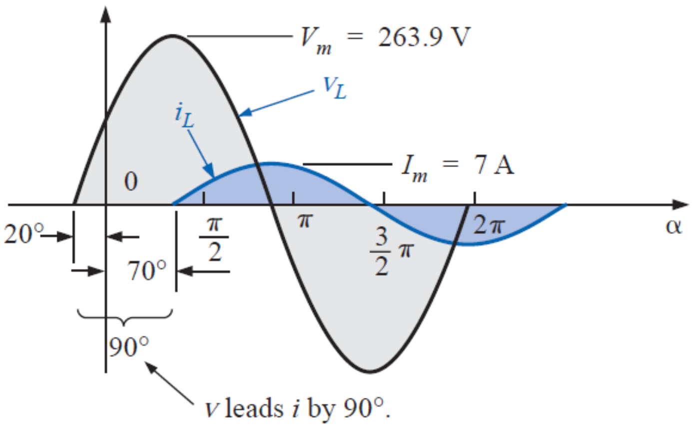


b)

$V_m = I_m X_L = (7 \text{ A})(37.7\ \Omega) = 263.9 \text{ V}$

v leads i by 90° $\quad v = 263.9 \sin(377t - 70° + 90°)$

$$v = \mathbf{263.9 \sin(377t + 20°)}$$



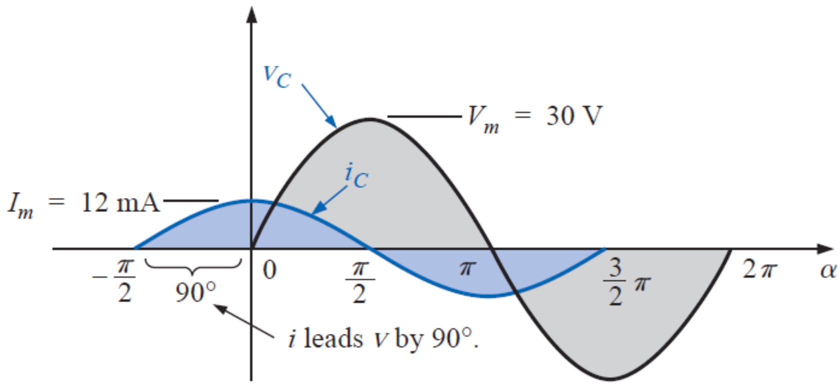
EXAMPLE: The voltage across a 1-μF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.
v =30 sin 400t

Solutions:

$$X_C = \frac{1}{\omega C} = \frac{1}{(400\ \text{rad/s})(1 \times 10^{-6}\ \text{F})} = \frac{10^6\ \Omega}{400} = 2500\ \Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{30\ \text{V}}{2500\ \Omega} = 0.0120\ \text{A} = 12\ \text{mA}$$

i leads v by 90° $\quad i = \mathbf{12 \times 10^{-3} \sin(400t + 90°)}$



EXAMPLE: At what frequency will the reactance of a 200-mH inductor match the resistance level of a 5-kΩ resistor?

Solutions:

$$5000\ \Omega = X_L = 2\pi f L = 2\pi L f$$
$$= 2\pi(200 \times 10^{-3}\ \text{H})f = 1.257f$$

$$f = \frac{5000\ \text{Hz}}{1.257} \cong \mathbf{3.98\ kHz}$$
