Electrical Engineering Fundamentals

First class

AC

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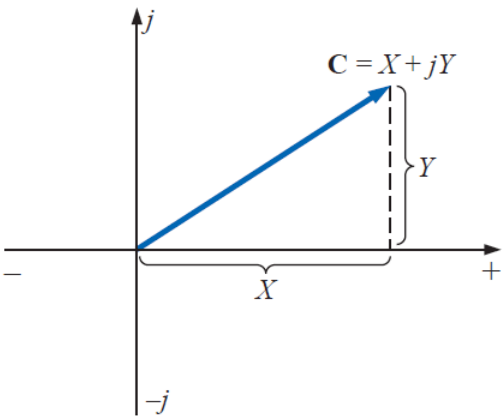
COMPLEX NUMBERS

A **complex number** represents a point in a two-dimensional plane located with reference to two distinct axes. This point can also determine a radius vector drawn from the origin to the point. The horizontal axis is called the *real* axis, while the vertical axis is called the *imaginary* axis.
Two forms are used to represent a complex number: **rectangular** and **polar.**

1) RECTANGULAR FORM

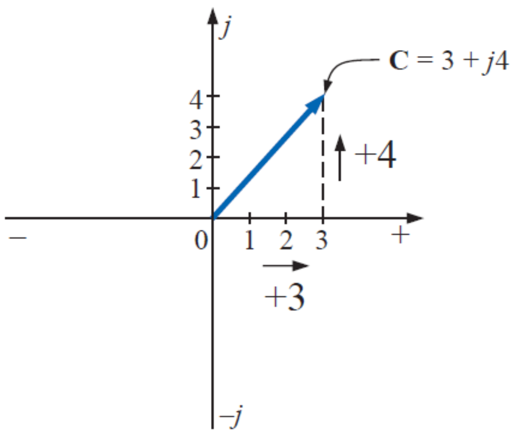
The format for the **rectangular form** is

$$\boxed{\mathbf{C} = X + jY}$$

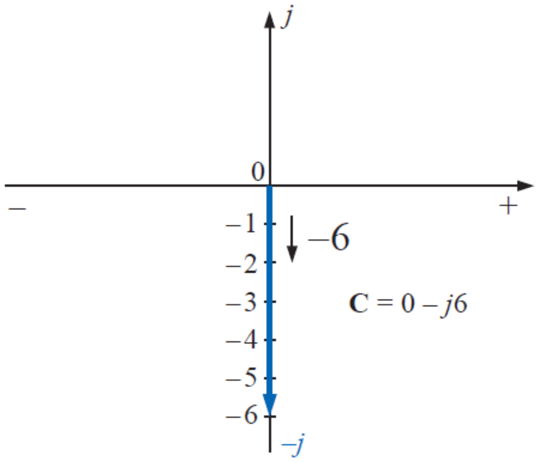


EXAMPLE: Sketch the following complex numbers in the complex plane:
a) $\mathbf{C} = 3 + j\,4$
b) $\mathbf{C} = 0 - j\,6$
c) $\mathbf{C} = -10 - j20$

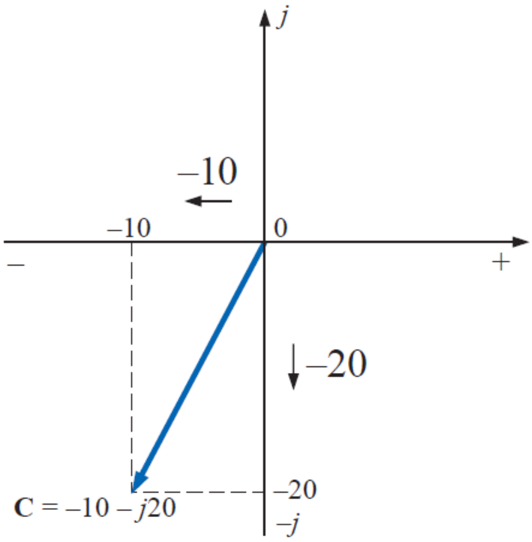
Solutions:
a)



b)



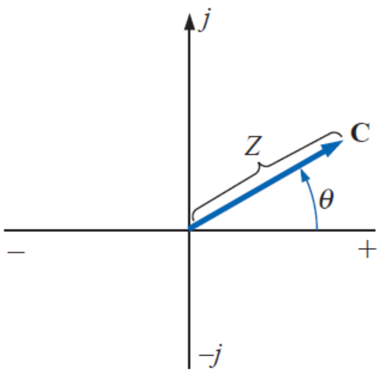
c)



2) POLAR FORM

The format for the **polar form** is

$$\boxed{\mathbf{C} = Z \angle \theta}$$



$$\boxed{-\mathbf{C} = -Z \angle \theta = Z \angle \theta \pm 180^\circ}$$

EXAMPLE: Sketch the following complex numbers in the complex plane:

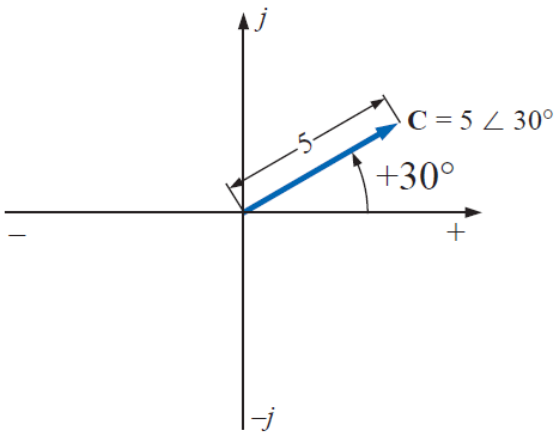
a) $\mathbf{C} = 5 \angle 30°$

b) $\mathbf{C} = 7 \angle -120°$

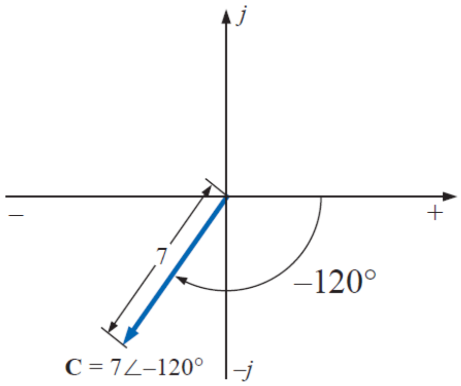
c) $\mathbf{C} = -4.2 \angle 60°$

Solutions:

a)

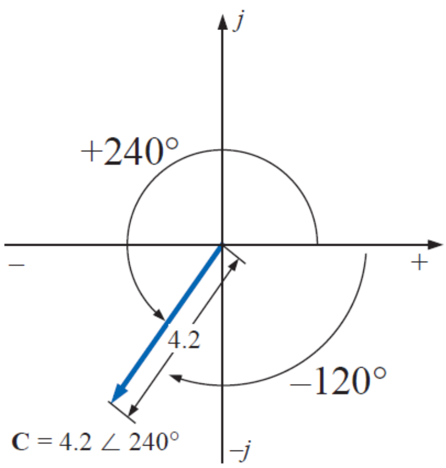


b)

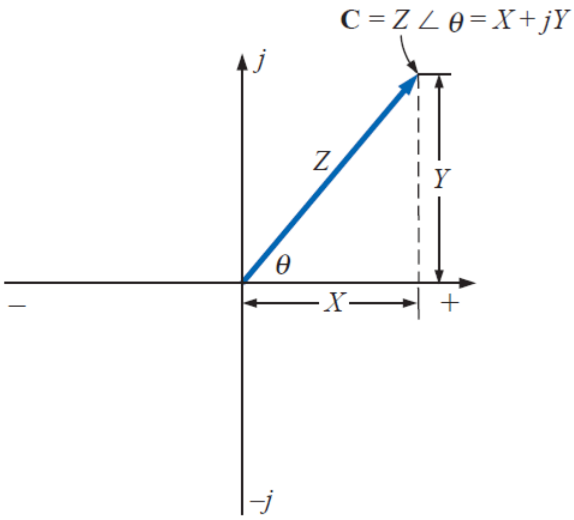


c)

$$\mathbf{C} = -4.2 \angle 60° = 4.2 \angle 60° + 180°$$
$$= 4.2 \angle +240°$$



CONVERSION BETWEEN FORMS



1) **Rectangular to Polar**

$$\boxed{Z = \sqrt{X^2 + Y^2}}$$

$$\boxed{\theta = \tan^{-1}\frac{Y}{X}}$$

2) **Polar to Rectangular**

$$\boxed{X = Z\cos\theta}$$

$$\boxed{Y = Z\sin\theta}$$

EXAMPLE: Convert the following from rectangular to polar form:

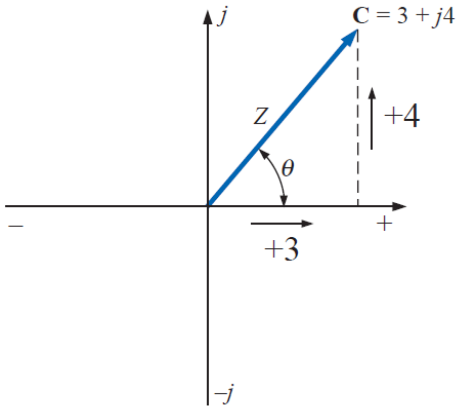
$$\mathbf{C} = 3 + j\,4$$

Solutions:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

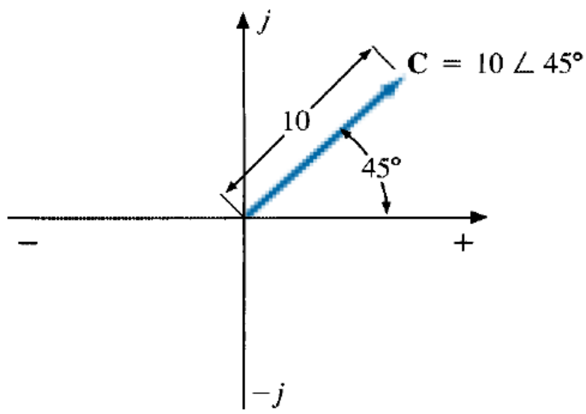
$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

$$\mathbf{C = 5 \angle 53.13^\circ}$$



EXAMPLE: Convert the following from polar to rectangular form:

$$\mathbf{C} = 10 \angle 45^\circ$$



Solutions:

$$X = 10 \cos 45^\circ = (10)(0.707) = 7.07$$

$$Y = 10 \sin 45^\circ = (10)(0.707) = 7.07$$

$$\mathbf{C = 7.07 + j7.07}$$

Problem 5

Convert the following from polar to rectangular form $C = 10\ \angle 30$

MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

Complex numbers lend themselves readily to the basic mathematical operations of addition, subtraction, multiplication, and division. A few basic rules and definitions must be understood before considering these operations.

$$\boxed{j = \sqrt{-1}}$$

$$\boxed{j^2 = -1} \qquad \boxed{\frac{1}{j} = -j}$$

1) Addition

$$\mathbf{C}_1 = \pm X_1 \pm j\,Y_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm j\,Y_2$$

$$\boxed{\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j\,(\pm Y_1 \pm Y_2)}$$

EXAMPLE:

a. Add $\mathbf{C}_1 = 2 + j4$ and $\mathbf{C}_2 = 3 + j1$.
b. Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.

Solutions:

a)
$\mathbf{C}_1 + \mathbf{C}_2 = (2 + 3) + j(4 + 1) = \mathbf{5 + \mathit{j}5}$

b)
$\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + \mathit{j}9}$

2) Subtraction

$$\mathbf{C}_1 = \pm X_1 \pm j\,Y_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm j\,Y_2$$

$$\boxed{\mathbf{C}_1 - \mathbf{C}_2 = [\pm X_2 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)]}$$

EXAMPLE:

a. Subtract $\mathbf{C}_2 = 1 + j4$ from $\mathbf{C}_1 = 4 + j6$.
b. Subtract $\mathbf{C}_2 = -2 + j5$ from $\mathbf{C}_1 = +3 + j3$.

Solutions:

a)

$\mathbf{C}_1 - \mathbf{C}_2 = (4 - 1) + j(6 - 4) = \mathbf{3 + j2}$

b)

$\mathbf{C}_1 - \mathbf{C}_2 = [3 - (-2)] + j(3 - 5) = \mathbf{5 - j2}$

Problem 6

Given A = 2 + j1 and B = 1+ j3 . Determine their sum and difference analytically

3) Multiplication

$$\mathbf{C}_1 = X_1 + jY_1 \quad \text{and} \quad \mathbf{C}_2 = X_2 + jY_2$$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)}$$

$$\mathbf{C}_1 = Z_1 \angle\theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle\theta_2$$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1Z_2 \,\angle\theta_1 + \theta_2}$$

EXAMPLE:

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 5 \angle 20^\circ \quad \text{and} \quad \mathbf{C}_2 = 10 \angle 30^\circ$$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 2 \angle -40^\circ \quad \text{and} \quad \mathbf{C}_2 = 7 \angle +120^\circ$$

Solutions:

a. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (5 \angle 20^\circ)(10 \angle 30^\circ) = (5)(10) \angle 20^\circ + 30^\circ = \mathbf{50 \angle 50^\circ}$

b. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (2 \angle -40^\circ)(7 \angle +120^\circ) = (2)(7) \angle -40^\circ + 120^\circ$
$= \mathbf{14 \angle +80^\circ}$

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4) **Division**

$$\mathbf{C}_1 = Z_1 \angle\theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle\theta_2$$

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{Z_1}{Z_2} \angle\theta_1 - \theta_2}$$

EXAMPLE:

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 15 \angle 10^\circ$ and $\mathbf{C}_2 = 2 \angle 7^\circ$.

b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 8 \angle 120^\circ$ and $\mathbf{C}_2 = 16 \angle -50^\circ$.

Solutions:

a. $\dfrac{\mathbf{C}_1}{\mathbf{C}_2} = \dfrac{15\angle 10^\circ}{2\angle 7^\circ} = \dfrac{15}{2}\,\underline{/10^\circ - 7^\circ} = \mathbf{7.5\angle 3^\circ}$

b. $\dfrac{\mathbf{C}_1}{\mathbf{C}_2} = \dfrac{8\angle 120^\circ}{16\angle -50^\circ} = \dfrac{8}{16}\,\underline{/120^\circ - (-50^\circ)} = \mathbf{0.5\angle 170^\circ}$

$$\boxed{\frac{1}{Z\angle\theta} = \frac{1}{Z}\angle -\theta}$$

Problem 7

Given $A = 3\angle 35^\circ$ and $B = 2\angle -20^\circ$, determine $A.B$ and A/B