Electrical Engineering Fundamentals

First class

AC

Lecture 12 & 13

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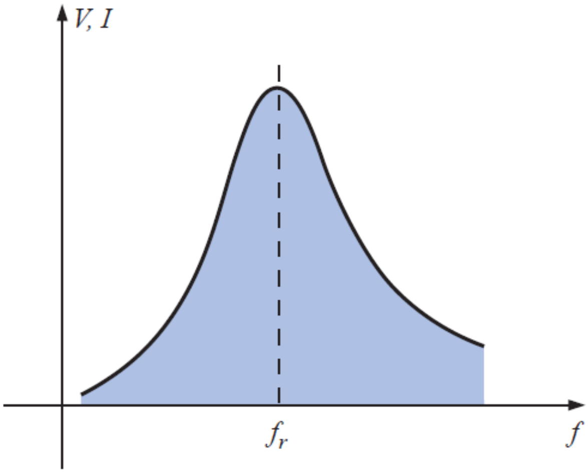
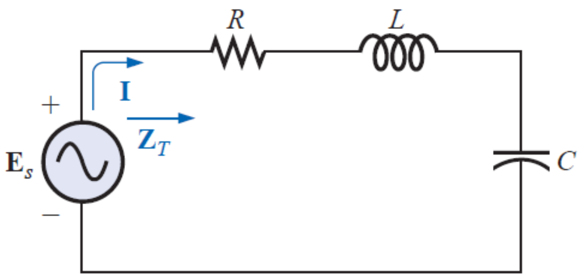
Resonance

The resonant circuit is a combination of R, L, and C elements having a frequency response characteristic similar to the one appearing in Figure below. Note in the figure that the response is a maximum for the frequency fr, decreasing to the right and left of this frequency.



1-SERIES RESONANT CIRCUIT



$$\mathbf{Z}_T = R + j\,X_L - j\,X_C = R + j\,(X_L - X_C)$$

$$\boxed{X_L = X_C}$$

$$\boxed{\mathbf{Z}_{T_s} = R}$$

$$\omega L = \frac{1}{\omega C} \quad \text{and} \quad \omega^2 = \frac{1}{LC}$$

$$\boxed{\omega_s = \frac{1}{\sqrt{LC}}}$$

$$\boxed{f_s = \frac{1}{2\pi\sqrt{LC}}} \qquad \begin{aligned} f &= \text{hertz (Hz)} \\ L &= \text{henries (H)} \\ C &= \text{farads (F)} \end{aligned}$$

$$\boxed{V_{L_s} = V_{C_s}}$$

$$F_p = \cos\theta = \frac{P}{S}$$

$$\boxed{F_{p_s} = 1}$$

THE QUALITY FACTOR (Q)

The quality factor Q of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance; that is,

$$\boxed{Q_s = \frac{\text{reactive power}}{\text{average power}}}$$

$$Q_s = \frac{I^2 X_L}{I^2 R}$$

$$\boxed{Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R}}$$

$$Q_s = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi}{R}\left(\frac{1}{2\pi\sqrt{LC}}\right) L$$

$$= \frac{L}{R}\left(\frac{1}{\sqrt{LC}}\right) = \left(\frac{\sqrt{L}}{\sqrt{L}}\right)\frac{L}{R\sqrt{LC}}$$

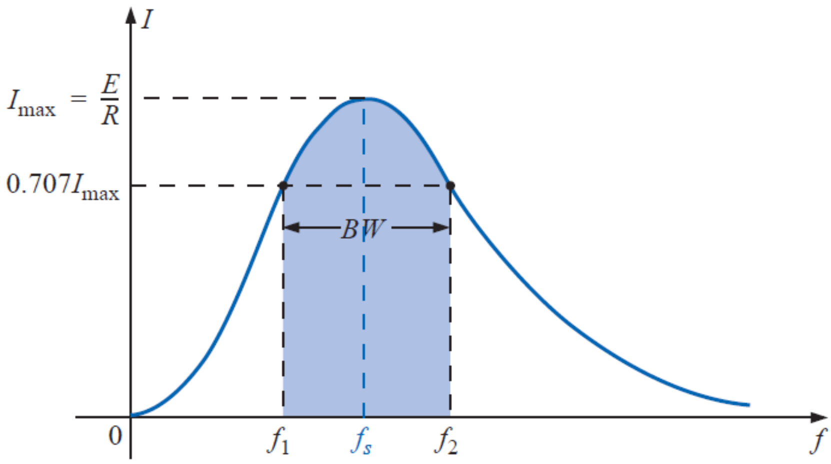
$$\boxed{Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$V_{L_s} = Q_s E$$

$$V_{C_s} = Q_s E$$

SELECTIVITY

There is a definite range of frequencies at which the current is near its maximum value and the impedance is at a minimum. Those frequencies corresponding to 0.707 of the maximum current are called the **band frequencies**, **cutoff frequencies**, or **half-power frequencies**. They are indicated by f1 and f2 in Figure below. The range of frequencies between the two is referred to as the bandwidth (abbreviated BW) of the resonant circuit.



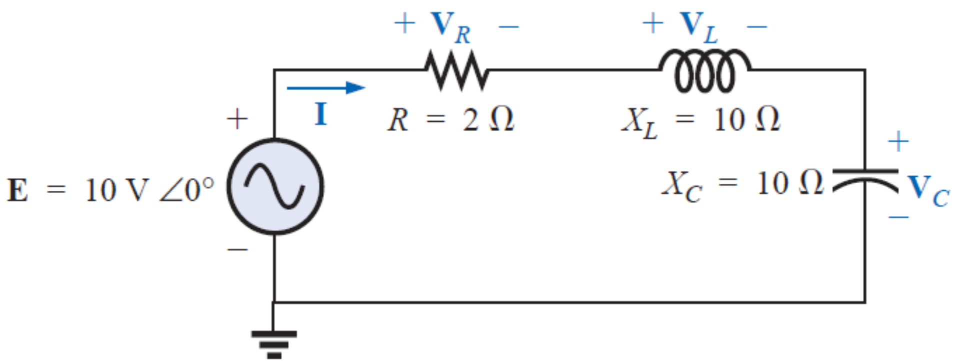
$$P_{\text{HPF}} = \frac{1}{2} P_{\max}$$

$$BW = f_2 - f_1 = \frac{R}{2\pi L}$$

$$BW = \frac{f_s}{Q_s}$$

EXAMPLE:

a. For the series resonant circuit, find **I**, **V***R*, **V***L*, and **V***C* at resonance.
b. What is the *Qs* of the circuit?
c. If the resonant frequency is 5000 Hz, find the bandwidth.
d. What is the power dissipated in the circuit at the half-power frequencies?



Solutions:

a. $\mathbf{Z}_{T_s} = R = 2\ \Omega$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_{T_s}} = \frac{10\text{ V}\angle 0^\circ}{2\ \Omega\angle 0^\circ} = \mathbf{5\ A\angle 0^\circ}$$

$$\mathbf{V}_R = \mathbf{E} = 10\text{ V}\angle 0^\circ$$

$$\mathbf{V}_L = (I\angle 0^\circ)(X_L\angle 90^\circ) = (5\text{ A}\angle 0^\circ)(10\ \Omega\angle 90^\circ) = \mathbf{50\ V\angle 90^\circ}$$

$$\mathbf{V}_C = (I\angle 0^\circ)(X_C\angle -90^\circ) = (5\text{ A}\angle 0^\circ)(10\ \Omega\angle -90^\circ) = \mathbf{50\ V\angle -90^\circ}$$

b. $Q_s = \dfrac{X_L}{R} = \dfrac{10\ \Omega}{2\ \Omega} = \mathbf{5}$

c. $BW = f_2 - f_1 = \dfrac{f_s}{Q_s} = \dfrac{5000\text{ Hz}}{5} = \mathbf{1000\ Hz}$

d. $P_{\text{HPF}} = \dfrac{1}{2}P_{\max} = \dfrac{1}{2}I_{\max}^2 R = \left(\dfrac{1}{2}\right)(5\text{ A})^2(2\ \Omega) = \mathbf{25\ W}$

EXAMPLE: The bandwidth of a series resonant circuit is 400 Hz.

a. If the resonant frequency is 4000 Hz, what is the value of Q_s?

b. If $R = 10\,\Omega$, what is the value of X_L at resonance?

c. Find the inductance L and capacitance C of the circuit.

Solutions:

a. $BW = \dfrac{f_s}{Q_s}$ or $Q_s = \dfrac{f_s}{BW} = \dfrac{4000\text{ Hz}}{400\text{ Hz}} = \mathbf{10}$

b. $Q_s = \dfrac{X_L}{R}$ or $X_L = Q_s R = (10)(10\ \Omega) = \mathbf{100\ \Omega}$

c. $X_L = 2\pi f_s L$ or $L = \dfrac{X_L}{2\pi f_s} = \dfrac{100\ \Omega}{2\pi(4000\text{ Hz})} = \mathbf{3.98\ mH}$

$X_C = \dfrac{1}{2\pi f_s C}$ or $C = \dfrac{1}{2\pi f_s X_C} = \dfrac{1}{2\pi(4000\text{ Hz})(100\ \Omega)}$

$= \mathbf{0.398\ \mu F}$

EXAMPLE: A series *R-L-C* circuit has a series resonant frequency of 12,000 Hz.
a. If $R = 5\ \Omega$, and if XL at resonance is $300\ \Omega$, find the bandwidth.
b. Find the cutoff frequencies.

Solutions:

a. $$Q_s = \frac{X_L}{R} = \frac{300\ \Omega}{5\ \Omega} = 60$$

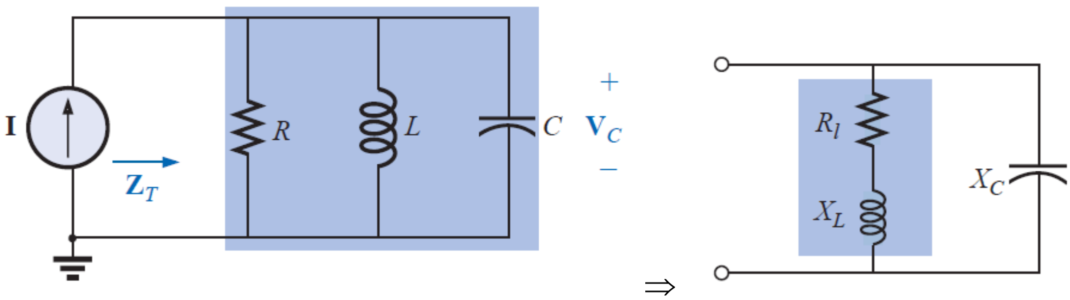
$$BW = \frac{f_s}{Q_s} = \frac{12{,}000\text{ Hz}}{60} = \mathbf{200\ Hz}$$

b. Since $Q_s \geq 10$, the bandwidth is bisected by f_s. Therefore,

$$f_2 = f_s + \frac{BW}{2} = 12{,}000\text{ Hz} + 100\text{ Hz} = \mathbf{12{,}100\ Hz}$$

and $$f_1 = 12{,}000\text{ Hz} - 100\text{ Hz} = \mathbf{11{,}900\ Hz}$$

2-PARALLEL RESONANT CIRCUIT



$$\boxed{R_p = \frac{R_l^2 + X_L^2}{R_l}}$$

$$\boxed{X_{L_p} = \frac{R_l^2 + X_L^2}{X_L}}$$

$$\boxed{X_{L_p} = X_C}$$

$$\boxed{f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}}$$

$$\boxed{f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}}}$$

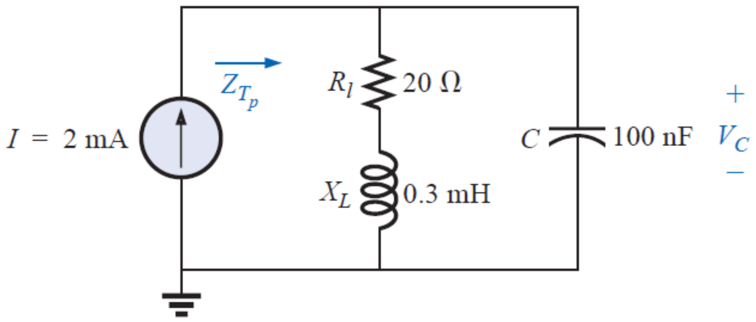
SELECTIVITY CURVE FOR PARALLEL RESONANT CIRCUITS

$$\boxed{Q_p = \frac{X_L}{R_l} = Q_l}$$

$$\boxed{BW = f_2 - f_1 = \frac{f_r}{Q_p}}$$

EXAMPLE: For the parallel resonant circuit

a. Determine *fs*, and *fp*, and compare their levels.
b. Determine the quality factor *Qp*.
c. Calculate the bandwidth.



Solutions:

a. $f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3\text{ mH})(100\text{ nF})}} = \mathbf{29.06\text{ kHz}}$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}} = (29.06\text{ kHz})\sqrt{1 - \left[\frac{(20\ \Omega)^2(100\text{ nF})}{0.3\text{ mH}}\right]}$$

$$= \mathbf{27.06\text{ kHz}}$$

b.

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = Q_l = \frac{X_L}{R_l}$$

$$= \frac{2\pi(27.06\ \text{kHz})(0.3\ \text{mH})}{20\ \Omega} = \frac{51\ \Omega}{20\ \Omega} = \mathbf{2.55}$$

c.

$$BW = \frac{f_p}{Q_p} = \frac{27.06\ \text{kHz}}{2.55} = \mathbf{10.61\ kHz}$$