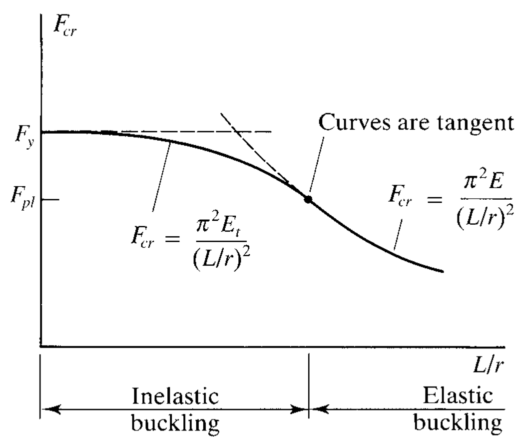
The stress–strain curve shown in Figure 4.5 is different from those shown earlier for ductile steel (in Figures 1.3 and 1.4) because it has a pronounced region of nonlinearity. This curve is typical of a compression test of a short length of W-shape called a *stub column,* rather than the result of a tensile test. The nonlinearity is primarily because of the presence of residual stresses in the W-shape. When a hot-rolled shape cools after rolling, all elements of the cross section do not cool at the same rate. The tips of the flanges, for example, cool faster than the junction of the flange and the web. This uneven cooling induces stresses that remain permanently. Other factors, such as welding and cold-bending to create curvature in a beam, can contribute to the residual stress, but the cooling process is its chief source.

Note that E_t is smaller than E and for the same L/r corresponds to a smaller critical load, P_{cr}. Because of the variability of E_t, computation of P_{cr} in the inelastic range by the use of Equation 4.5 is difficult. In general, a trial-and-error approach must be used, and a compressive stress–strain curve such as the one shown in Figure 4.5 must be used to determine E_t for trial values of P_{cr}. For this reason, most design specifications, including the AISC Specification, contain empirical formulas for inelastic columns.

Engesser's tangent modulus theory had its detractors, who pointed out several inconsistencies. Engesser was convinced by their arguments, and in 1895 he refined his theory to incorporate a reduced modulus, which has a value between E and E_t. Test results, however, always agreed more closely with the tangent modulus theory. Shanley (1947) resolved the apparent inconsistencies in the original theory, and today the tangent modulus formula, Equation 4.5, is accepted as the correct one for inelastic buckling. Although the load predicted by this equation is actually a lower bound on the true value of the critical load, the difference is slight (Bleich, 1952).

For any material, the critical buckling stress can be plotted as a function of slenderness, as shown in Figure 4.6. The tangent modulus curve is tangent to the Euler curve at the point corresponding to the proportional limit of the material. The composite curve, called a *column strength curve,* completely describes the strength of any column of a given material. Other than F_y, E, and E_t, which are properties of the material, the strength is a function only of the slenderness ratio.

FIGURE 4.6



Effective Length

Both the Euler and tangent modulus equations are based on the following assumptions:

1. The column is perfectly straight, with no initial crookedness.
2. The load is axial, with no eccentricity.
3. The column is pinned at both ends.

The first two conditions mean that there is no bending moment in the member before buckling. As mentioned previously, some accidental moment will be present, but in most cases it can be ignored. The requirement for pinned ends, however, is a serious limitation, and provisions must be made for other support conditions. The pinned-end condition requires that the member be restrained from lateral translation, but not rotation, at the ends. Constructing a frictionless pin connection is virtually impossible, so even this support condition can only be closely approximated at best. Obviously, all columns must be free to deform axially.

Other end conditions can be accounted for in the derivation of Equation 4.3. In general, the bending moment will be a function of *x,* resulting in a nonhomogeneous differential equation. The boundary conditions will be different from those in the original derivation, but the overall procedure will be the same. The form of the resulting equation for P_{cr} will also be the same. For example, consider a compression member pinned at one end and fixed against rotation and translation at the other, as shown in Figure 4.7. The Euler equation for this case, derived in the same manner as Equation 4.3, is

$$P_{cr} = \frac{2.05\pi^2 EI}{L^2}$$

or

$$P_{cr} = \frac{2.05\pi^2 EA}{(L/r)^2} = \frac{\pi^2 EA}{(0.70L/r)^2}$$

FIGURE 4.7