Laminar and Turbulent Flows

The flow is called laminar flow if the liquid particles appear to move in definite smooth paths and the flow appears to be as a movement of thin layers on top of each other. In turbulent flow, the liquid particles move in irregular paths which are not fixed with respect to either time or space. The relative magnitude of viscous and inertial forces determines whether the flow is laminar or turbulent: The flow is laminar if the viscous forces dominate, and the flow is turbulent if the inertial forces dominate. The ratio of viscous and inertial forces is defined as the Reynolds number,

$$Re = V\ L\ \nu \qquad (1-1)$$

in which Re = Reynolds number; V = mean flow velocity; L = a characteristic length; and ν = kinematic viscosity of the liquid. Unlike pipe flow in which the pipe diameter is usually used for the characteristic length, either hydraulic depth or hydraulic radius may be used as the characteristic length in freesurface flows. Hydraulic depth is defined as the flow area divided by the top water-surface width and the hydraulic radius is defined as the flow area divided by the wetted perimeter. The transition from laminar to turbulent flow in freesurface flows occurs for Re of about 600, in which Re is based on the hydraulic radius as the characteristic length.

In real-life applications, laminar free-surface flows are extremely rare. A smooth and glassy flow surface may be due to surface velocity being less than that required to form capillary waves and may not necessarily be due to the fact that the flow is laminar. Care should be taken while selecting geometrical scales for the hydraulic model studies so that the flow depth on the model is not very small. Very small depth may produce laminar flow on the model even though the prototype flow to be modelled is turbulent. The results of such a model are not reliable.

Subcritical, Supercritical, and Critical Flows

A flow is called critical if the flow velocity is equal to the velocity of a gravity wave having small amplitude. A gravity wave may be produced by a change in the flow depth. The flow is called subcritical flow, if the flow velocity is less than the critical velocity, and the flow is called supercritical flow if the flow velocity is greater than the critical velocity. The Froude number, Fr, is equal to the ratio of inertial and gravitational forces and, for a rectangular channel, it is defined as

$$Fr = V \sqrt{gy} \qquad (1-2)$$

in which y = flow depth. General expressions for Fr are presented in Section 3-2. Depending upon the value of Fr, flow is classified as subcritical if $Fr < 1$; critical if $Fr = 1$; and supercritical if $Fr > 1$.

1-4 Terminology

Channels may be natural or artificial. Various names have been used for the artificial channels: A long channel having mild slope usually excavated in the ground is called a canal. A channel supported above ground and built of wood, metal, or concrete is called a flume. A chute is a channel having very steep bottom slope and almost vertical sides. A tunnel is a channel excavated through a hill or a mountain. A short channel flowing partly full is referred to as a culvert.

A channel having the same cross section and bottom slope throughout is referred to as a prismatic channel, whereas a channel having varying crosssection and/or bottom slope is called a non-prismatic channel. A long channel may be comprised of several prismatic channels. A cross section taken normal to the direction of flow (e.g., Section BB in Fig. 1-8) is called a channel section. The depth of flow, y, at a section is the vertical distance of the lowest point of the channel section from the free surface. The depth of flow section, d, is the depth of flow normal to the direction of flow. The stage, Z, is the elevation or vertical distance of free surface above a specified datum (Fig. 1-8). The top width, B, is the width of channel section at the free surface. The flow area, A, is the cross-sectional area of flow normal to the direction of flow. The wetted perimeter, P is defined as the length of line of intersection of channel wetted surface with a cross-sectional plane normal to the flow direction. The hydraulic radius, R, and hydraulic depth, D, are defined as

$$R = A/P$$

$$D = A/B \qquad (1-3)$$

Expressions for A, P, D and R for typical channel cross sections are presented in Table 1-1.

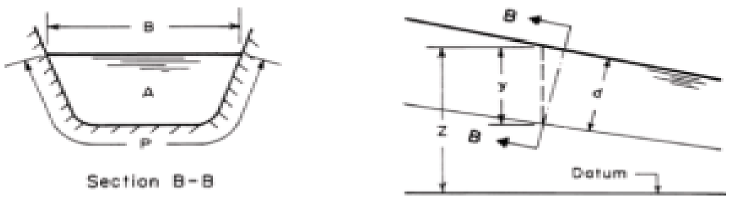
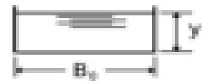
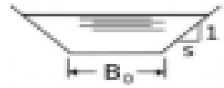
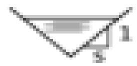
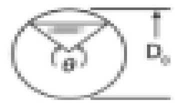


Fig. 1-8. Definition sketch

1-5 Velocity Distribution

The flow velocity in a channel section varies from one point to another. This is due to shear stress at the bottom and at the sides of the channel and due to the presence of free surface. Fig. 1-9 shows typical velocity distributions in different channel cross sections. The flow velocity may have components in all three Cartesian coordinate directions. However, the components of velocity in the vertical and transverse directions are usually small and may be neglected. Therefore, only the flow velocity in the direction of flow needs to be considered. This velocity component varies with depth from the free surface. A typical variation of velocity with depth is shown in Fig. 1-10.

Table 1-1. Properties of typical channel cross sections

Section	Area, A	Wetted Perimeter, P	Hydraulic radius, R	Top width, B	Hydraulic depth, D	
Rectangular	$B_o y$	$B_o + 2y$	$\frac{B_o y}{B_o + 2y}$	B_o	y	
Trapezoidal	$(B_o + sy)y$	$B_o + 2y\sqrt{1+s^2}$	$\frac{(B_o + sy)y}{B_o + 2y\sqrt{1+s^2}}$	$B_o + 2sy$	$\frac{(B_o + sy)y}{B_o + 2sy}$	
Triangular	sy^2	$2y\sqrt{1+s^2}$	$\frac{sy}{2\sqrt{1+s^2}}$	$2sy$	$0.5y$	
Circular	$\frac{1}{8}(\theta - \sin\theta)D_o^2$	$\frac{1}{2}\theta D_o$	$\frac{1}{4}\left(1 - \frac{\sin\theta}{\theta}\right) D_o$	$D_o \sin\frac{1}{2}\theta$	$\left(\frac{\theta - \sin\theta}{\sin\frac{1}{2}\theta}\right)\frac{D_o}{8}$	

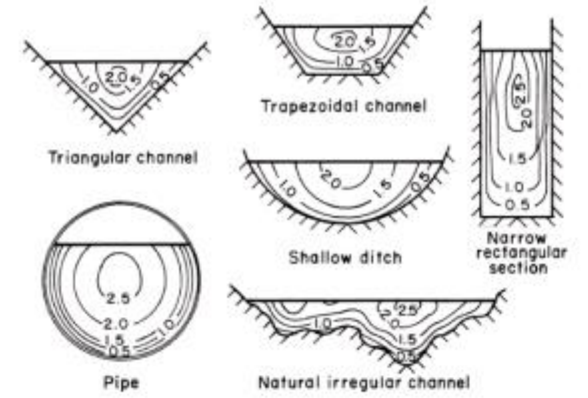


Fig. 1-9. Velocity distribution in different channel sections
(After Chow [1959])

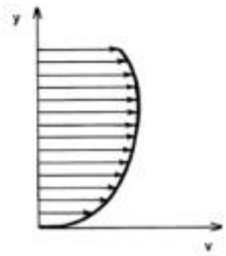


Fig. 1-10. Typical velocity variation with depth

Energy Coefficient

As discussed in the previous paragraphs, the flow velocity in a channel section usually varies from one point to another. Therefore, the mean velocity head in a channel section, $(V^2/2g)$m, is not the same as the velocity head, $V^2_m/(2g)$, computed by using the mean flow velocity, Vm, in which the subscript m refers to the mean values. This difference may be taken into consideration by introducing an energy coefficient, α, which is also referred to as the velocityhead, or Coriolis coefficient. An expression for this coefficient is derived in the following paragraphs

Referring to Fig. 1-11, the mass of liquid flowing through area ΔA per unit time = ρV ΔA, in which ρ = mass density of the liquid. Since, the kinetic energy of mass m traveling at velocity V is (1/2)mV 2, we can write

Kinetic energy transfer through area ΔA per unit time

$= 1/2\ \rho V\ \Delta A V^{2}$

$$= 1/2\ \rho V^{3} \Delta A \qquad (1-4)$$

Hence,

Kinetic energy transfer through area A per unit time

$$= \frac{1}{2}\rho \int V^{3}\, dA \qquad (1-5)$$

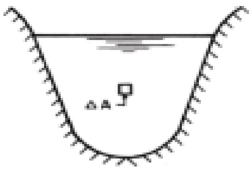


Fig. 1-11. Definition sketch

It follows from Eq. 1-4 that the kinetic energy transfer through area ΔA per unit time may be written as (γV ΔA)V 2/(2g) = weight of liquid passing through area ΔA per unit time × velocity head, in which γ = specific weight of the liquid. Now, if Vm is the mean flow velocity for the channel section, then the weight of liquid passing through total area per unit time =γVm dA; and the velocity head for the channel section =αV 2 m/(2g), in which α = velocityhead coefficient. Therefore, we can write

Kinetic energy transfer through area per unit time

$$= \rho\alpha Vm\ V\ 2\ m\ 2\ \ dA \qquad (1-6)$$

Hence, it follows from Eqs. 1-5 and 1-6 that

$$\alpha = \ V\ 3dA\ V\ 3\ m\ \ dA \qquad (1-7)$$

Figure 1-12 shows a typical cross section of a natural river comprising of the main river channel and the flood plain on each side of the main channel. The flow velocity in the floodplain is usually very low as compared to that in the main section. In addition, the variation of flow velocity in each subsection is small. Therefore, each subsection may be assumed to have the same flow velocity throughout. In such a case, the integration of various terms of Eq. 1-7 may be replaced by summation as follows:

$\alpha = V\ 3\ 1\ A1 + V\ 3\ 2\ A2 + V\ 3\ 3\ A3\ V\ 3\ m(A1 + A2 + A3)$ (1 − 8)

in which

$Vm = V1A1 + V2A2 + V3A3\ A1 + A2 + A3$ (1 − 9)

By substituting Eq. 1-9 into Eq. 1-8 and simplifying, we obtain

$\alpha = (V\ 3\ 1\ A1 + V\ 3\ 2\ A2 + V\ 3\ 3\ A3)(A1 + A2 + A3)2\ (V1A1 + V2A2 + V3A3)3$ (1 − 10)

Note that Eq. 1-10 is written for a section which may be divided into three subsections each having uniform velocity distribution. For a general case in which total area A may be subdivided into N such subareas each having uniform velocity, an equation similar to Eq. 1-10 may be written as

$\alpha = N\ i{=}1(V\ 3\ i\ Ai).(\ Ai)\ 2\ (\ ViAi)^3$ (1 − 11)

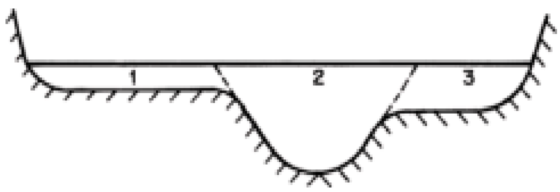


Fig. 1-12. Typical river cross section