جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

كلية العلوم

قسم الأمن السيبراني

Lecture: (3)

Sets of numbers, Finite sets and counting principle

Subject: Discrete Structures

First Stage: Semester II

Lecturer: BAQER KAREEM SALIM

Set of numbers:

Several sets are used so often, they are given special symbols.

N = the set of natural numbers or positive integers

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

Z = the set of all integers: . . . ,–2,–1, 0, 1, 2, . . .

$$\mathbb{Z} = \mathbb{N} \cup \{\ldots, -2, -1\}$$

Q = the set of rational numbers

$$\mathbb{Q} = \mathbb{Z} \cup \{\ldots, -1/3, -1/2, 1/2, 1/3, \ldots, 2/3, 2/5, \ldots\}$$

Where $Q = \{ a/b : a , b \in Z, b \neq 0\}$

R = the set of real numbers

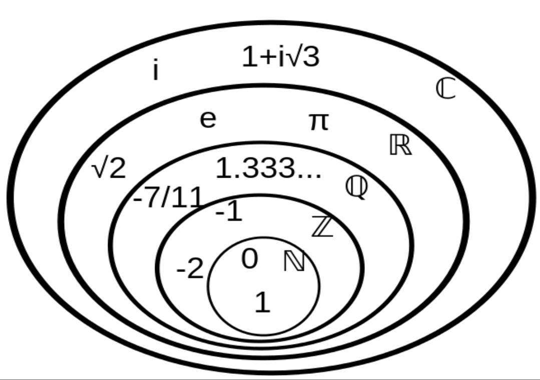
$$\mathbb{R} = \mathbb{Q} \cup \{\ldots, -\pi, -\sqrt{2}, \sqrt{2}, \pi, \ldots\}$$

C = the set of complex numbers

$$\mathbb{C} = \mathbb{R} \cup \{i, 1+i, 1-i, \sqrt{2}+\pi i, \ldots\}$$

Where $C=\{ x + iy \,;\, x , y \in R;\, i = \sqrt{-1}\}$

Observe that $N \subset Z \subset Q \subset R \subset C$.



Finite Sets and Counting Principle:

A set is said to be finite if it contains exactly m distinct elements, where m denotes some nonnegative integer. Otherwise, a set is said to be infinite.

For example:

- The empty set ∅ and the set of letters of English alphabet are finite sets,
- The set of even positive integers, {2,4,6,…..}, is infinite.
- If a set A is finite, we let n(A) or #(A) denote the number of elements of A.

Example:

If A ={1,2,a,w} then

$$n(A) = \#(A) = |A| = 4$$

Lemma: If A and B are finite sets and disjoint Then $A \cup B$ is finite set and:

$$n(A \cup B) = n(A) + n(B)$$

Theorem (Inclusion–Exclusion Principle): Suppose A and B are finite sets. Then

$A \cup B$ and $A \cap B$ are finite and

$$|A \cup B| = |A| + |B| - |A \cap B|$$

That is, we find the number of elements in A or B (or both) by first adding n(A) and n(B) (inclusion) and then subtracting $n(A \cap B)$ (exclusion) since its elements were counted twice.

We can apply this result to obtain a similar formula for three sets:

Corollary:
If A, B, C are finite sets then
$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$

Example (1) :
$A= \{1,2,3\}$
$B= \{3,4\}$
$C= \{5,6\}$
$A \cup B \cup C = \{1,2,3,4,5,6\}$
$|A \cup B \cup C| = 6$

$|A| =3$, $|B| = 2$, $|C| = 2$
$A \cap B = \{3\}$, $| A \cap B | = 1$
$A \cap C = \{ \}$, $| A \cap C | = 0$
$B \cap C = \{ \}$, $| B \cap C | = 0$
$A \cap B \cap C = \{ \}$, $| A \cap B \cap C | = 0$

$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$
$|A \cup B \cup C | = 3 + 2 +2 -1 – 0 – 0 + 0 = 6$

Example (2):
Suppose a list A contains the 30 students in a mathematics class, and a list B contains the 35 students in an English class, and suppose there are 20 names on both lists. Find the number of students:

(a) only on list A
(b) only on list B
(c) on list $A \cup B$

Solution:

(a) List A has 30 names and
20 are on list B;
hence $30 - 20 = 10$ names are only on list A.

(b) Similarly, $35 - 20 = 15$ are only on list B.

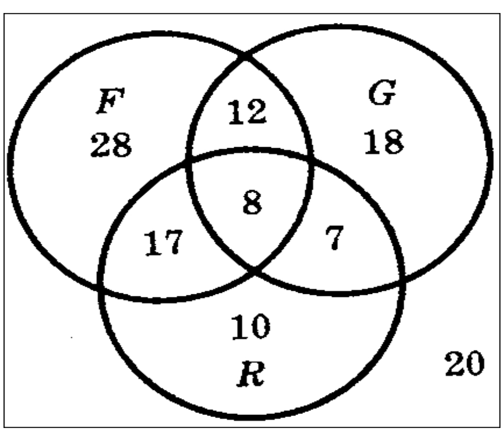
(c) We seek $n(A \cup B)$. By inclusion–exclusion,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 30 + 35 - 20 = 45.$$

Example (3):

Suppose that 100 of 120 computer science students at a college take at least one of languages: French, German, and Russian:
65 study French (F).
45 study German (G).
42 study Russian (R).
20 study French & German $F \cap G$.
25 study French & Russian $F \cap R$.
15 study German & Russian $G \cap R$.
Find the number of students who study:

1) All three languages ($F \cap G \cap R$)
2) The number of students in each of the eight regions of the Venn diagram



Solution:

$|F \cup G \cup R| = |F| + |G| + |R| - |F \cap G| - |F \cap R| - |G \cap R| + |F \cap G \cap R|$

$100 = 65 + 45 + 42 - 20 - 25 - 15 + |F \cap G \cap R|$

$100 = 92 + |F \cap G \cap R|$

$\therefore |F \cap G \cap R| = 8$ students study the 3 languages

$20 – 8 = 12$ $(F \cap G) - R$

$25 – 8 = 17$ $(F \cap R) - G$

$15 – 8 = 7$ $(G \cap R) – F$

65 – 12 – 8 – 17 = 28students study French only

45 – 12 – 8 7 = 18 students study German only

42 – 17 – 8 7 = 10 students study Russian only

120 – 100 = 20 students do not study any language