Thus this compression member has the same load capacity as a column that is pinned at both ends and is only 70% as long as the given column. Similar expressions can be found for columns with other end conditions.

The column buckling problem can also be formulated in terms of a fourth-order differential equation instead of Equation 4.2. This proves to be convenient when dealing with boundary conditions other than pinned ends.

For convenience, the equations for critical buckling load will be written as

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 E_t A}{(KL/r)^2} \tag{4.6a/4.6b}$$

where KL is the *effective length,* and K is the *effective length factor*. The effective length factor for the fixed-pinned compression member is 0.70. For the most favorable condition of both ends fixed against rotation and translation, $K = 0.5$. Values of K for these and other cases can be determined with the aid of Table C-A-7.1 in the Commentary to AISC Specification Appendix 7. The three conditions mentioned thus far are included, as well as some for which end translation is possible. Two values of K are given: a theoretical value and a recommended design value to be used when the ideal end condition is approximated. Hence, unless a "fixed" end is perfectly fixed, the more conservative design values are to be used. Only under the most extraordinary circumstances would the use of the theoretical values be justified. Note, however, that the theoretical and recommended design values are the same for conditions (d) and (f) in Commentary Table C-A-7.1. The reason is that any deviation from a perfectly frictionless hinge or pin introduces rotational restraint and tends to reduce K. Therefore, use of the theoretical values in these two cases is conservative.

The use of the effective length KL in place of the actual length L in no way alters any of the relationships discussed so far. The column strength curve shown in Figure 4.6 is unchanged except for renaming the abscissa KL/r. The critical buckling stress corresponding to a given length, actual or effective, remains the same.

4.3 AISC REQUIREMENTS

The basic requirements for compression members are covered in Chapter E of the AISC Specification. The nominal compressive strength is

$$P_n = F_{cr} A_g \qquad \text{(AISC Equation E3-1)}$$

For LRFD,

$$P_u \leq \phi_c P_n$$

where

P_u = sum of the factored loads
ϕ_c = resistance factor for compression = 0.90
$\phi_c P_n$ = design compressive strength

For ASD,

$$P_a \leq \frac{P_n}{\Omega_c}$$

where

P_a = sum of the service loads
Ω_c = safety factor for compression = 1.67
P_n/Ω_c = allowable compressive strength

If an allowable stress formulation is used,

$$f_a \leq F_a$$

where

f_a = computed axial compressive stress = P_a/A_g
F_a = allowable axial compressive stress

$$= \frac{F_{cr}}{\Omega_c} = \frac{F_{cr}}{1.67} = 0.6F_{cr} \tag{4.7}$$

In order to present the AISC expressions for the critical stress F_{cr}, we first define the Euler load as

$$P_e = \frac{\pi^2 EA}{(KL/r)^2}$$

This is the critical buckling load according to the Euler equation. The Euler stress is

$$F_e = \frac{P_e}{A} = \frac{\pi^2 E}{(KL/r)^2} \qquad \text{(AISC Equation E3-4)}$$

With a slight modification, this expression will be used for the critical stress in the elastic range. To obtain the critical stress for elastic columns, the Euler stress is reduced as follows to account for the effects of initial crookedness:

$$F_{cr} = 0.877F_e \tag{4.8}$$

For inelastic columns, the tangent modulus equation, Equation 4.6b, is replaced by the exponential equation

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \tag{4.9}$$

With Equation 4.9, a direct solution for inelastic columns can be obtained, avoiding the trial-and-error approach inherent in the use of the tangent modulus equation. At the