The *variance, v*, is a measure of the overall variation of the data from the mean and is defined as

$$v = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

The *standard deviation s* is the square root of the variance, or

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Like the variance, the standard deviation is a measure of the overall variation, but it has the same units and the same order of magnitude as the data. The *coefficient of variation, V*, is the standard deviation divided by the mean, or

$$V = \frac{s}{\bar{x}}$$

If the actual frequency distribution is replaced by a theoretical continuous function that closely approximates the data, it is called a *probability density function*. Such a function is illustrated in Figure 2.2. Probability functions are designed so that the total area under the curve is unity. That is, for a function $f(x)$,

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1.0$$

which means that the probability that one of the sample values or events will occur is 1.0. The probability of one of the events between a and b in Figure 2.2 equals the area under the curve between a and b, or

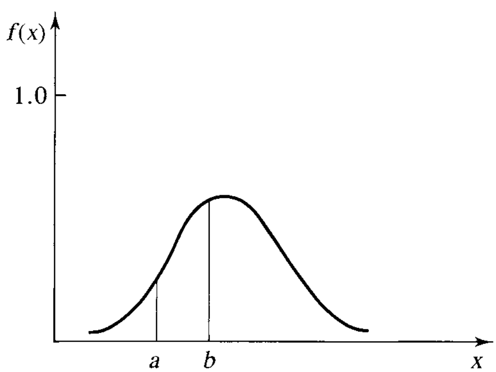
$$\int_{a}^{b} f(x)\,dx$$

When a theoretical probability density function is used, the following notation is conventional:

μ = mean

σ = standard deviation

FIGURE 2.2



The probabilistic basis of the load and resistance factors used by AISC is presented in the ASCE structural journal and is summarized here (Ravindra and Galambos, 1978). Load effects, Q, and resistances, R, are random variables and depend on many factors. Loads can be estimated or obtained from measurements and inventories of actual structures, and resistances can be computed or determined experimentally. Discrete values of Q and R from observations can be plotted as frequency distribution histograms or represented by theoretical probability density functions. We use this latter representation in the material that follows.

If the distributions of Q and R are combined into one function, $R..Q$, positive values of $R..Q$ correspond to survival. Equivalently, if a probability density function of R/Q, the factor of safety, is used, survival is represented by values of R/Q greater than 1.0. The corresponding probability of failure is the probability that R/Q is less than 1; that is,

$$P_F = P \quad \frac{R}{Q} < 1$$

Taking the natural logarithm of both sides of the inequality, we have

$$P_F = P \;\; \ln \frac{R}{Q} < \ln 1 \;\; = P \;\; \ln \frac{R}{Q} < 0$$

The frequency distribution curve of $\ln(R/Q)$ is shown in Figure 2.3. The *standardized* form of the variable $\ln(R/Q)$ can be defined as

$$U = \frac{\ln \frac{R}{Q} \qquad \ln \frac{R}{Q}_{\,m}}{\sigma_{\ln(R/Q)}}$$

FIGURE 2.3