Pressure Distribution

The pressure distribution in a channel section depends upon the flow conditions. Let us consider several possible cases, starting with the simplest one and then proceeding progressively to more complex situations.

Static Conditions

Let us consider a column of liquid having cross-sectional area ΔA, as shown in Fig. 1-13. The horizontal and vertical components of the resultant force acting on the liquid column are zero, since the liquid is stationary. If p = pressure intensity at the bottom of the liquid column, then the force due to pressure at the bottom of the column acting vertically upwards = $p\Delta A$. The weight of the liquid column acting vertically downwards = $\rho g y \Delta A$. Since the vertical component of the resultant force is zero, we can write

$$p\Delta A = \rho g y \Delta A$$

or

$$p = \rho g y \qquad (1-15)$$

In other words, the pressure intensity is directly proportional to the depth below the free surface. Since ρ is constant for typical engineering applications, the relationship between the pressure intensity and depth plots as a straight line, and the liquid rises to the level of the free surface in a piezometer, as shown in Fig. 1-13. The linear relationship, based on the assumption that ρ is constant, is usually valid except at very large depths, where large pressures result in increased density.

Horizontal, Parallel Flow

Let us now consider the forces acting on a vertical column of liquid flowing in a horizontal, frictionless channel (Fig. 1-14).

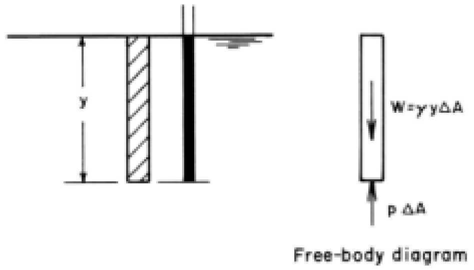

Fig. 1-13. **Pressure in stationary fluid**

Let us assume that there is no acceleration in the direction of flow and the flow velocity is parallel to the channel bottom and is uniform over the channel section. Thus the streamlines are parallel to the channel bottom. Since there is no acceleration in the direction of flow, the component of the resultant force in this direction is zero. Referring to the free-body diagram shown in Fig. 1-14 and noting that the vertical component of the resultant force acting on the column of liquid is zero, we may write

$$\rho g y \Delta A = p \Delta A$$

or

$$p = \rho g y = \gamma y \qquad (1-16)$$

in which $\gamma = \rho g$ = specific weight of the liquid. Note that this pressure distribution is the same as if the liquid were stationary; it is, therefore, referred to as the hydrostatic pressure distribution.

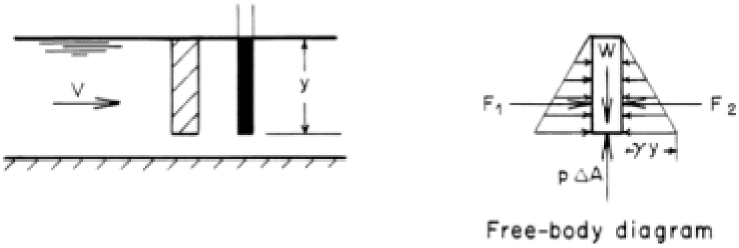

Fig. 1-14. Horizontal, parallel flow

Parallel Flow in Sloping Channels

Let us now consider the flow conditions in a sloping channel such that there is no acceleration in the flow direction, the flow velocity is uniform at a channel cross section and is parallel to the channel bottom; i.e., the streamlines are parallel to the channel bottom. Figure 1-15 shows the free-body diagram of a column of liquid normal to the channel bottom. The cross-sectional area of the column is ΔA. If θ = slope of the channel bottom, then the component of the weight of column acting along the column is $\rho g d \Delta A \cos\theta$ and the force acting at the bottom of the column is $p\Delta A$. There is no acceleration in a direction along the column length, since the flow velocity is parallel to the channel bottom. Hence, we can write $p\Delta A = \rho g d \Delta A \cos\theta$, or $p = \rho g d \cos\theta = \gamma d \cos\theta$. By substituting $d = y \cos\theta$ into this equation (y = flow depth measured vertically, as shown in Fig. 1-15), we obtain

$$p = \gamma y \cos^2\theta \qquad (1-17)$$

Note that in this case the pressure distribution is not hydrostatic in spite of

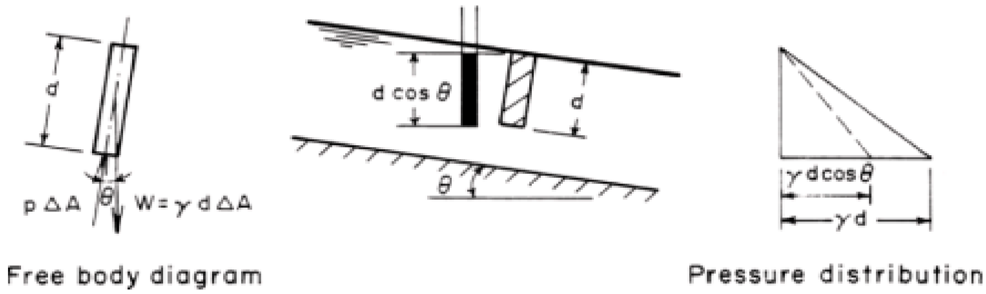

Fig. 1-15. **Parallel flow in a sloping channel**

the fact that we have parallel flow and there is no acceleration in the direction of flow. However, if the slope of the channel bottom is small, then $\cos\theta \approx 1$ and $d \approx y$. Hence,

$$p \approx \rho g d \approx \rho g y \qquad (1-18)$$

In several derivations in the subsequent chapters we assume that the slope of the channel bottom is small. With this assumption, the pressure distribution may be assumed to be hydrostatic if the streamlines are almost parallel and straight, and the flow depths measured vertically or normal to the channel bottom are approximately the same.

Curvilinear Flow

In the previous three cases, the streamlines were straight and parallel to the channel bottom. However, in several real-life situations, the streamlines have pronounced curvature. To determine the pressure distribution in such flows, let us consider the forces acting in the vertical direction on a column of liquid with cross-sectional area ΔA, as shown in Fig. 1-16.

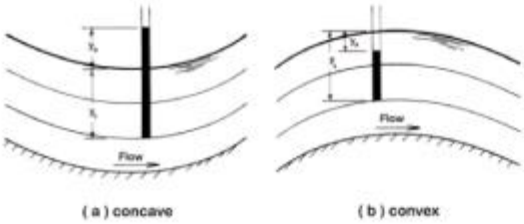

Fig. 1-16. Curvilinear flow

1-7 Reynolds Transport Theorem

The Reynolds transport theorem relates the flow variables for a specified fluid mass to that of a specified flow region. We will utilize it in later chapters to derive the governing equations for steady and unsteady flow conditions. To simplify the presentation of its application, we include a brief description in this section; for details, see Roberson and Crowe [1997].

We will call a specified fluid mass the system and a specified region, the control volume. The boundaries of a system separate it from its surroundings and the boundaries of a control volume are referred to as the control surface. The three well-known conservation laws of mass, momentum, and energy de scribe the interaction between a system and its surroundings. However, in hydraulic engineering, we are usually interested in the flow in a region as compared to following the motion of a fluid particle or the motion of a quantity of mass. The Reynolds transport theorem relates the flow variables in a control volume to those of a system.

Let the extensive property of a system be B and the corresponding intensive property be β. The intensive property is defined as the amount of B per unit mass, m, of a system, i.e.,

$$\beta = \lim \Delta m \rightarrow 0 \ \Delta B \ \Delta m \qquad (1-24)$$

Thus, the total amount of B in a control volume

$$B_{cv} = \int_{cv} \beta\rho dV \qquad (1-25)$$

in which ρ = mass density and dV = differential volume of the fluid, and the integration is over the control volume. We will consider mainly one-dimensional flows in this book. The control volume will be fixed in space and will not change its shape with respect to time, i.e., it will not stretch or contract. For such a control volume for one dimensional flow, the following equation relates the system properties to those in the control volume:

$$dB_{sys}/dt = \frac{d}{dt} \int_{cv} \beta\rho dV + (\beta\rho AV)_{out} - (\beta\rho AV)_{in} \qquad (1-26)$$

in which the subscripts in and out refer to the quantities for the inflow and outflow from the control volume and V = flow velocity. The system is assumed to occupy the entire control volume, i.e., the system boundaries coincide with the control surface.

Let us now discuss the application of this equation to a control volume. As an example, the time rate of change of momentum of a system is equal to the sum of the forces exerted on the system by its surroundings (New□ton's second law of motion).To use this equation to describe the conservation of momentum of the water of mass m in a control volume, the extensive property B is the momentum of water = mV and the corresponding intensive property, $\beta = \lim_{\Delta m \to 0} V (\Delta m/\Delta m) = V$. To describe the conservation of mass, B is the mass of water and the corresponding intensive property $\beta = \lim_{\Delta m \to 0}(\Delta m/\Delta m)=1$.