ALMUSTAQBAL UNIVERSITY

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

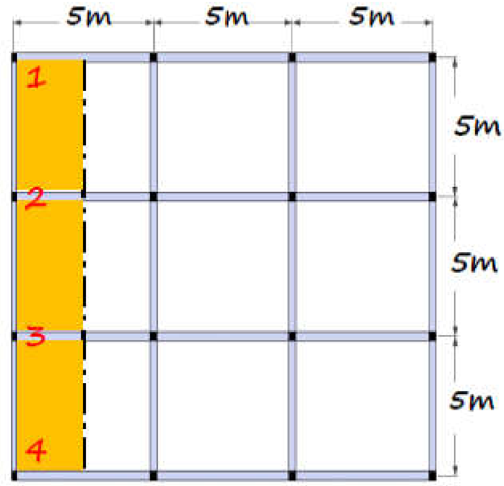
ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

EXAMPLES IN THE EQUIVALENT FRAME METHOD

II

EXAMPLE 5: For the edge equivalent frame of the roof slab shown below, storey height is 3m, beams are of $250 \times 500mm$, columns are dimensioned as $250 \times 250mm$, $t = 150mm$, $f_c' = 25MPa$, find K_c, K_t, K_{ce} and distribution factors DF at each point.

SOLUTION:



1. **Calculating K_{sb}**

$$\frac{C_{1A}}{l_1} = \frac{C_{1B}}{l_1} = \frac{250}{5000} = 0.05$$

$$\therefore \ k_{sb} = 4.05, FEM = 0.084, COF = 0.503$$

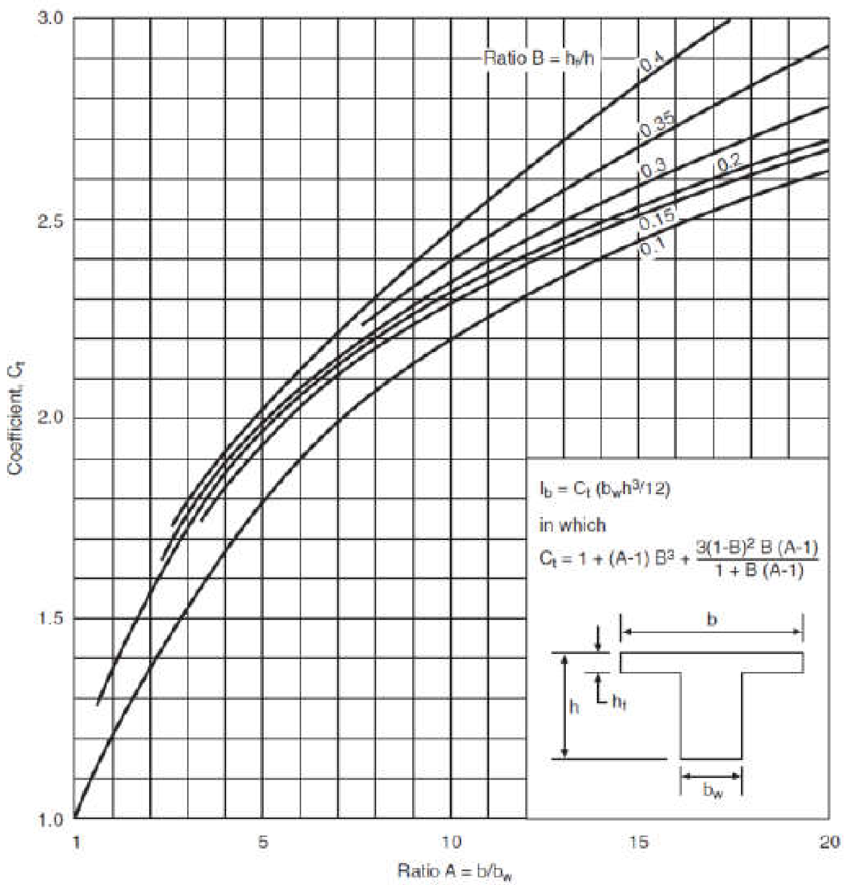
$$K_{sb} = k.\frac{E(I_s + I_b)}{L_1}$$

$$I_s = \frac{h^3 L_2}{12}$$

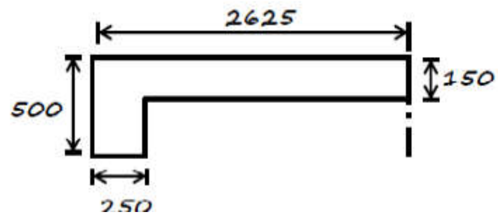
$$L_2 = \frac{5000}{2} + \frac{250}{2} = 2625mm,$$

$$\therefore \ I_s = \frac{150^3 \times 2625}{12} = 0.738 \times 10^9 mm^4$$

$$I_b = C_t.\frac{bh^3}{12}, \quad C_t \ is \ taken \ from \ the \ chart \ shown \ below:$$



$\frac{h_f}{h} = \frac{150}{500} = 0.3,\ \frac{b}{b_w} = \frac{2625}{250} = 10.5$



From the chart above $C_t = 2.35$

$$I_b = 2.35\frac{250 \times 500^3}{12} = 6.12 \times 10^9 mm^4$$

$$K_{sb} = k.\frac{E(I_s + I_b)}{L_1} = 4.05 \times \frac{23500(0.738 \times 10^9 + 6.12 \times 10^9)}{5000} = 1.31 \times 10^{11} N.mm$$

2. **Calculate K_c:**

$K_c = k.\frac{EI_c}{l_c}$,

$$\frac{C_1 A}{l_c} = \frac{500}{3000} = 0.167$$

$$k_{bot} = 7.64 + \frac{0.167 - 0.15}{0.05}(9.69 - 7.64) = 8.33$$

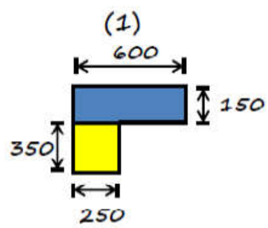
$$I_c = \frac{{c_1}^3 c_2}{12} = \frac{250^4}{12} = 0.326 \times 10^9 mm^4.$$

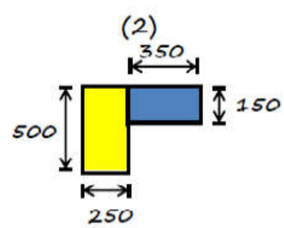
$$K_{c\ BOT} = k.\frac{EI_c}{L_c} = 8.33 \times \frac{23500 \times 0.326 \times 10^9}{3000} = 2.12 \times 10^{10} N.mm$$

3. **Calculate K_t:**

$@\ exterior\ joint$:

$$K_t = \frac{9EC}{l_2(1 - \frac{c_2}{l_2})^3}\ , \qquad C = \sum(1 - 0.63\frac{x}{y})\frac{x^3 y}{3}$$





$$C1 = \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} = \left(1 - 0.63 \times \frac{150}{600}\right) \times \frac{150^3 \times 350}{3} + \left(1 - 0.63 \times \frac{250}{350}\right) \times \frac{250^3 \times 350}{3}$$
$$= 1.57 \times 10^9 mm^4$$

$$C2 = \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} = \left(1 - 0.63 \times \frac{150}{350}\right) \times \frac{150^3 \times 350}{3} + \left(1 - 0.63 \times \frac{250}{500}\right) \times \frac{250^3 \times 500}{3}$$
$$= \mathbf{2.07 \times 10^9 mm^4}$$

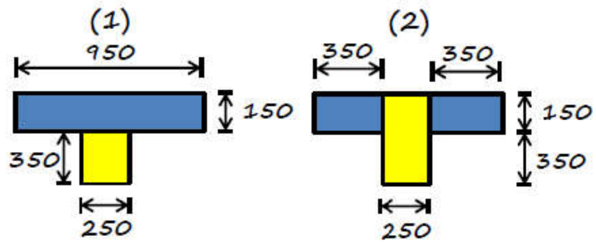
$$\therefore \boldsymbol{C = 2.07 \times 10^9 mm^4}$$

$$K_t = \frac{9 \times 23500 \times 2.07 \times 10^9}{5000(1 - \frac{250}{5000})^3} = 10.17 \times 10^{10} N.mm$$

Since there is a beam in the direction of the frame, then we use K_{ta}

$$K_{ta} = K_t \frac{I_{sb}}{I_s} = 10.17 \times 10^{10} \times \frac{6.12 \times 10^9}{0.738 \times 10^9} = 84.4 \times 10^{10} N.mm$$

$@\ interior\ joint$:



$$C1 = \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} = \left(1 - 0.63 \times \frac{150}{950}\right) \times \frac{150^3 \times 950}{3} + \left(1 - 0.63 \times \frac{250}{350}\right) \times \frac{250^3 \times 350}{3}$$
$$= 1.96 \times 10^9 mm^4$$

$$C2 = \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} = 2 \times \left(1 - 0.63 \times \frac{150}{350}\right) \times \frac{150^3 \times 350}{3} + \left(1 - 0.63 \times \frac{250}{500}\right) \times \frac{250^3 \times 500}{3}$$
$$= \mathbf{2.36 \times 10^9 mm^4}$$

$$\therefore C = 2.36 \times 10^9 mm^4$$
$$K_t = \frac{9 \times 23500 \times 2.36 \times 10^9}{5000(1 - \frac{250}{5000})^3} = 11.59 \times 10^{10} N.mm$$
$$\therefore\ K_{ta} = K_t \frac{I_{sb}}{I_s} = 11.59 \times 10^{10} \times \frac{6.12 \times 10^9}{0.738 \times 10^9} = 96.2 \times 10^{10} N.mm$$

4. Calculate K_{ec}

@ exterior joints:

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t} = \frac{1}{2.12 \times 10^{10}} + \frac{1}{84.4 \times 10^{10}} \rightarrow K_{ec} = 2.068 \times 10^{10} N.mm$$

@ interior joints:

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t} = \frac{1}{2.12 \times 10^{10}} + \frac{1}{96.2 \times 10^{10}} \rightarrow K_{ec} = 2.074 \times 10^{10} N.mm$$

5. Calculate DF:

Exterior joints:

$$DF = \frac{K_{sb}}{K_{sb} + K_{ec}} = \frac{8.69}{8.69 + 2.068} = 0.81$$

$$DF_{ec} = \frac{K_{ec}}{K_{sb} + K_{ec}} = \frac{2.068}{8.69 + 2.068} = 0.19$$

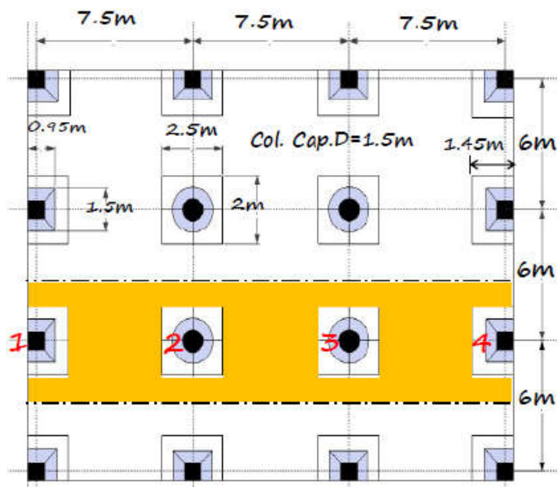
Interior Joints:

$$DF_{sb\ 2-1} = DF_{sb2-3} = \frac{K_{sb2-1}}{K_{sb2-1} + K_{sb2-3} + K_{ec}} = \frac{8.69}{8.69 \times 2 + 2.074} = 0.45$$

$$DF_{ec} = \frac{K_{ec}}{K_{sb2-1} + K_{sb2-3} + K_{ec}} = \frac{2.074}{8.69 \times 2 + 2.074} = 0.1$$

EXAMPLE 6: for the flat slab floor shown below, determine K_c, K_t, K_{ce} and distribution factors DF at each point. Given for this slab are the storey height of $3m$ $t = 180mm$, $t_{drop} = 45mm$, circular columns are of $450mm$, square columns are of $400mm$, and concrete strength at 28 days is 25MPa.

SOLUTION:



1. ***FIND Kc:***

Interior columns:

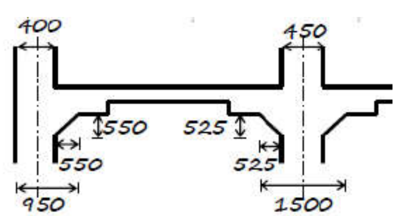
$$\frac{C_1 A}{l_c} = \frac{180 + 45 + 525}{3000} = 0.250$$

From the table: $k_{bot} = 12.44, k_{top} = 5.33$

$$I_c = \frac{\pi D^4}{64} = \frac{\pi 450^4}{64} = 2.012 \times 10^9 mm^4$$

$$E = 4700\sqrt{f_c{}'} = 4700\sqrt{25} = 23500MPa$$

$$K_c = k.\frac{EI_c}{l_c}$$



$$K_{cbot} = 12.44 \times \frac{23500 \times 2.012 \times 10^9}{3000} = 19.61 \times 10^{10} N.mm$$

$$K_{ctop} = 5.33 \times \frac{23500 \times 2.012 \times 10^9}{3000} = 8.4 \times 10^{10} N.mm$$

Exterior columns:

$$\frac{C_1 A}{l_c} = \frac{180 + 45 + 550}{3000} = 0.258 \cong 0.25$$

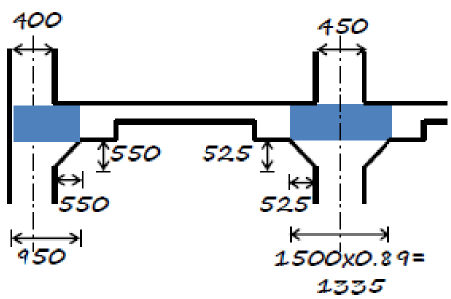
From the table: $k_{bot} = 12.44, k_{top} = 5.33$

$$I_c = \frac{C_2{C_1}^3}{12} = \frac{400^4}{12} = 2.133 \times 10^9 mm^4$$

$$K_{cbot} = 12.44 \times \frac{23500 \times 2.133 \times 10^9}{3000} = 20.79 \times 10^{10} N.mm$$

$$K_{ctop} = 5.33 \times \frac{23500 \times 2.133 \times 10^9}{3000} = 8.9 \times 10^{10} N.mm$$

2. ***Find Kt***



<u>Interior joints (2&3):</u>

$$C = \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} = \left(1 - 0.63\frac{225}{1335}\right)\frac{225^3 \times 1335}{3} = 4.53 \times 10^9 mm^4$$

$$K_t = \frac{9EC}{L_2(1 - \frac{C_2}{L_2})^3} = \frac{9 \times 23500 \times 4.53 \times 10^9}{6000(1 - \frac{1335}{6000})^3} = 33.9 \times 10^{10} N.mm$$

<u>Exterior joints (1&4):</u>

$$C = \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} = \left(1 - 0.63\frac{225}{950}\right)\frac{225^3 \times 950}{3} = 3.06 \times 10^9 mm^4$$

$$K_t = \frac{9EC}{L_2(1 - \frac{C_2}{L_2})^3} = \frac{9 \times 23500 \times 3.06 \times 10^9}{6000(1 - \frac{1500}{6000})^3} = 4.55 \times 10^{10} N.mm$$

3. ***Find Kec:***

<u>Interior joints:</u>

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t}$$

$$\frac{1}{K_{ec}} = \frac{1}{(19.61 + 8.4) \times 10^{10}} + \frac{1}{2 \times 33.9 \times 10^{10}} \rightarrow \therefore K_{ec} = 19.8 \times 10^{10} N.mm$$

Exterior joints:

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t}$$

$$\frac{1}{K_{ec}} = \frac{1}{(20.79 + 8.9) \times 10^{10}} + \frac{1}{2 \times 4.55 \times 10^{10}} \rightarrow \therefore K_{ec} = 6.96 \times 10^{10} N.mm$$

4. *Find DF:*

Exterior joints:

$$DF_{sb\ 1-2} = \frac{K_{sb\ 1-2}}{K_{sb\ 1-2} + K_{ec}} = \frac{4.717}{4.717 + 6.96} = 0.403$$

$$DF_{ec1} = \frac{K_{ec}}{K_{sb\ 1-2} + K_{ec}} = \frac{6.96}{4.717 + 6.96} = 0.596$$

Interior joints:

$$DF_{sb\ 1-2} = \frac{K_{sb\ 1-2}}{K_{sb\ 1-2} + K_{sb\ 2-3} + K_{ec}} = \frac{4.87}{4.87 + 4.938 + 19.8} = 0.164$$

$$DF_{sb\ 2-3} = \frac{K_{sb\ 2-3}}{K_{sb\ 1-2} + K_{sb\ 2-3} + K_{ec}} = \frac{4.938}{4.87 + 4.938 + 19.8} = 0.166$$

$$DF_{sb\ 2-3} = \frac{K_{sb\ 2-3}}{K_{sb\ 1-2} + K_{sb\ 2-3} + K_{ec}} = \frac{19.8}{4.87 + 4.938 + 19.8} = 0.668$$

IMPORTANT NOTES:

- For BRICK WALL $K_t = 0, K_{ec} = 0, C_1A = 0, DF = 0.$
- For CONCRETE WALL $K_t = \infty, K_{ec} = \sum K_c$.