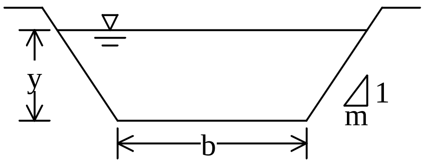
8.2.2.2 Trapezoidal section



$$B = b+2*m*y$$

$$A = (b+m*y)*y$$

$$P = b+2*y*\sqrt{1+m^2}$$

By eliminating b from P,

$$P = \frac{A}{y} + (2*\sqrt{1+m^2} - m)*y$$

For a minimum value of P, $\delta P = 0$,

i.e. $\frac{dP}{dy} = 0$ and $\frac{dP}{dm} = 0$

From $\frac{dP}{dy} = 0$, $y^2 = \frac{A}{\sqrt{3}}$

From $\frac{dP}{dm} = 0$, $m = \frac{1}{\sqrt{3}}$

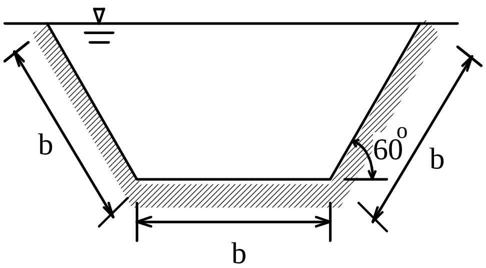
It implies the side slope of the channel is 60° to horizontal.

$$b = \frac{A}{y} - my = \sqrt{3}y - \frac{y}{\sqrt{3}} = \frac{2\sqrt{3}}{3}y$$

and $$P = \frac{2\sqrt{3}}{3}y + \frac{4}{\sqrt{3}}y = 2\sqrt{3}y$$

i.e. $P = 3*b$

The optimum section is given as follow:

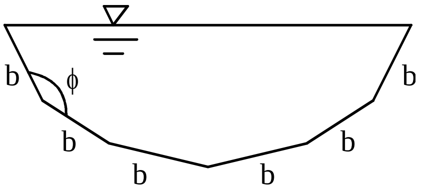


8.2.2.3 Other sections

N-side Channel

- from the conclusion of the previous two sections
  - reflection of the rectangular optimum section about the water surface will form a **square** of side b.
  - reflection of the trapezoidal optimum section about the water surface will form a regular **hexagon** of side b.

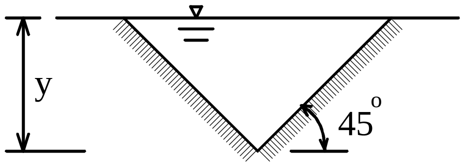
- For a N-side channel, the optimum hydraulic section should be in a form of half a **2N-side regular polygon**.



$$\phi = \left(\frac{N-1}{N}\right) * 180°$$

Triangular Section

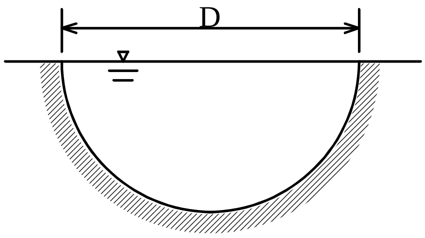
- N = 2, hence $\phi = 90°$



Circular Section

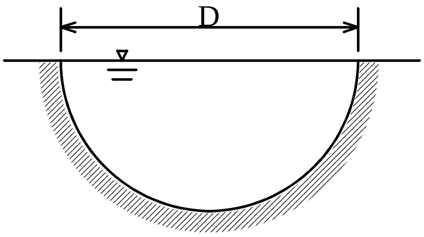
- From the result of N-side channel, it can be concluded that the optimum section of a circular channel is a **semi-circle**.

- It is the most optimum section for all the possible open-channel cross-section.



Worked examples

1. An open channel is to be designed to carry 1m^3/s at a slope of 0.0065. The channel material has an n value of 0.011. Find the optimum hydraulic cross-section for a semi-circular section.



Answer

The optimum circular section is a semi-circular section with diameter D which can discharge 1 m^3/s.

For a semi-circular section,

$$A = \pi * D^2/8$$
$$P = \pi * D/2$$
$$R = A/P = D/4$$

As n = 0.011, S = 0.0065 and Q = 1 m^3/s.

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

i.e.

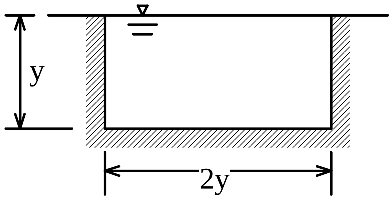
$$1 = \frac{\pi * D^2}{8 * 0.011} * \left(\frac{D}{4}\right)^{2/3} * \sqrt{0.0065}$$

$$D^{8/3} = \frac{8 * 0.011}{\pi} * 4^{\frac{2}{3}} * 0.0065^{-\frac{1}{2}}$$

$$D = 0.951 \text{ m}$$

The diameter of this optimum section is 951mm.

2. Find the optimum rectangular section from the last example.



Answer

A = $2*y^2$
P = $4*y$
R = $A/P = y/2$

By Manning equation,

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

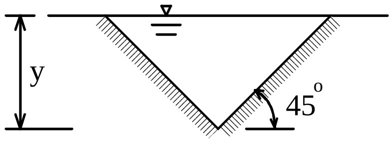
$$1 = \frac{2*y^2}{0.011} * \left(\frac{y}{2}\right)^{\frac{2}{3}} * \sqrt{0.0065}$$

$$y^{8/3} = \frac{1*0.011*2^{\frac{2}{3}}}{2*\sqrt{0.0065}}$$

y = 0.434 m

The optimum rectangular section has dimension of width 0.868m and depth 0.434m.

3. Find the optimum triangular section from the last example.



Answer

$$A = y^2$$
$$P = 2\sqrt{2} * y$$
$$R = A/P = y/2\sqrt{2}$$

By Manning equation,

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

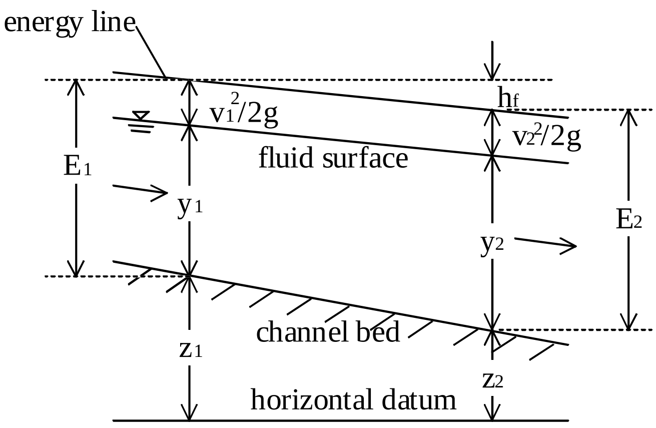
$$1 = \frac{y^2}{0.011} * \left(\frac{y}{2\sqrt{2}}\right)^{\frac{2}{3}} * \sqrt{0.0065}$$

$$y^{8/3} = \frac{0.011 * (2\sqrt{2})^{\frac{2}{3}}}{\sqrt{0.0065}}$$

$$y = 0.614 \text{ m}$$

The optimum triangular section is a right angle triangle with depth 0.614 m.

8.3 Non-Uniform flow - Specific Energy in Open Channel & Critical Flow



- In open channel, the solution of many problems are greatly assisted by the concept of **specific energy**, i.e.

$$E = \frac{v^2}{2g} + y \qquad (8.3)$$

In terms of flow rate, Q,

$$E = \frac{1}{2g}\left(\frac{Q}{A}\right)^2 + y \qquad (8.4)$$

- The minimum energy will be given as

$$\frac{dE}{dy} = 0 \qquad (8.5)$$

8.3.1 Rectangular Channel

- Let $q = \frac{Q}{b} = v*y \qquad (8.6)$

q - the discharge per unit width of a rectangular channel

$$\therefore \quad E = \frac{q^2}{2gy^2} + y \qquad (8.7)$$

- By assuming q is constant

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0 \qquad (8.8)$$

or $$y = y_c$$

$$= \left(\frac{q^2}{g}\right)^{1/3} \qquad (8.9)$$

y_c - **critical depth** at which the energy is minimum.

- The corresponding energy, E is

$$E_{min} = \frac{3}{2} y_c \qquad (8.10)$$

- From (8.6), $v = \frac{q}{y}$.

  Substitute into (8.8),

$$1 - \frac{v_c^2}{gy_c} = 0$$

$$\frac{v_c^2}{gy_c} = 1 \qquad (8.11)$$

or $$v_c = \sqrt{gy_c} \qquad (8.12)$$

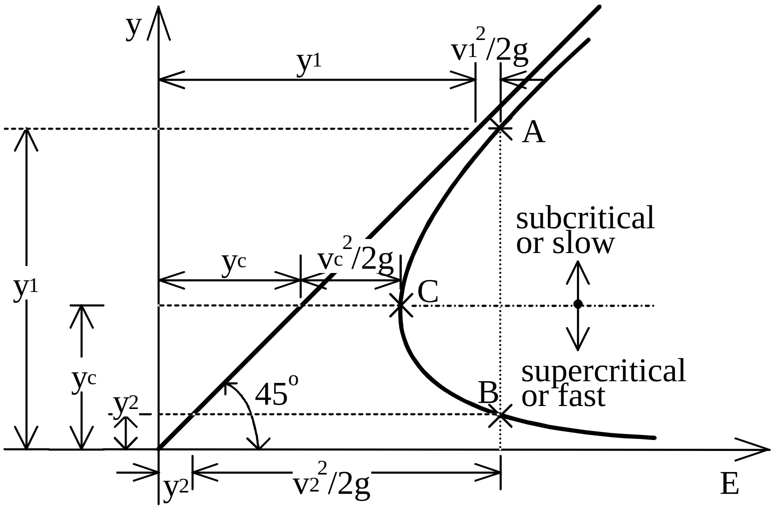
- Since Froude number, Fr is defined as

$$Fr = \frac{v}{\sqrt{gy_{ave}}} \qquad (8.13)$$

  Hence, the minimum energy is occurred when

$$Fr^2 = 1 \qquad (8.14)$$

- For a given discharge, Q, if the flow is such that E is a min., the flow is **critical flow.**
  - critical flow - flow with E_{min}
  - critical depth, yc - the depth of the critical flow
  - critical velocity - $v_c = \sqrt{gy_c}$



- If the flow with $E > E_{min}$, there are two possible depths (y_1, y_2).
  - (y_1, y_2) are called **alternate depths.**

- C divides the curve AB into AC and CB regions.
  - AC - **subcritical** flow region
  - CB - **supercritical** flow region

	Subcritical	Critical	Supercritical
Depth of flow	$y > y_c$	$y = y_c$	$y < y_c$
Velocity of flow	$v < v_c$	$v = v_c$	$v > v_c$
Slope	Mild $S < S_c$	Critical $S = S_c$	Steep $S > S_c$
Froude number	$Fr < 1.0$	$Fr = 1.0$	$Fr > 1.0$
Other	$\frac{v^2}{2g} < \frac{y_c}{2}$	$\frac{v^2}{2g} = \frac{y_c}{2}$	$\frac{v^2}{2g} > \frac{y_c}{2}$

8.3.2 Non - Rectangular Channel

◆ If the channel width varies with y, the specific energy must be written in the form

$$E = \frac{Q^2}{2gA^2} + y \tag{8.15}$$

◆ The minimum energy also occurs where

$$\frac{dE}{dy} = 0 \quad \text{at constant } Q$$

◆ Since A = A(y), therefore (8.15) becomes

$$1 - \frac{2Q^2A^{-3}}{2g}\frac{dA}{dy} = 0$$

or

$$\frac{dA}{dy} = \frac{gA^3}{Q^2} \tag{8.16}$$

◆ Since $\frac{dA}{dy} = B$ - the channel width at the free surface,

$$\therefore \quad B = \frac{gA^3}{Q^2}$$

or

$$A = \left(\frac{BQ^2}{g}\right)^{1/3} \tag{8.17}$$

$$v_c = \frac{Q}{A}$$

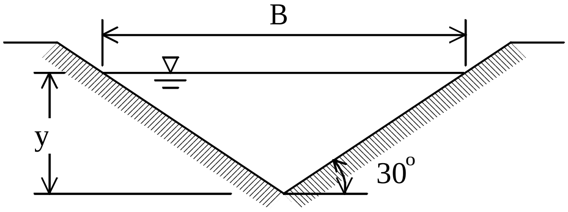
$$= \left(\frac{gA}{B}\right)^{1/2} \tag{8.18}$$

◆ For a given channel shape, A(y) & B(y), and a given Q, (8.17) & (8.18) have to be solved by trial and error to find the A and then v_c.

◆ If a critical channel flow is also moving uniformly (at constant depth), it must correspond to a critical slope, S_c, with $y_n = y_c$. This condition can be analysed by Manning formula.

Worked examples:

1. A triangular channel with an angel of 120° made by 2 equal slopes. For a flow rate of 3 m^3/s, determine the critical depth and hence the maximum depth of the flow.



Answer

For critical flow,

$$v^2 = g*y_{ave}$$

$$Q^2 = g*y_{ave}*A^2$$

$$= \frac{gA^3}{B} \qquad (y_{ave} = \frac{A}{B})$$

For critical flow,

$$B = 2*y*\cot 30°$$

$$\& \quad A = y^2*\cot 30°$$

$$\therefore \quad Q^2 = \frac{3g}{2}y^5$$

Hence

$$y = (\frac{2Q^2}{3g})^{1/5}$$

$$= (\frac{2*3^2}{3*9.81})^{1/5} \text{ m}$$

$$= \underline{0.906 \text{ m}}$$

The maximum depth is 0.906 m.

~~The critical depth, y_c =~~ $y_{ave} = \frac{A}{B} = \frac{(B*y)/2}{B} = \frac{y}{2} = \frac{0906}{2} = 0.453m$.