AL MUSTAQBAL UNIVERSITY

ENGINEERING TECHNICAL COLLEGE

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGIES

ENGINEERING PHYSICS

FIRST CLASS

LECTURE NO. 2

ASST. LECTURER

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UNITS AND DIMENSIONS

1.4 DEFINITION OF DIMENSIONS

Dimensions: The powers, to which the fundamental units of mass, length and time written as M, L and T are raised, which include their nature and not their magnitude.

For example,

Area = Length x Breadth

$= [L^1] \times [L^1] = [L^2] = [M^0L^2T^0]$

Power (0,2,0) of fundamental units are called dimensions of area in mass, length and time respectively.

e.g. Density = mass/volume

$= [M^1]/[L^3]$

$= [M^1L^{-3}T^0]$

1.5 DIMENSIONAL FORMULAE AND SI UNITS OF PHYSICAL QUANTITIES

Dimensional Formula: An expression along with power of mass, length & time which indicates how physical quantity depends upon fundamental physical quantity.

e.g. Speed = Distance/Time

$= [L^1]/[T^1] = [M^0L^1T^{-3}]$

It tells us that speed depends upon L & T. It does not depend upon M.

Dimensional Equation: An equation obtained by equating the physical quantity with its dimensional formula is called dimensional equation.

The dimensional equation of area, density & velocity are given as under-

Area = $[M^0L^2T^0]$

Density = $[M^1L^{-3}T^0]$

Velocity = $[M^0L^1T^{-1}]$

Dimensional formula SI& CGS unit of Physical Quantities

Sr. No.	Physical Quantity	Formula	Dimensions	Name of S.I unit
1	Force	Mass × acceleration	$[M^1L^1T^{-2}]$	Newton (N)
2	Work	Force × distance	$[M^1L^2 T^{-2}]$	Joule (J)
3	Power	Work / time	$[M^1L^2 T^{-3}]$	Watt (W)
4	Energy (all form)	Stored work	$[M^1L^2T^{-2}]$	Joule (J)
5	Pressure, Stress	Force/area	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
6	Momentum	Mass × velocity	$[M^1L^1T^{-1}]$	$Kgms^{-1}$
7	Moment of force	Force × distance	$[M^1L^2T^{-2}]$	Nm
8	Impulse	Force × time	$[M^1L^1T^{-1}]$	Ns
9	Strain	Change in dimension / Original dimension	$[M^0L^0T^0]$	No unit
10	Modulus of elasticity	Stress / Strain	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
11	Surface energy	Energy / Area	$[M^1L^0T^{-2}]$	$Joule/m^2$
12	Surface Tension	Force / Length	$[M^1L^0T^{-2}]$	N/m
13	Co-efficient of viscosity	Force × Distance/ Area × Velocity	$[M^1L^{-1}T^{-1}]$	N/m^2
14	Moment of inertia	Mass × (radius of gyration)^2	$[M^1L^2T^0]$	$Kg\text{-}m^2$
15	Angular Velocity	Angle / time	$[M^0L^0T^{-1}]$	Rad. per sec
16	Frequency	1/Time period	$[M^0L^0T^{-1}]$	Hertz
17	Area	Length × Breadth	$[M^0L^2T^0]$	$Metre^2$
18	Volume	Length × breadth × height	$[M^0L^3T^0]$	$Metre^3$

Classification of Physical Quantity: Physical quantity has been classified into following four categories on the basis of dimensional analysis.

1. Dimensional Constant: These are the physical quantities which possess dimensions and have constant (fixed) value.

e.g. Planck's constant, gas constant, universal gravitational constant etc.

2. Dimensional Variable: These are the physical quantities which possess dimensions and do not have fixed value. e.g. velocity, acceleration, force etc.

3. Dimensionless Constant: These are the physical quantities which do not possess dimensions but have constant (fixed) value.

e.g. e, π, $numbers\ like$ 1,2,3,4,5 etc.

4. **Dimensionless Variable**: These are the physical quantities which do not possess dimensions and have variable value.

e.g. angle, strain, specific gravity etc.

Example.1 Derive the dimensional formula of following Quantity & write down their dimensions.

(i) Density (ii) Power (iii) Co-efficient of viscosity (iv) Angle

Sol.

(i) **Density** = mass/volume

$=[M]/[L^3] = [M^1L^{-3}T^0]$

(ii) **Power** = Work/Time

=Force x Distance/Time

$= [M^1L^1T^{-2}] \times [L^1]/[T^1]$

$= [M^1L^2T^{-3}]$

(iii) **Co-efficient of viscosity** = [Force x Distance] / [Area x Velocity]

= [Mass x Acceleration x Distance x time] / [length x length x Displacement]

$=[M] \times [LT^{-2}] \times [L]\ [T]/[L^2] \times [L]$

$=[M^1L^{-1}T^{-1}]$

(iv) **Angle** = arc (length)/radius (length)

$= [L]/[L]$

$=[M^0L^0T^0]$ = no dimension

Example.2 Explain which of the following pair of physical quantities have the same dimension: (i) Work &Power (ii) Stress & Pressure (iii) Momentum &Impulse

Sol.

(i) Dimension of work = force x distance = $[M^1L^2T^{-2}]$

Dimension of power = work / time = $[M^1L^2T^{-3}]$

Work and Power have not the same dimensions.

(ii) Dimension of stress = force / area = $[M^1L^1T^{-2}]/[L^2] = [M^1L^{-1}T^{-2}]$

Dimension of pressure = force / area = $[M^1L^1T^{-2}]/[L^2] = [M^1L^{-1}T^{-2}]$

Stress and pressure have the same dimension.

(iii) Dimension of momentum = mass x velocity= $[M^1L^1T^{-1}]$

Dimension of impulse = force x time = $[M^1L^1T^{-1}]$

Momentum and impulse have the same dimension.

1.6 PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

It states that **the dimensions of all the terms on both sides of an equation must be the same**. According to the principle of homogeneity, the comparison, addition & subtraction of all physical quantities is possible only if they are of the same nature i.e., they have the same dimensions.

If the power of M, L and T on two sides of the given equation are same, then the physical equation is correct otherwise not. Therefore, this principle is very helpful to check the correctness of a physical equation.

Example: A physical relation must be dimensionally homogeneous, i.e., all the terms on both sides of the equation must have the same dimensions.

In the equation, $S = ut + \frac{1}{2} at^2$

The length (S) has been equated to velocity (u) & time (t), which at first seems to be meaningless, but if this equation is dimensionally homogeneous, i.e., the dimensions of all the terms on both sides are the same, then it has physical meaning.

Now, dimensions of various quantities in the equation are:

Distance, $S = [L^1]$

Velocity, $u = [L^1T^{-1}]$

Time, $t = [T^1]$

Acceleration, $a = [L^1T^{-2}]$

½ is a constant and has no dimensions.

Thus, the dimensions of the term on L.H.S. is $S=[L^1]$ and

Dimensions of terms on R.H.S.

$ut + \frac{1}{2} at^2 = [L^1T^{-1}]\,[T^1] + [L^1T^{-2}]\,[T^2] = [L^1] + [L^1]$

Here, the dimensions of all the terms on both sides of the equation are the same. Therefore, the equation is dimensionally homogeneous.

1.7 DIMENSIONAL EQUATIONS, APPLICATIONS OF DIMENSIONAL EQUATIONS;

Dimensional Analysis: A careful examination of the dimensions of various quantities involved in a physical relation is called dimensional analysis. The analysis of the dimensions of a physical quantity is of great help to us in a number of ways as discussed under the uses of dimensional equations.

Uses of dimensional equation: The principle of homogeneity & dimensional analysis has put to the following uses:

(i) Checking the correctness of physical equation.

(ii) To convert a physical quantity from one system of units into another.

(iii) To derive relation among various physical quantities.

1. **To check the correctness of Physical relations**: According to principle of Homogeneity of dimensions a physical relation or equation is correct, if the dimensions of all the terms on both sides of the equation are the same. If the dimensions of even one term differs from those of others, the equation is not correct.

Example 3. Check the correctness of the following formulae by dimensional analysis.

(i) $F=mv^2/r$ (ii) $t=2\pi\sqrt{l/g}$

Where all the letters have their usual meanings.

Sol. $\boldsymbol{F=mv^2/r}$

Dimensions of the term on **L.H.S**

Force, F = $[M^1L^1T^{-2}]$

Dimensions of the term on **R.H.S**

$\boldsymbol{mv2/r}$ = $[M^1]\,[L^1T^{-1}]^2/[L]$

= $[M^1L^2T^{-2}]/[L]$

= $[M^1L^1T^{-2}]$

The dimensions of the term on the **L.H.S** are equal to the dimensions of the term on **R.H.S**. Therefore, the relation is **correct**.

(ii) $\boldsymbol{t=2\pi\sqrt{l/g}}$

Here, Dimensions of L.H.S, t = $[T^1] = [M^0L^0T^1]$

Dimensions of the terms on R.H.S

Dimensions of (length) = $[L^1]$

Dimensions of g (acc due to gravity) = $[L^1T^{-2}]$

2π being constant have no dimensions.

Hence, the dimensions of terms $2\pi\sqrt{l/g}$ on R.H.S

= $(L^1/ L^1T^{-2}])$ ½= $[T^1]$ = $[M^0L^0T^1]$

Thus, the dimensions of the terms on both sides of the relation are the same i.e., $[M^0L^0T^1]$. Therefore, the relation is correct.

Example 4. Check the correctness of the following equation on the basis of dimensional analysis, $V=\sqrt{Ed}$. Here V is the velocity of sound, *E* is the elasticity and *d* is the density of the medium.

Sol. Here, Dimensions of the term on L.H.S

V = $[M^0L^1T^{-1}]$

Dimensions of elasticity, E = $[M^1L^{-1}T^{-2}]$

& Dimensions of density, *d* = $[M^1L^{-3}T^0]$

Therefore, Dimensions of the terms on R.H.S

$\sqrt{\boldsymbol{Ed}}$ **=** $[M^1L^{-1}T^{-2}/ M^1L^{-1}T^{-2}]$ ½ = $[M^0L^1T^{-1}]$

Thus, dimensions on both sides are the same, therefore the equation is correct.

Example 5. Using Principle of Homogeneity of dimensions, check the correctness of equation, h = 2Td /rgCos$\boldsymbol{\theta}$.

Sol. The given formula is, h = 2Td /rgCos$\boldsymbol{\theta}$.

Dimensions of term on L.H.S

Height (h) = $[M^0L^1T^0]$

Dimensions of terms on R.H.S

T= surface tension = $[M^1L^0T^{-2}]$

D= density = $[M^1L^{-3}T^0]$

r =radius = $[M^0L^1T^0]$

g=acc. due to gravity = $[M^0L^1T^{-2}]$

Cos$\boldsymbol{\theta}$ = $[M^0L^0T^0]$ = no dimensions

So, Dimensions of 2Td/rgCos$\boldsymbol{\theta}$ = $[M^1L^0T^{-2}]$ x $[M^1L^{-3}T^0]$ / $[M^0L^1T^0]$ x $[M^0L^1T^{-2}]$

= $[M^2L^{-5}T^0]$

Dimensions of terms on L.H.S are not equal to dimensions on R.H.S. Hence, formula is not correct.