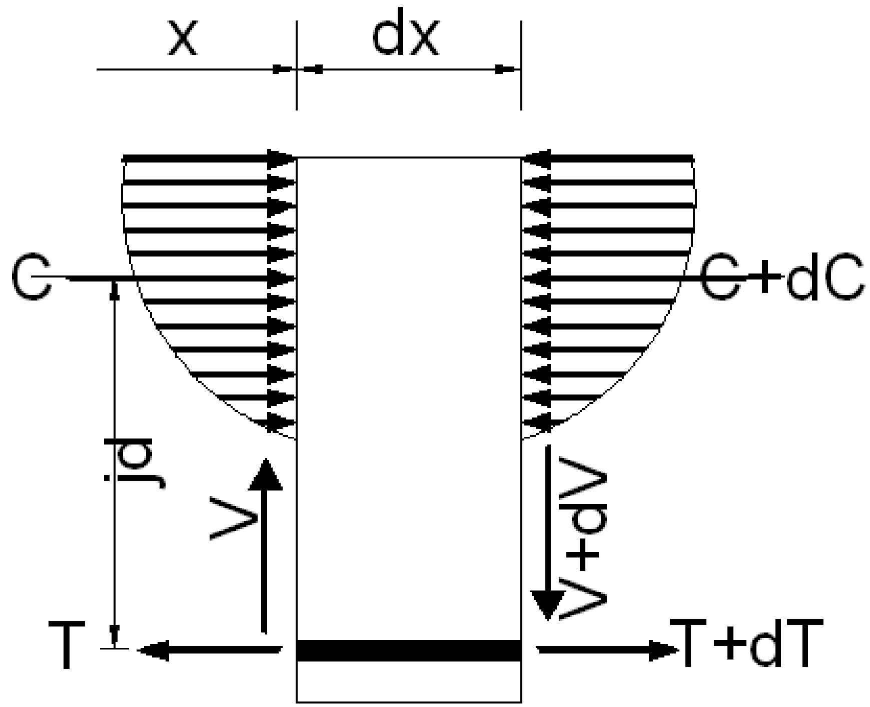
Bond and development length:

Bond:

$$M_{@x} = T * jd$$

$$M_{@x+dx} = M + dM$$

$$= (T + dT) * jd$$

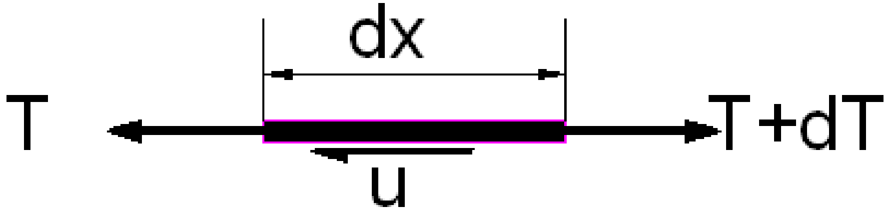


$$dM = dT * jd \rightarrow dT = \frac{dM}{jd} \ldots .1$$

This change in bar force is resisted at the contact surface between steel and concrete by equal and opposite force produced by bond.

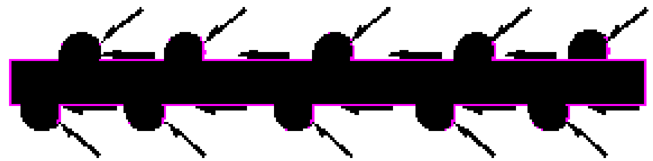
If U=bond force per unit length

of bar.



Bond force consist of :

1. Adhesion
2. Friction
3. Mechanical interlock of the deformations with surrounding concrete



$$\sum fx = 0 \rightarrow T + U\ dX = T + dT \rightarrow U = \frac{dT}{dX} \ldots\ldots\ldots\ldots..2$$

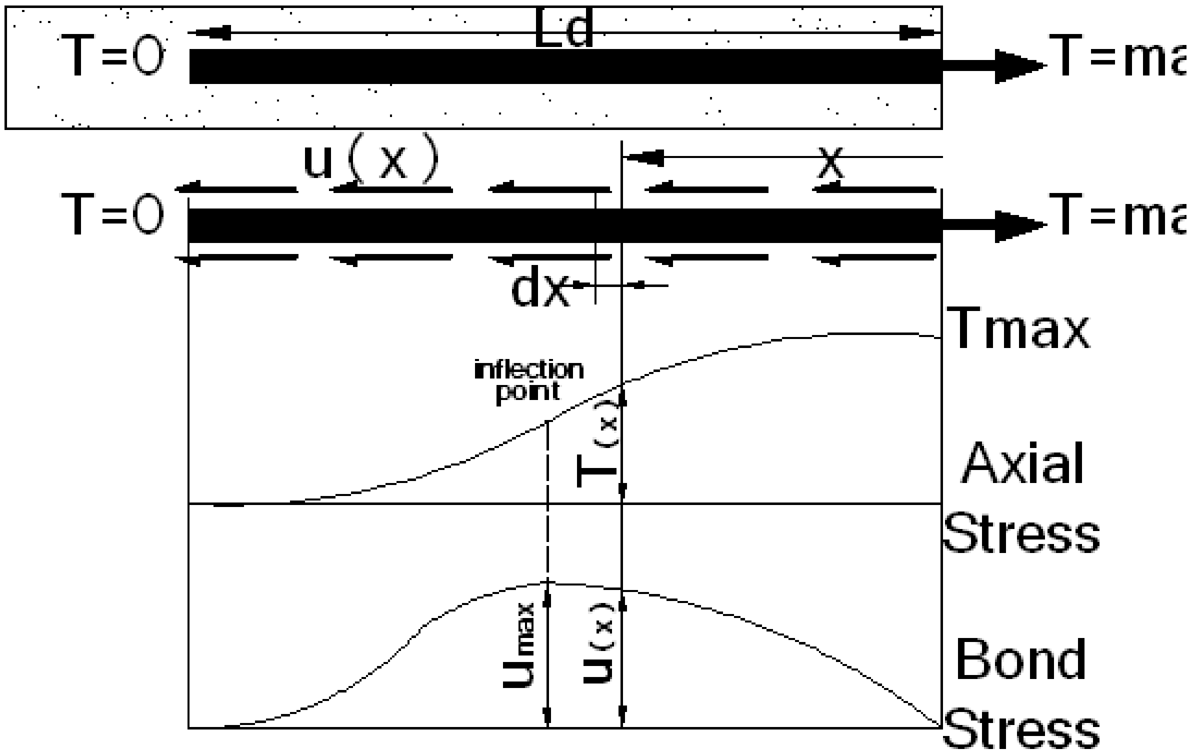
Substitute eq1 into eq2

$$U = \frac{1}{jd}\frac{dM}{dX} = \frac{V}{jd} \quad apply\ to\ tension\ bars\ only,$$

$$V: shear\ force$$

Development length: that length of embedment necessary to develop the full tensile strength of the bar.

Factors influencing development length:



1. Concrete tensile strength, fc_t
2. Cover distance
3. Spacing of reinforcing bars
4. Transverse steel bars

ACI Code equations for development of *tension* bars:

- $\sqrt{fc'} \le \frac{25}{3}\ MPa$
- Ld≥300mm

a. Basic equation:

$$Ld = \left(\frac{3}{40} \frac{fy}{\sqrt{fc'}} \frac{\alpha\beta\gamma\lambda}{\frac{C+K_{tr}}{db}} \right) db$$, ACI eq.12-1,sec12.2.3,empirical eq.

$$\frac{C + K_{tr}}{db} \le 2.5$$

C:smaller of min. cover(measuring from center of the bar to the nearest concrete surface) or $\frac{1}{2}$ the center to center spacing of the bars.

K_{tr}:transverse reinforcement index=$\frac{A_{tr} f_{yt}}{10sn}$ it shall be permitted to use $K_{tr} = 0$ as a design simplification even transverse reinforcement(stirrups) is present.

A_{tr} : total cross sectional area of all transverse reinforcement that is within the spacing (S), mm^2.

S: max. center to center spacing of transverse reinforcement with Ld, mm.

n: number of bars being spliced or developed.

α : reinforcement location factor:

- Horizontal reinforcement so placed that more than 300mm of fresh concrete is cast in the member below the development length or splice…..1.3
- Other reinforcement …….1.0

β: coating factor:

- Epoxy coated bars with cover less than 3db or clear spacing less than 6db…………………………...1.5
- All other epoxy coated bars ……1.2
- Uncoated bars ……………………..1.0

Note: $\alpha*\beta \leq 1.7$

γ: reinforcement size factor:

- Bar diameter ≤ 19mm ……………0.8
- Bar diameter ≥22mm ……………1.0

λ: Light weight aggregate concrete factor:

- When Light weight aggregate concrete is used ….1.3

  However, when fc_t is specified, λ permitted to be taken as :

  $\frac{\sqrt{fc'}}{1.8\, fc_t}$ but not less than 1.0

- When normal weight concrete is used, …..1.0

b. Simplified equations:

Condition	Bar diameter ≤ 19mm	Bar diameter ≥22mm
• Clear spacing of bars being developed or spliced not less than bar diameter • Clear cover ≥db • Stirrups or ties through Ld≥Av_{min} OR • Clear spacing of bars being developed or spliced≥ 2db • Clear cover ≥db	$\left(\frac{12}{25}\frac{fy\ \alpha\beta\lambda}{\sqrt{fc'}}\right)db$	$\left(\frac{3}{5}\frac{fy\ \alpha\beta\lambda}{\sqrt{fc'}}\right)db$
Other cases	$\left(\frac{18}{25}\frac{fy\ \alpha\beta\lambda}{\sqrt{fc'}}\right)db$	$\left(\frac{9}{10}\frac{fy\ \alpha\beta\lambda}{\sqrt{fc'}}\right)db$
*Excess reinforcement→ $Ld * \frac{As_{required}}{As_{r \quad id}}$		

ACI Code equations for development of *compression* bars:ACI 12.3

- $\sqrt{fc'} \leq \frac{25}{3}\ MPa$

- Ld≥200mm

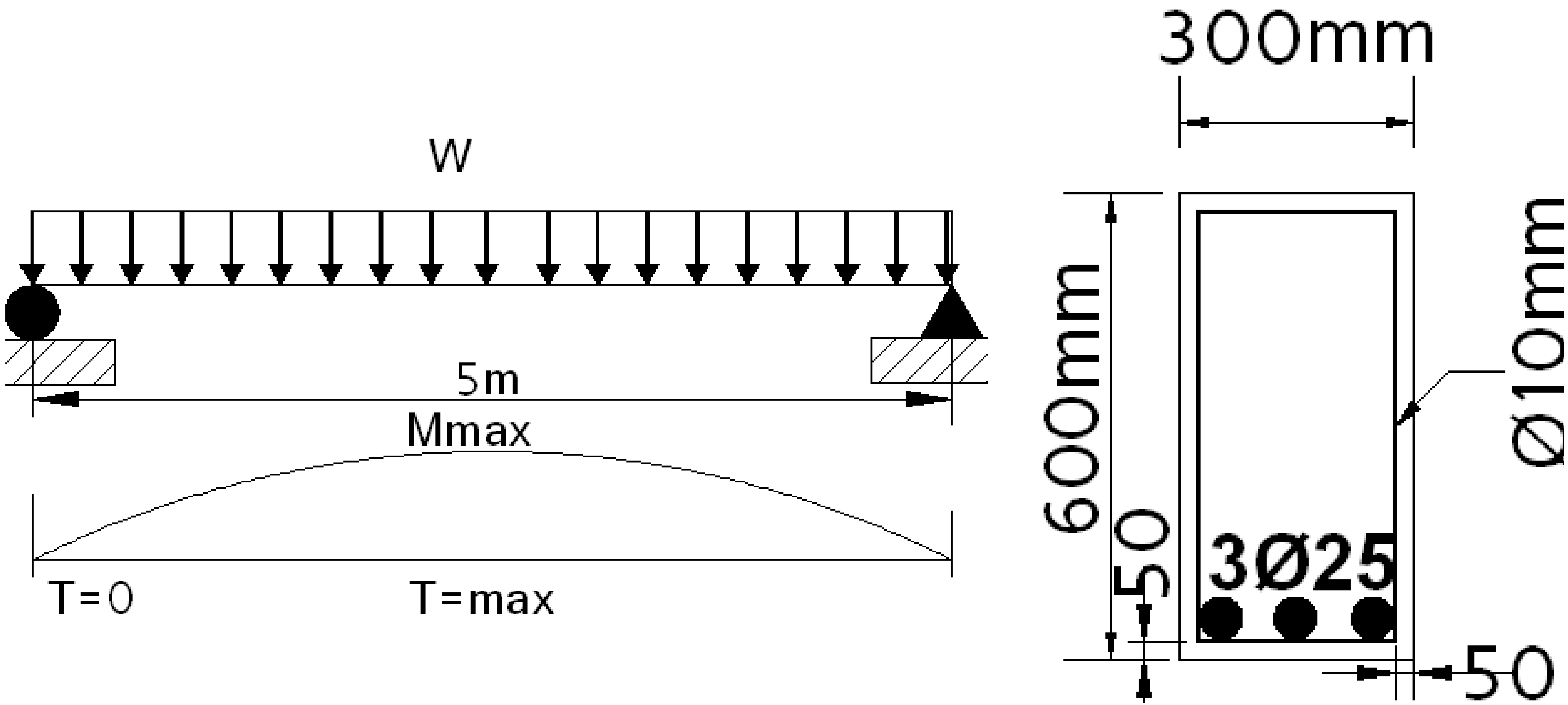
$$Ld_c = \max\left[\left(\frac{0.24fy}{\sqrt{fc'}}\right)db\ \ ,\ \ (0.043fy)db\right],$$

$$wher\ 0.043\ in\ \frac{mm^2}{N}$$

* Ld_c shall be permitted to be multiplied by the applicable factors for:

- Excess reinforcement ………………… $\frac{As_{required}}{As_{provided}}$
- Reinforcement enclosed within spiral reinforcement Ø≥6mm and pitch ≤100mm OR within ties conformance with sec. 7.10.5(column requirements) and spaced at not more than 100mm……………….0.75

EX1: fy= 400MPa, fc'=25MPa, As $_{required}$ =1250mm^2, check development length, Ld.



Solution:

- Clear spacing Sc=[300-2*50-2*10-3*25]/(3-1)=52.5mm > 2db=2*25=50mm
- Clear cover=50+10=60mm>db=25mm

Second conditions are satisfied

γ= 1.0 (distance =50+10=60mm<300mm)

β=1.0 uncoated reinforcement

λ=1.0 normal concrete

$$\emptyset = 25mm > 22mm \rightarrow Ld = \left(\frac{3}{5}\frac{fy\ \alpha\beta\lambda}{\sqrt{fc'}}\right)db$$

$$Ld = \left(\frac{3}{5} * \frac{400 * \ 1 * 1 * 1}{\sqrt{25}}\right) * 25 = 1200mm$$

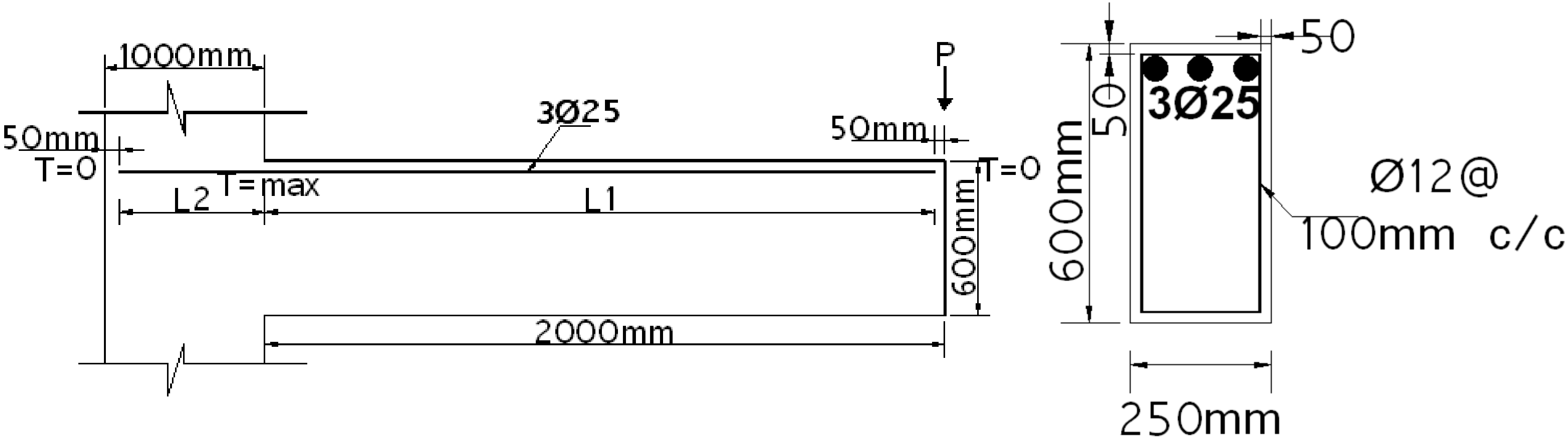
$$Ld = 1200 * \frac{As_{required}}{As_{provided}} = 1200 * \frac{1250}{3 * 25^2 * \pi/4} = 1018mm$$

$$> 300mm\ O.K$$

Available distance=(5000/2)-(50/1000)=2450mm>Ld=1018mm

O.K

EX2: fy= 400MPa, fc'=25MPa, check development length, Ld.



Solution:

- Clear spacing Sc=[250-2*50-2*12-3*25]/(3-1)=25.5mm > db=25mm
- Clear cover=50+12=62mm>db=25mm
- $Av_{min} = \frac{bw\ S}{3fy} = \frac{250*100}{3*400} = 21mm^2$

OR $Av_{min} = \frac{\sqrt{fc'}bw\ S}{16fy} = \frac{\sqrt{25}*250*100}{16*400} = 20mm^2$

$Av_{provided} = 2*113 = 226mm^2 \ > Av_{min}$

First conditions are satisfied

α=1.3 since (distance=600-50-12-25=513mm >300mm)

β=1.0 uncoated reinforcement

λ=1.0 normal concrete

$$\emptyset = 25mm > 22mm \rightarrow Ld = \left(\frac{3}{5}\frac{fy\ \alpha\beta\lambda}{\sqrt{fc'}}\right)db$$

$$Ld = \left(\frac{3}{5} * \frac{400 * \ 1.3 * 1 * 1}{\sqrt{25}}\right) * 25 = 1560mm > 300mm\ O.K$$

Available distance:

L1 provided=2000-50=1950mm>Ld=1560mm O.K

L2 provided=1000-50=950mm<Ld=1560mm N.G

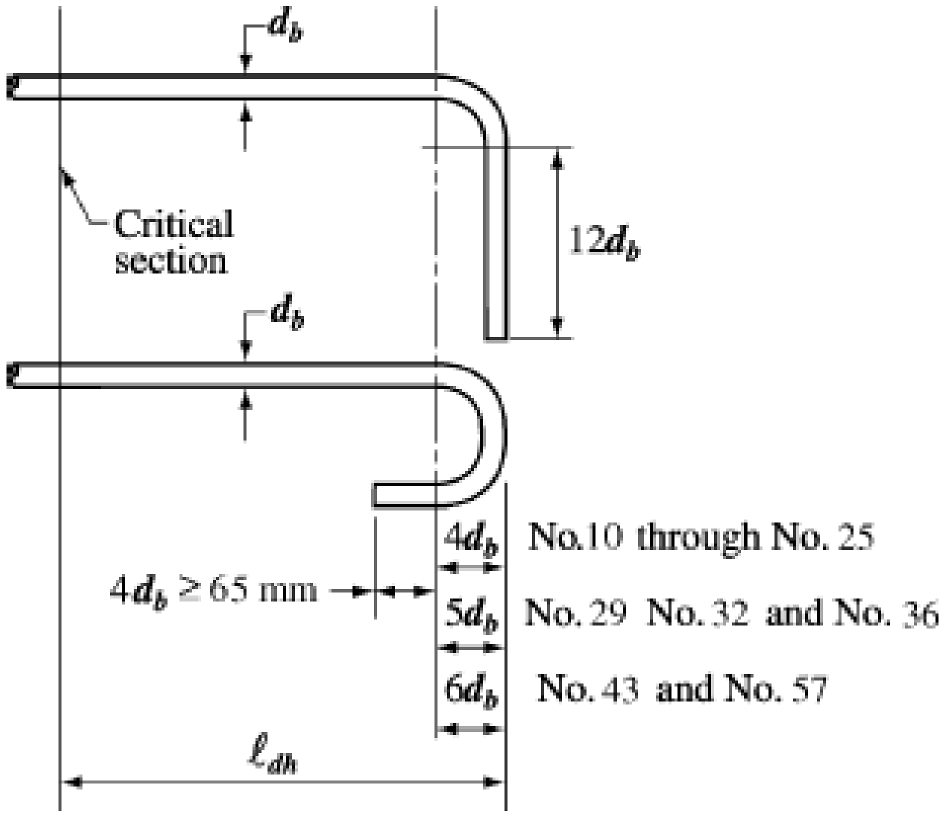
Alternate solutions:

1. Increase numbers of bars with smaller diameter
2. Use mechanical anchorage.

Mechanical anchorage:

It is effective only for bars in tension.

a. Standard hook of main reinforcement(flexural reinforcement) ACI 7.1,7.2:



Development of standard hook of main reinforcement(flexural reinforcement) ACI 12.5:

$$ld_h = \left(\frac{0.24 fy\, \beta\lambda}{\sqrt{fc'}}\right) db * factor \geq \max\,(8db, 150mm)$$

ACI 12.5.2

β=1.2 for epoxy coated reinforcement

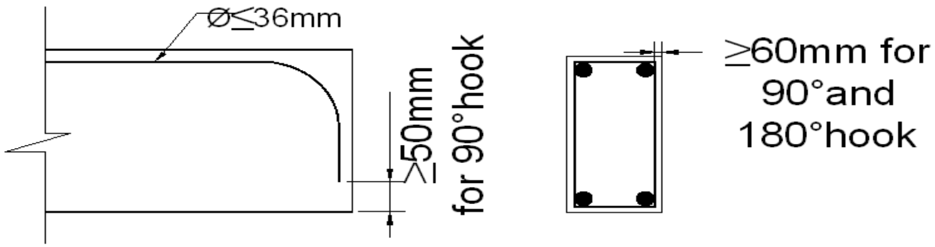
λ=1.3 for light weight concrete

for other cases β=λ=1.0

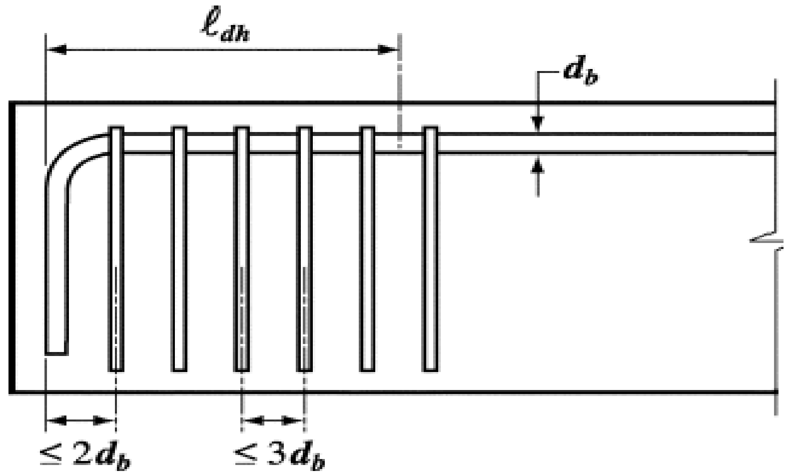
Factors :ACI 12.5.3

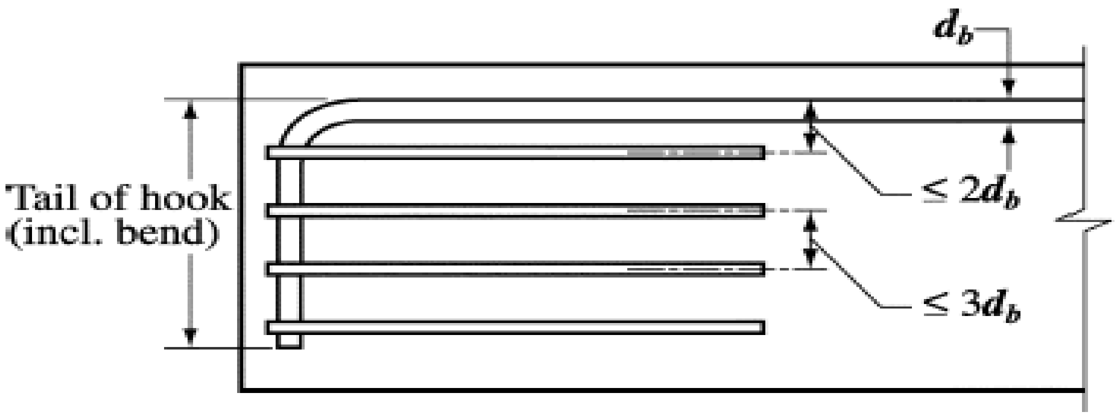
(a) For No. 36 bar and smaller hooks with side cover (normal to plane of hook) not less than 65 mm, and for 90 degree hook with cover on bar extension beyond hook not less than 50 mm 0.7

(b) For 90 degree hooks of No. 36 and smaller bars that are either enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along ℓ_{dh}; or enclosed within ties or stirrups parallel to the bar being developed, spaced not greater than $3d_b$ along the length of the tail extension of the hook plus bend 0.8



ACI 12.5.3a

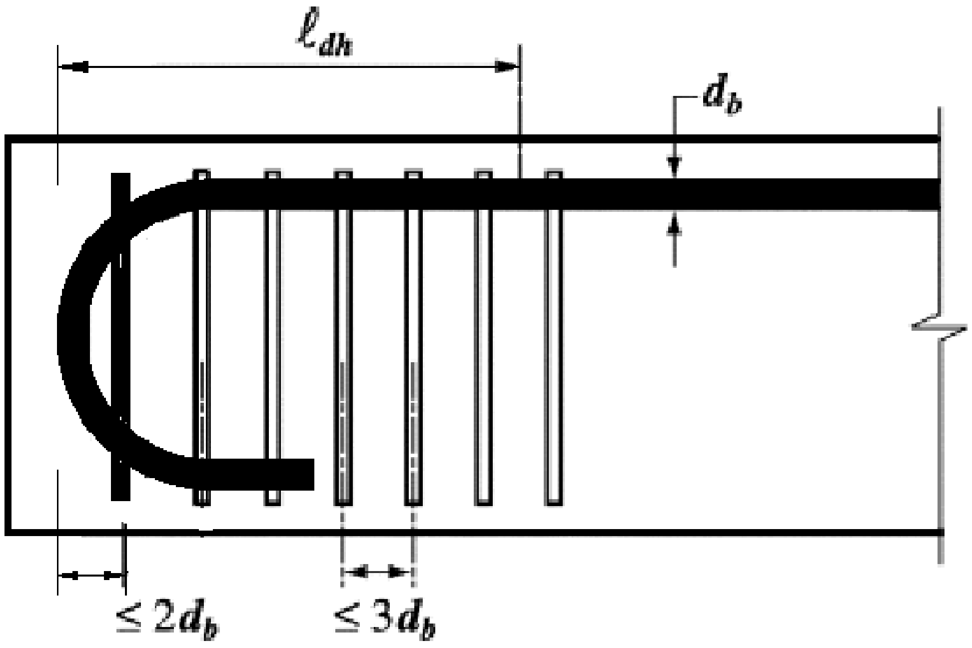




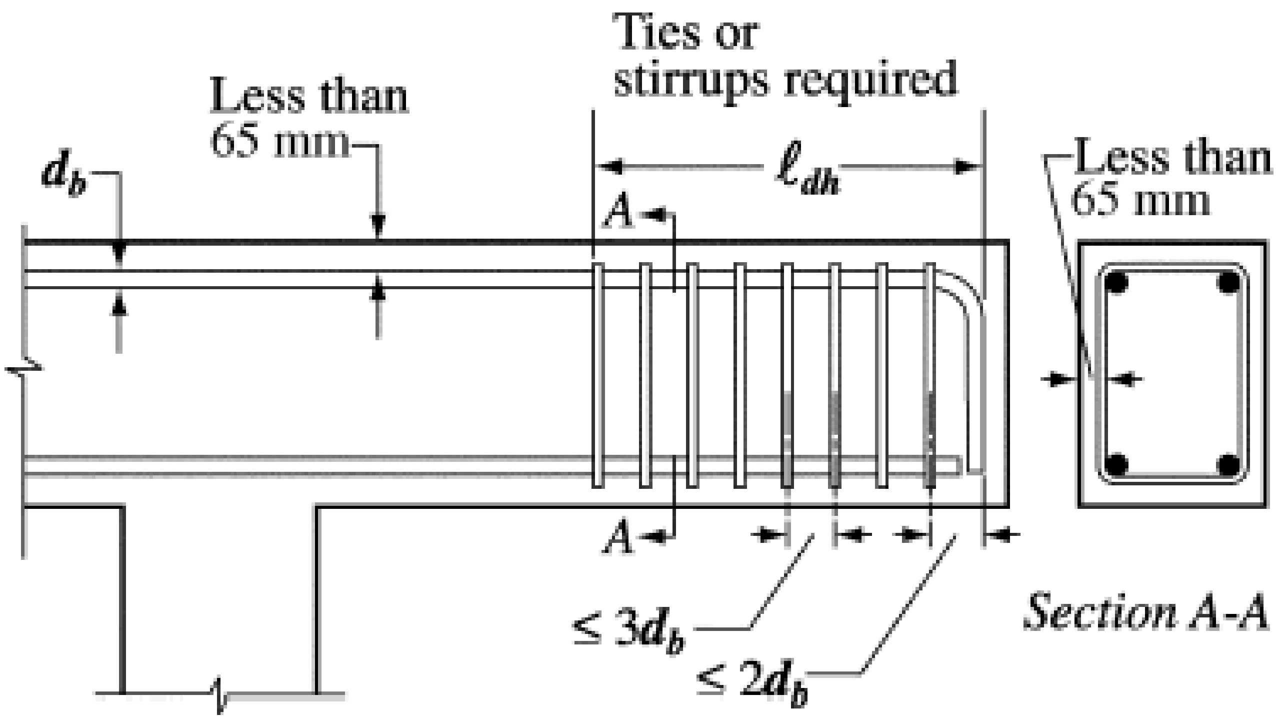
(c) For 180 degree hooks of No. 36 and smaller bars that are enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $\mathbf{3}d_b$ along ℓ_{dh} 0.8

(d) Where anchorage or development for f_y is not specifically required, reinforcement in excess of that required by analysis .... (A_s required)/(A_s provided)

In 12.5.3(b) and 12.5.3(c), d_b is the diameter of the hooked bar, and the first tie or stirrup shall enclose the bent portion of the hook, within $\mathbf{2}d_b$ of the outside of the bend.



12.5.4 — For bars being developed by a standard hook at discontinuous ends of members with both side cover and top (or bottom) cover over hook less than 65 mm, the hooked bar shall be enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along ℓ_{dh}. The first tie or stirrup shall enclose the bent portion of the hook, within $2d_b$ of the outside of the bend, where d_b is the diameter of the hooked bar. For this case, the factors of 12.5.3(b) and (c) shall not apply.



For previous example if use 90° hook:

β=1.0 uncoated reinforcement

λ=1.0 normal concrete

factor=1.0

$$ld_h = \left(\frac{0.24 fy\, \beta\lambda}{\sqrt{fc'}}\right) db * factor \geq \max\,(8db, 150mm)$$

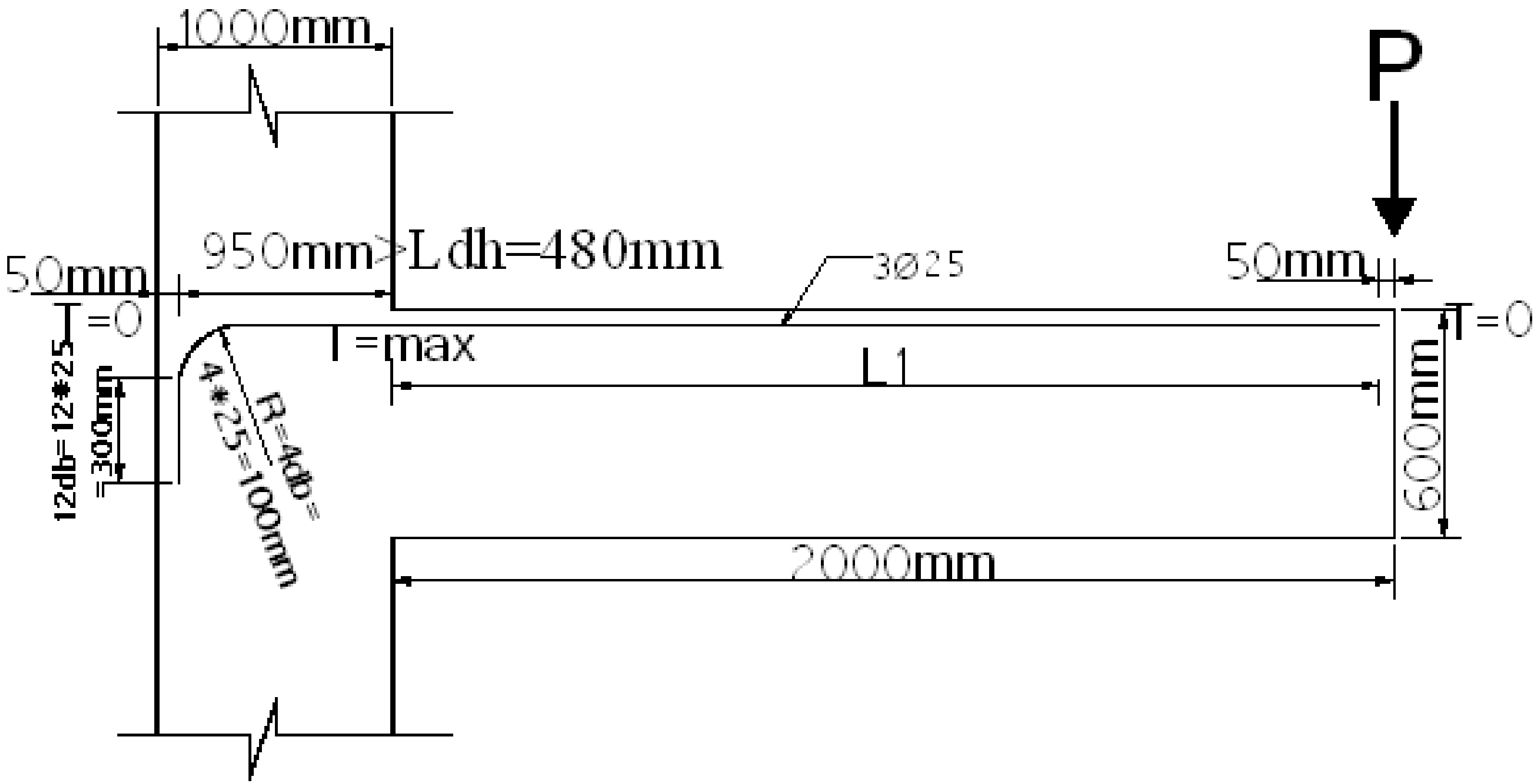
$$ld_h = \left(\frac{0.24 * 400 * 1 * 1}{\sqrt{25}}\right) 25 * 1 * 1 = 480mm$$

$$\geq \max(8db = 8 * 25 = 200{,}150mm) \rightarrow$$

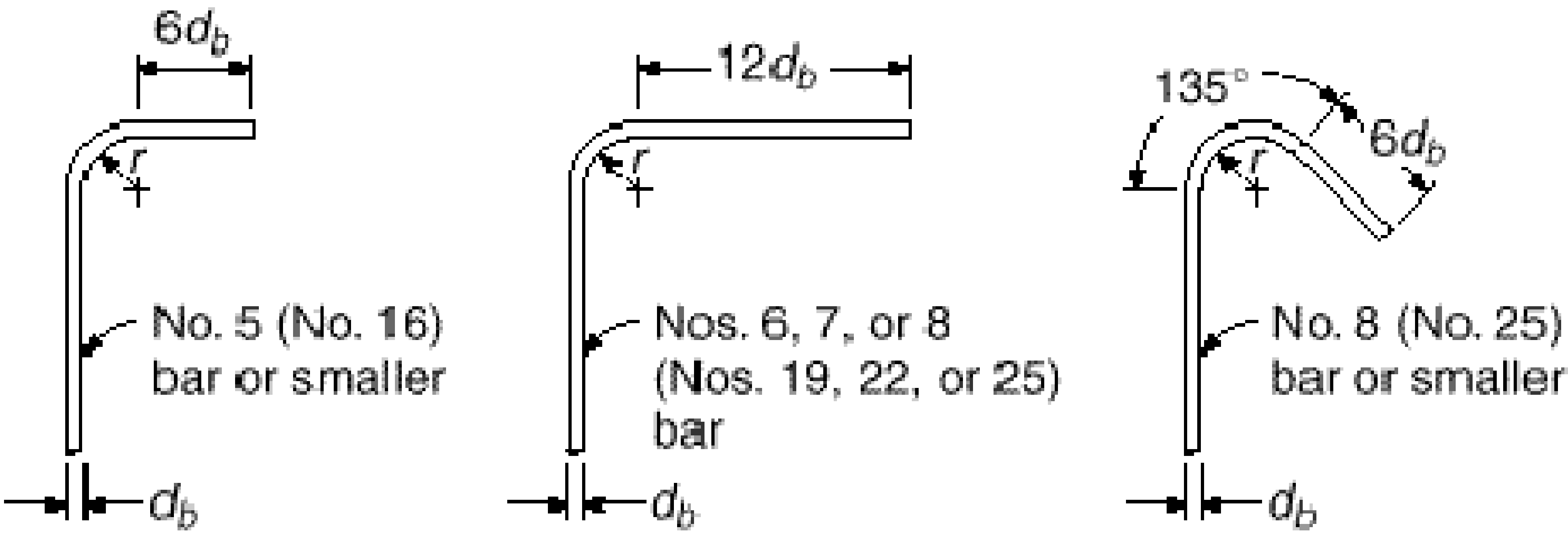
$$\therefore\ ld_h = 480mm$$

L2 provided=950mm>Ld_h=480mm O.K

r=4db=4*25=100mm



b.Standard hook of web reinforcement(Stirrups):ACI 7.1.3



r=2db

r=3db

$r=2d_b$ forØ≤16

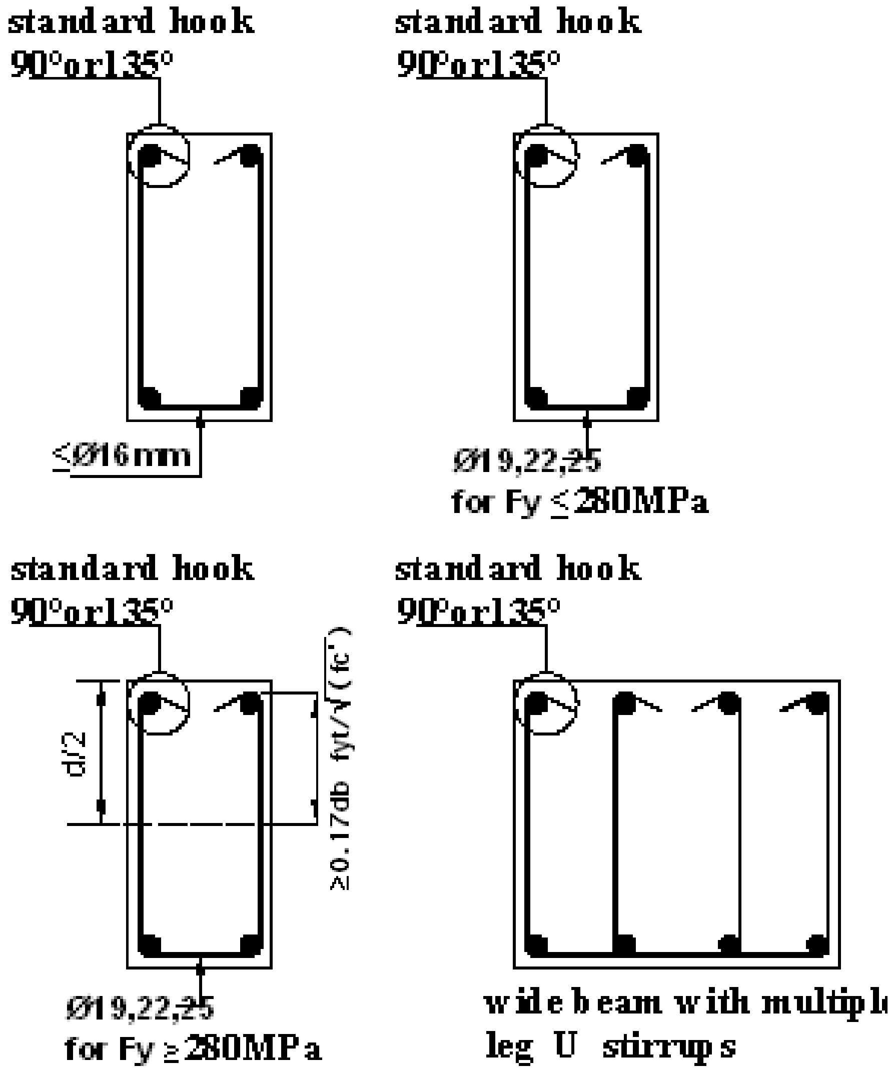
$r=3d_b$ for 16<Ø≤25

Development of web reinforcement(Stirrups):ACI 12.13

12.13.2.1 — For No. 16 bar and MD 200 wire, and smaller, and for No. 19, No. 22, and No. 25 bars with f_{yt} of 280 MPa or less, a standard hook around longitudinal reinforcement.

12.13.2.2 — For No. 19, No. 22, and No. 25 stirrups with f_{yt} greater than 280 MPa, a standard stirrup hook around a longitudinal bar plus an embedment between midheight of the member and the outside end of the hook equal to or greater than $0.17 d_b f_{yt} / \sqrt{f_c'}$.

12.13.3 — Between anchored ends, each bend in the continuous portion of a simple U-stirrup or multiple U-stirrup shall enclose a longitudinal bar.



Bar Splices:

a. Splice of deformed bars in tension ACI 12.15.

*min. lap length =300mm

*Ld, calculated as before, without Excess reinforcement factor $\left(\frac{As_{required}}{As_{provided}}\right)$

$\frac{As_{provided}}{As_{required}}$	Max. percent of As spliced within required lap length	
	50%	100%
≥ 2.0	Class A (1.0*Ld)	Class B (1.3*Ld)
< 2.0	Class B (1.3*Ld)	Class B (1.3*Ld)

b.Splice of deformed bars in compression ACI12.16.

fy,(MPa)	fc',(MPa)	Lap length	
≤420	≥21	$0.071\ fy\ d_b$	≥300mm
	<21	$1.33fy\ d_b$	
>420	≥21	$(0.13\ fy-24)d_b$	
	<21	$1.33(0.13\ fy-24)\ d_b$	

Bar cutoff and bend points in beams:

The tensile force to be resist by the reinforcement at any cross section is:

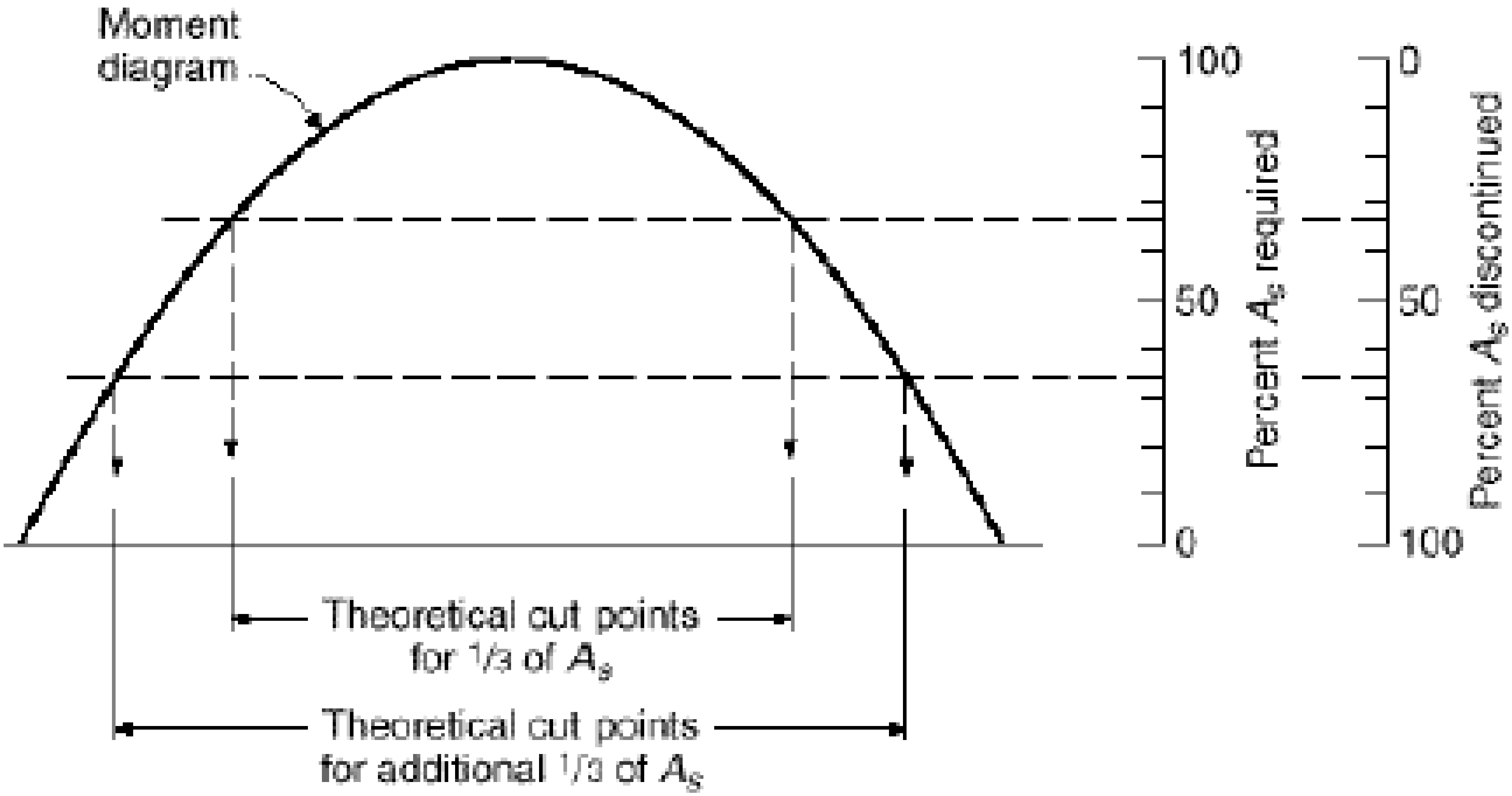
$$T = As\ Fs = \frac{M}{z}$$

M:the value of bending moment at that section

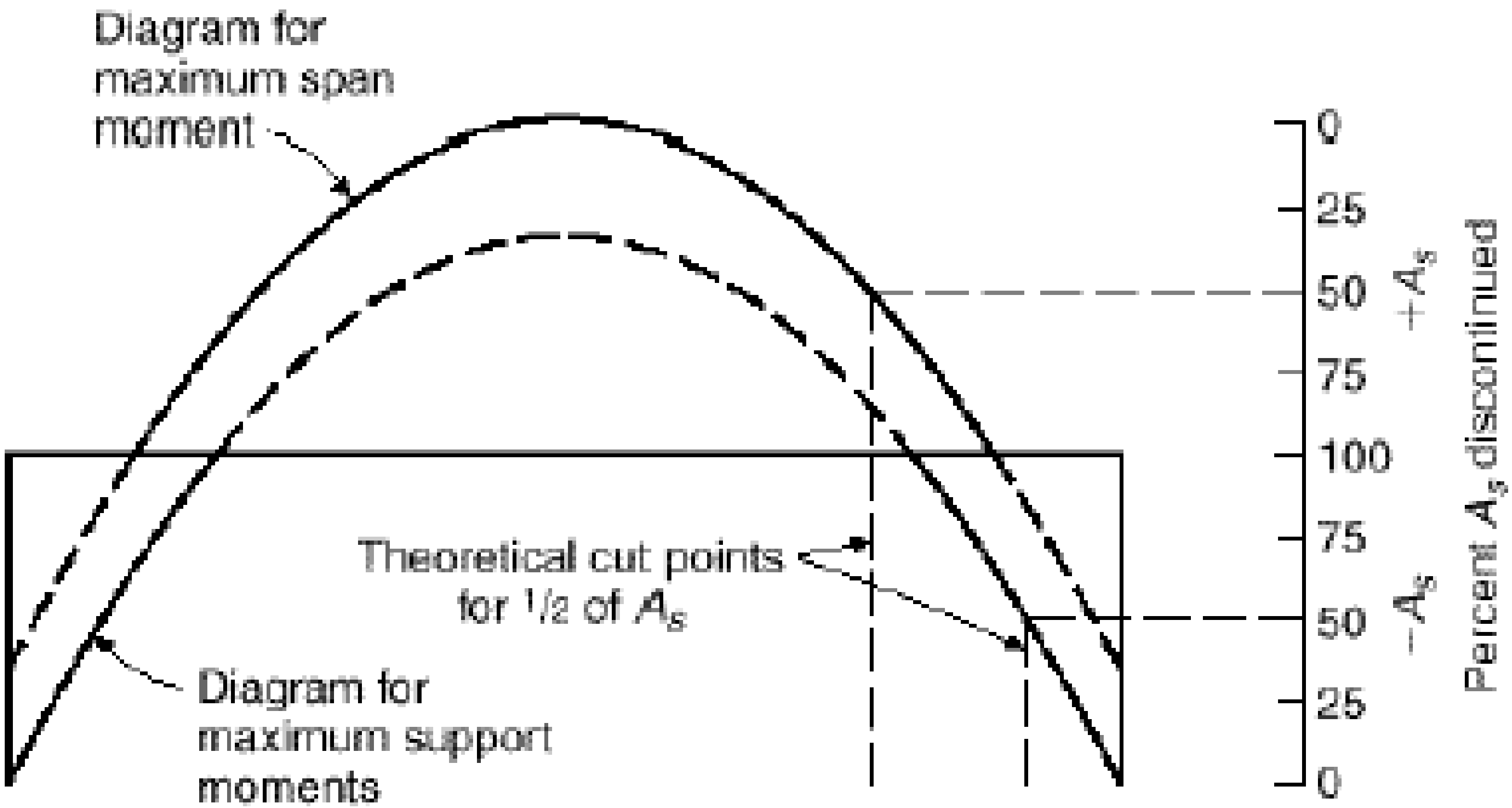
z: the internal lever arm of resisting moment.

The tensile force can be taken with good accuracy directly proportional to the bending moment. Since it is desirable to design that the steel everywhere in the beam is as nearly fully stressed as possible, it follows that the required steel area is very nearly proportional to the bending moment.

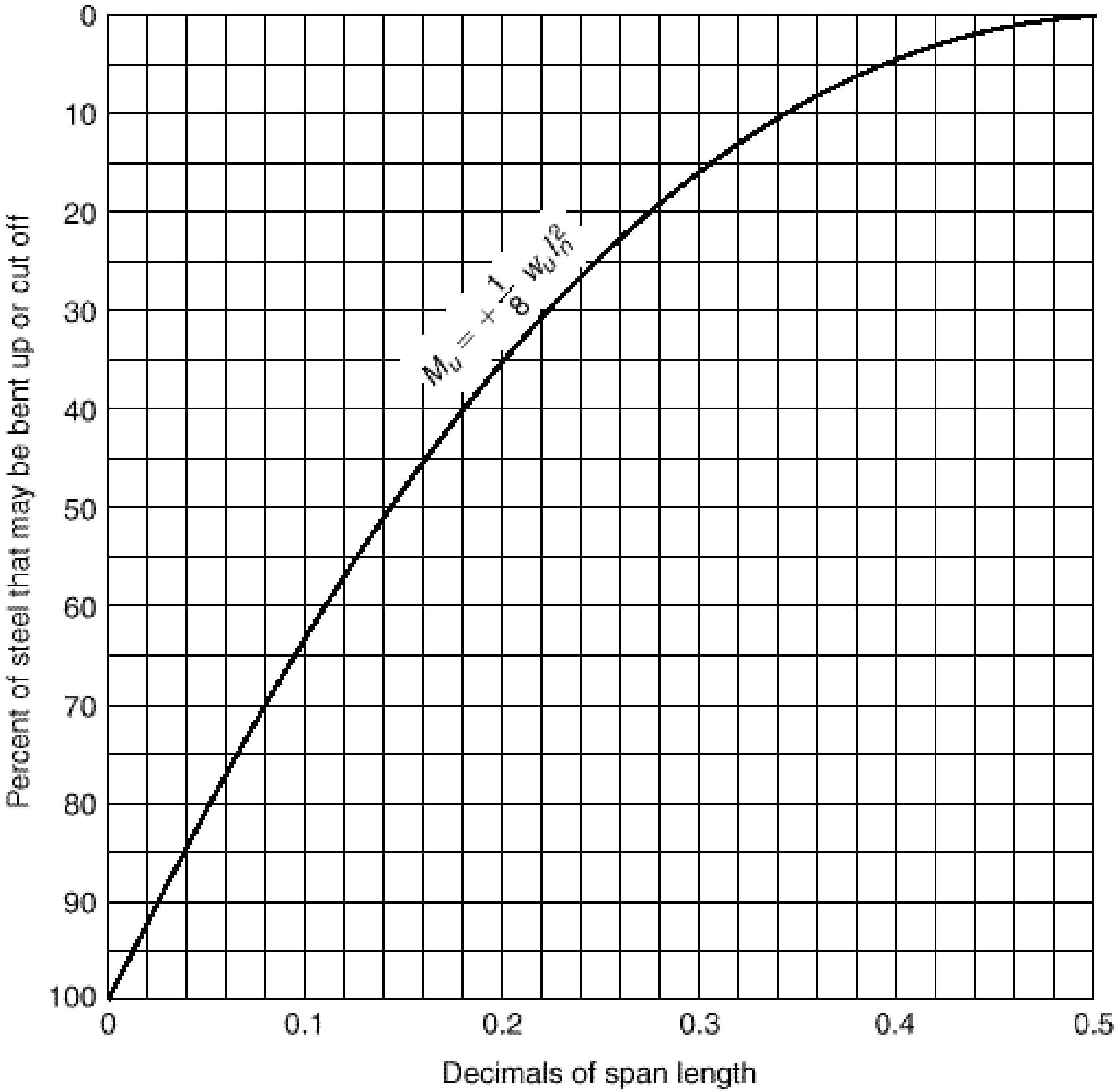
a. Theoretical points of cut off or bend



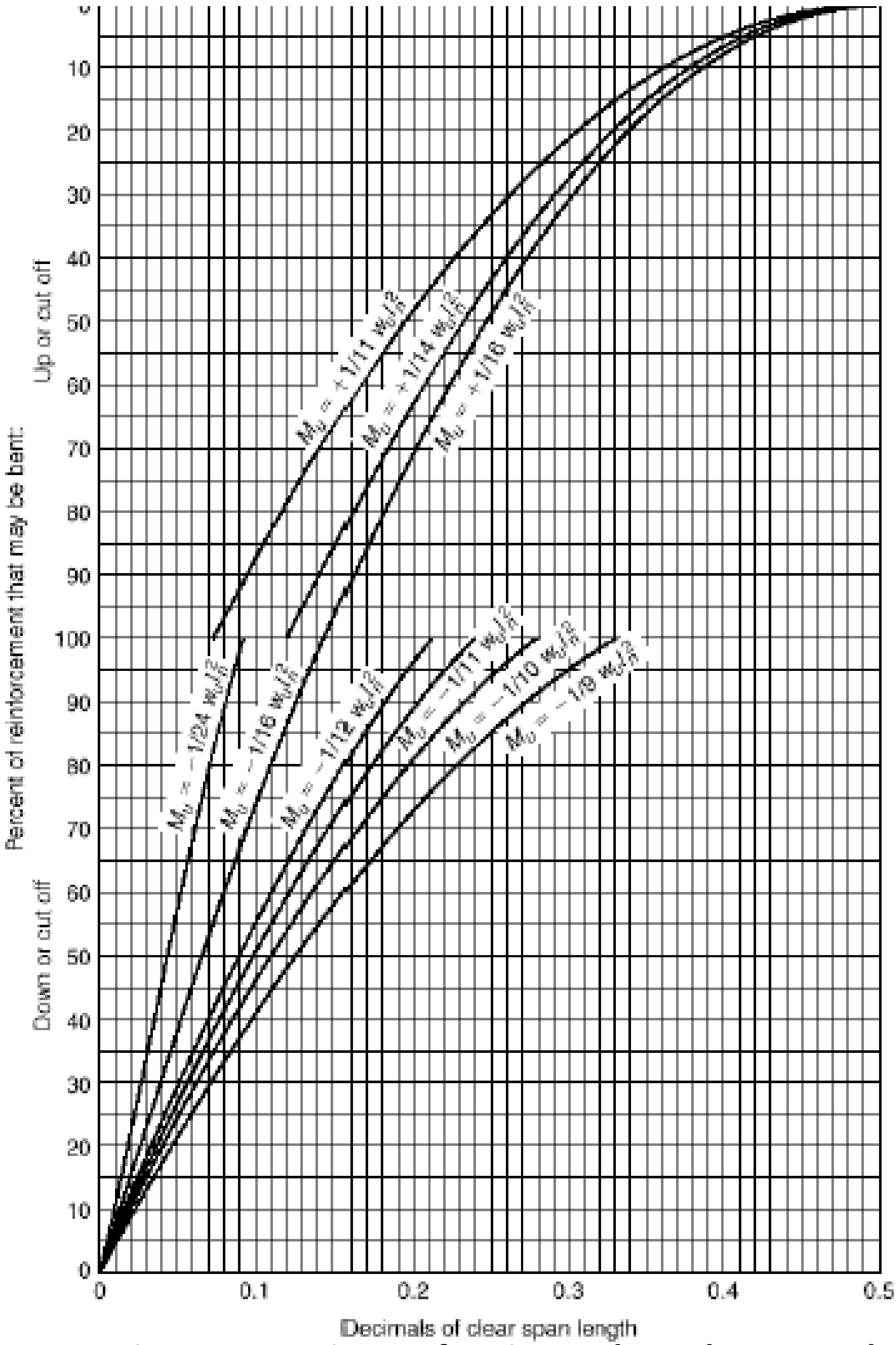
(a):Moment diagram for a uniformly loaded simply supported beam



(b):Moment diagram for a uniformly loaded continuous beam



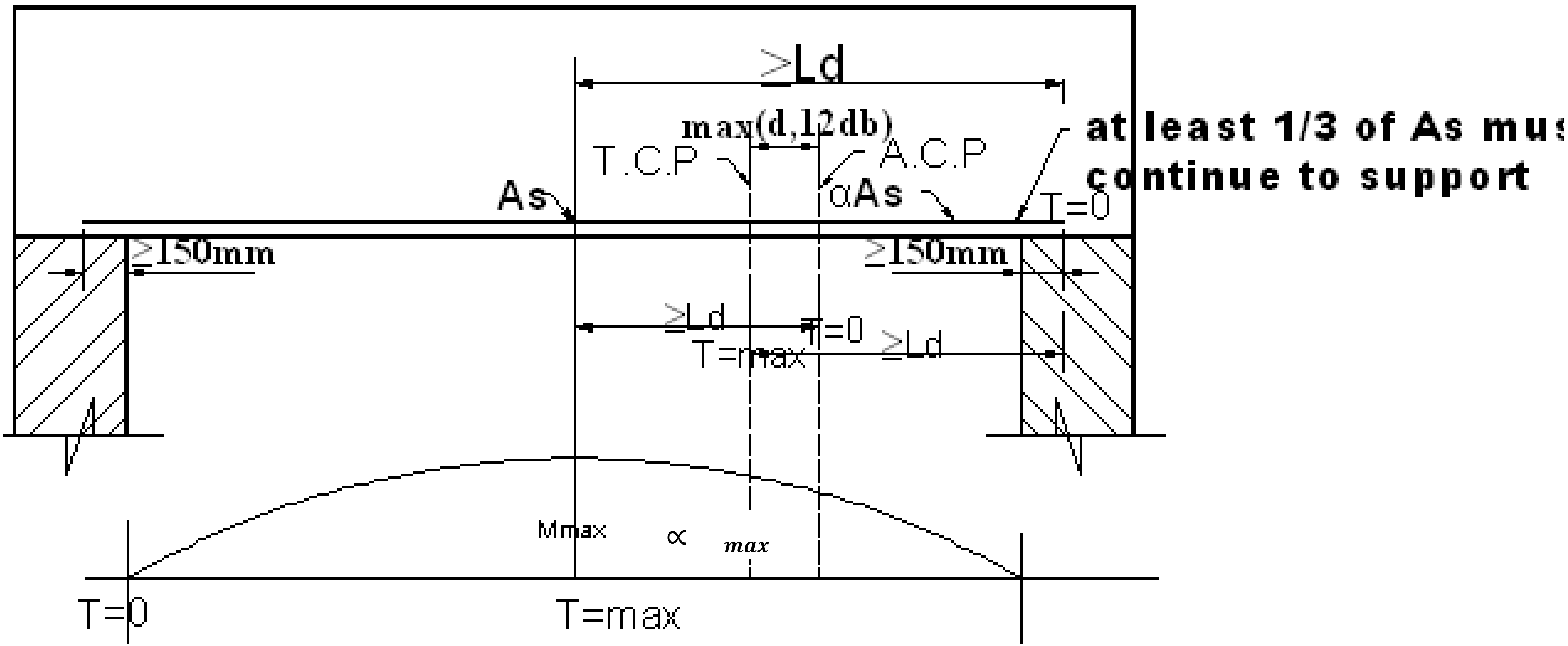
Location of points where bars can be bent up or cut off for simply supported beam uniformly loaded



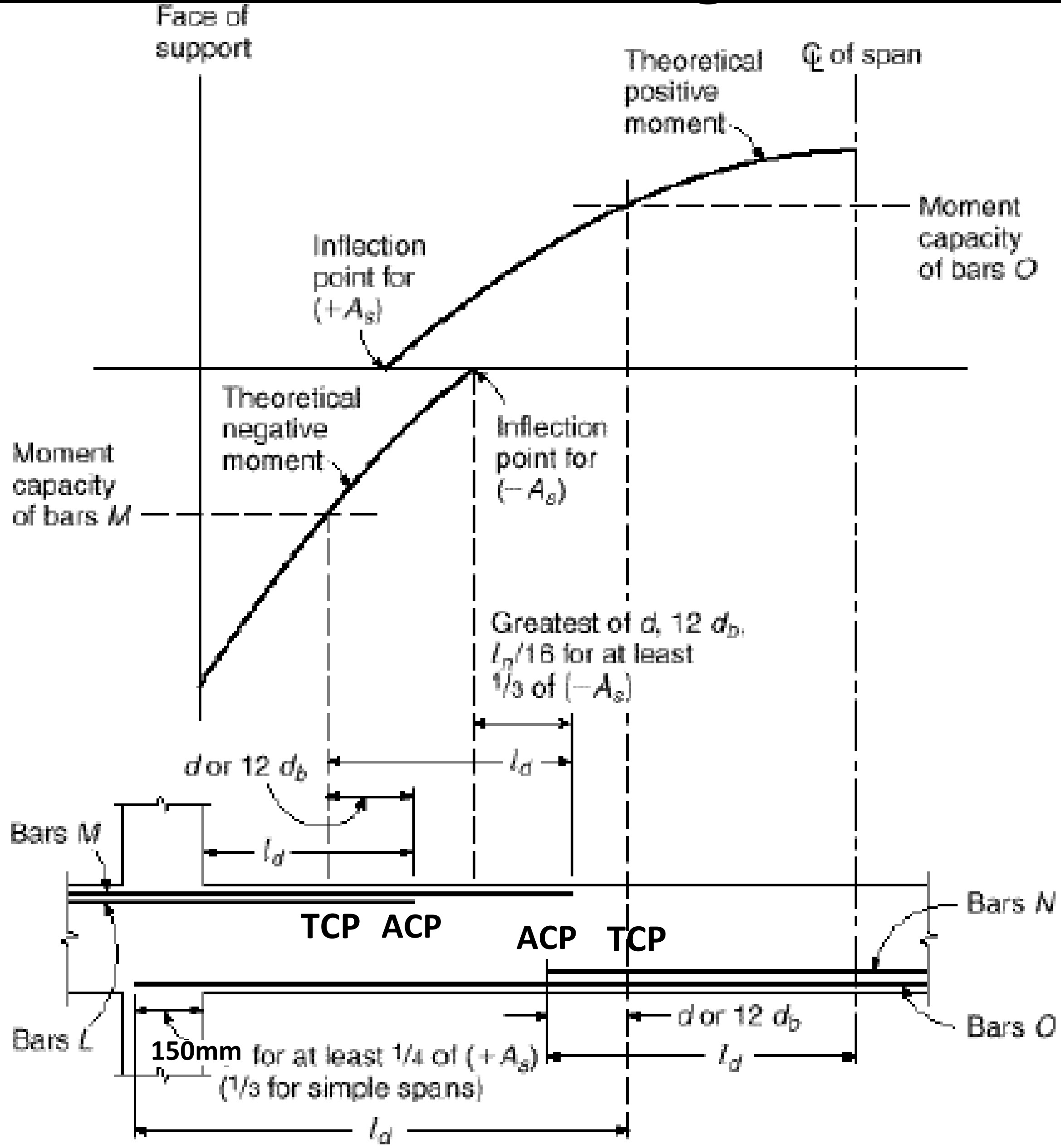
Approximate Locations of points where bars can be bent up or down or cut off for continuous beams uniformly loaded and built integrally with their supports according to the coefficients in the ACI Code.

b. Practical considerations and ACI code requirements: ACI 12.10, 12.11, 12.12

For simply supported span



For continuous beams : using ACI code moment coefficients



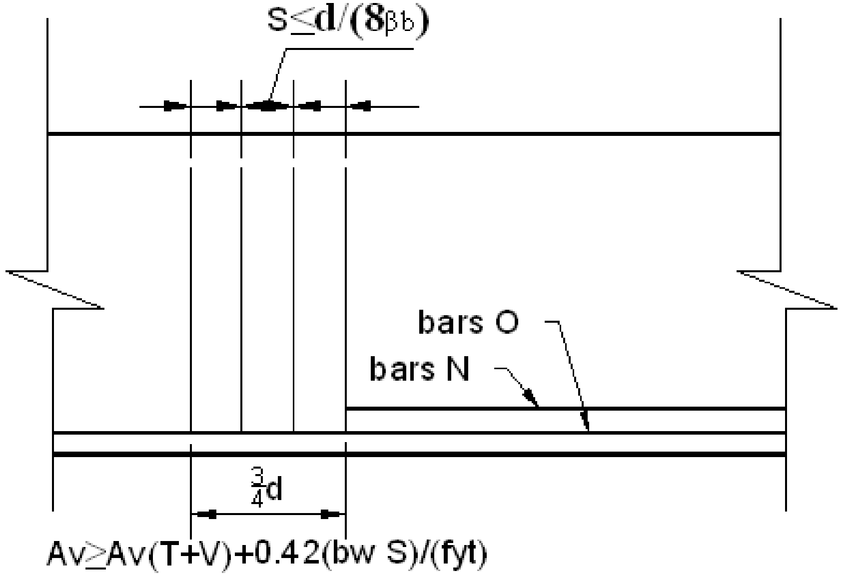
ACI code 12.10 requires special precautions

Flexural reinforcements shall not be terminated in a tension zone unless one of the following is satisfied:

12.10.5.1 — V_u at the cutoff point does not exceed $(2/3)\phi V_n$.

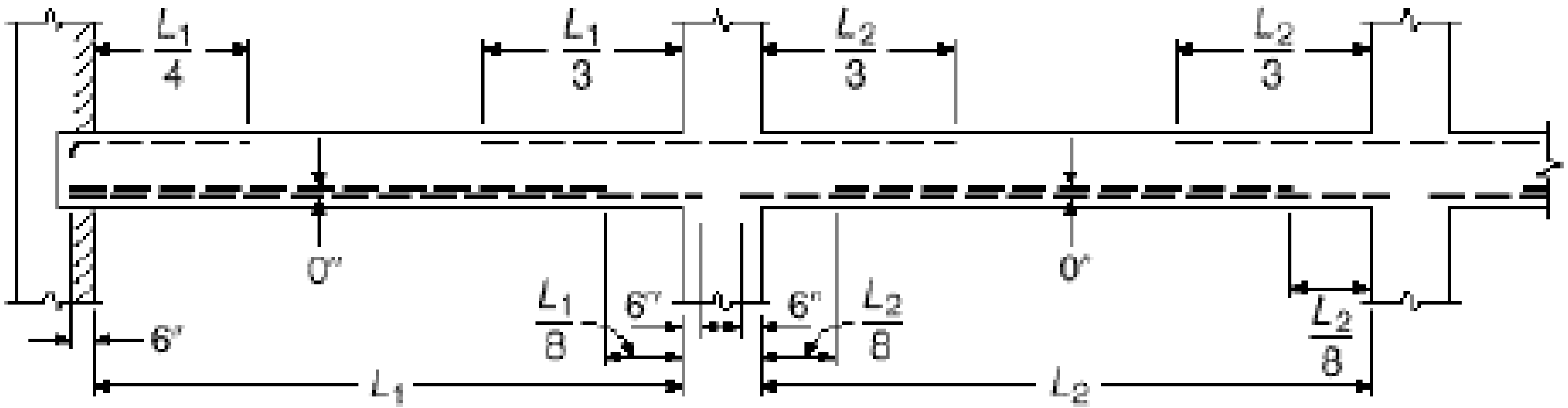
12.10.5.2 — Stirrup area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance **(3/4)*d*** from the termination point. Excess stirrup area shall be not less than $0.41 b_w s/f_{yt}$. Spacing ***s*** shall not exceed $d/(8\beta_b)$.

12.10.5.3 — For No. 36 bars and smaller, continuing reinforcement provides double the area required for flexure at the cutoff point and V_u does not exceed $(3/4)\phi V_n$.

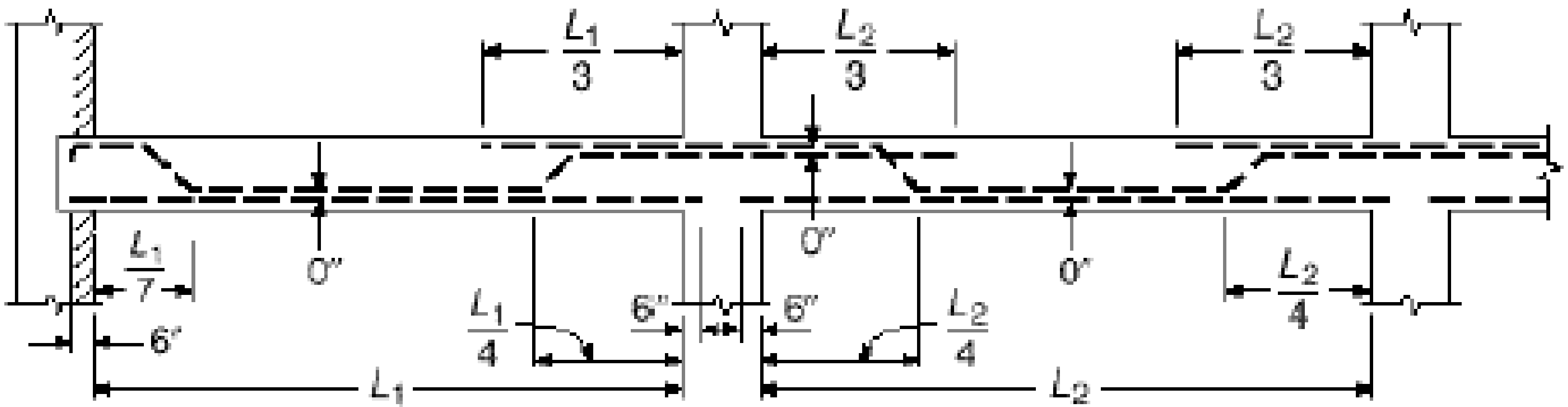


β_b: ratio of area of reinforcement cut off to total area of reinforcement at section

Standard cut off and bent points for bars in approximately equal spans with uniformly distributed loading:



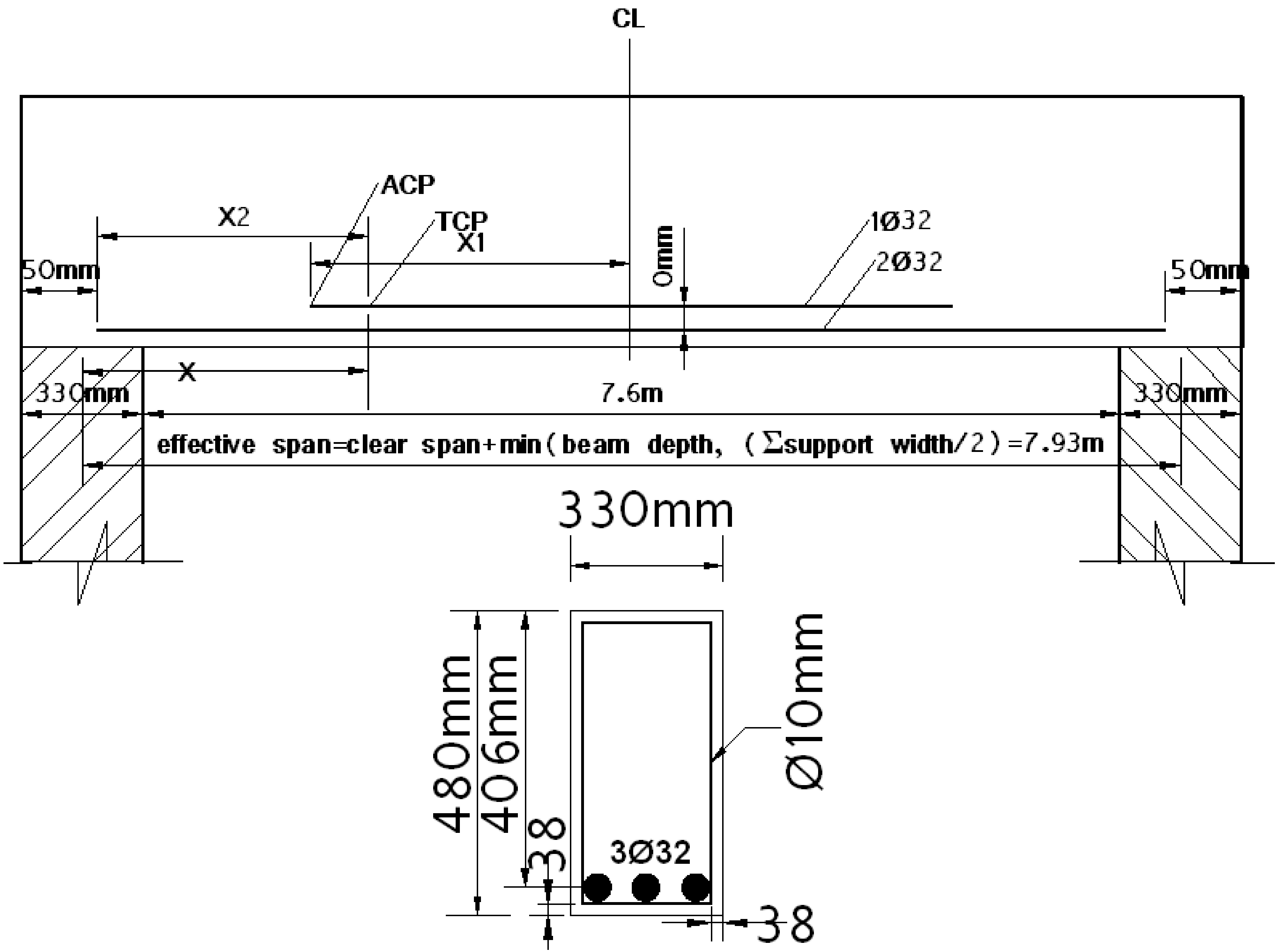
(a)



Not more than 1/2 the tensile steel is to be cut off or bent

EX:simply beam, clear span 7.6m, service load[DL=10.5kN/m(including its own weight), LL=15.8kN/m)], fc'=28 MPa, fy=414MPa, stirrups Ø10mm with a cover of 38mm at spacing less than ACI Code maximum, The reinforcement consists of three bars Ø32mm at effective depth of 406mm, one of which is to be discontinued where no longer needed.

1. Calculate the point where the center bar can be discontinued.
2. Check to be sure that adequate embedded length is provided for continued and discontinued bars.
3. If Ø10mm bars are used for transverse reinforcement, specify special reinforcing details in the vicinity where the Ø32mm bar is cut off.
4. Could two bars be discontinued rather than one.


CL
ACP
X2
TCP
X1
0mm
1Ø32
2Ø32
50mm
50mm
X
330mm
7.6m
330mm
effective span=clear span+min(beam depth, (Σsupport width/2)=7.93m
330mm
480mm
406mm
38
3Ø32
Ø10mm
38

Solution:

Span=min(7.6+0.48=8.08m , 7.6+0.33=7.93m)=7.93m

Wu=1.2*10.5+1.6*15.8=38kN/m

Ru=38*7.93/2=150.7kN

1. **As $_{\text{discontinued bars}}$ =2Ø32=1608** mm^2

$$\rightarrow \rho_{\text{discontinued}} = \frac{1608}{330*406} = 0.012 \begin{pmatrix} >\rho_{min} \\ <\rho_{max} \end{pmatrix} \rightarrow$$

$$Mu_{\text{discontinued}}$$

$$= 0.9 * 0.012 * 0.33 * 0.406^2$$

$$* 414\left(1 - 0.59 * 0.012 * \frac{414}{28}\right) = 0.218MN.m$$

Assume external ultimate moment=218kN.m at a distance=X from support.

$$Mu_{\text{external}} = Ru.X - \frac{Wu.X^2}{2} = 150.7X - \frac{38X^2}{2}$$

$$= 150.7X - 19X^2$$

$$Mu_{\text{external}} = Mu_{\text{internal}} \rightarrow 218 = 150.7X - 19X^2 \rightarrow X$$

$$= \binom{1.9m}{6.03m} \; T.C.P$$

A.C.P=1.9-max(d,12db) =1.9-max(0.406,12*32/1000=0.384)

=1.5m from support

2. Check Ld

- For 3Ø32 bars(discontinued)

*Sc=(330-2*38*2*10-3*32)/(3-1)=69mm>2db=2*32=64mm O.K

*clear cover=38+10=48mm>db=32mm O.K

Ø=32mm>22mm→ $l_d = \left(\frac{3}{5}\frac{fy\ \alpha\beta\lambda}{\sqrt{fc'}}\right) db = \left(\frac{3}{5} * \frac{414*1*1*1}{\sqrt{28}}\right) * 32 = 1502mm$

Available distance(X1)=L/2-ACP=7.93/2-1.5=2.465m>$l_d = 1.502m\ O.K$

- For 2Ø32 bars(continued)

*Sc=(330-2*38*2*10-2*32)/(2-1)=170mm>2db=2*32=64mm O.K

*clear cover=38+10=48mm>db=32mm O.K

Ø=32mm>22mm→ $l_d = \left(\frac{3}{5}\frac{fy\ \alpha\beta\lambda}{\sqrt{fc'}}\right)db = \left(\frac{3}{5} * \frac{414*1*1*1}{\sqrt{28}}\right) * 32 = 1502mm$

Available distance(X2)=TCP=1.9> $l_d = 1.502m\ O.K$

3. Vu at ACP=150.7-38*1.5=93.7kN

$$V_c = \frac{1}{6} * \sqrt{28} * 0.33 * 0.406 * 1000 = 118kN$$

$$Vu = \emptyset(Vc + Vs)$$

$$93.7 = 0.75(118 + Vs) \rightarrow Vs = 6.9kN \rightarrow Vs = \frac{A_v fy\, d}{S} = A_v$$

$$= \frac{Vs.S}{fy\, d} = \frac{6.9 * 1000S}{414 * 406} = 0.041S$$

$$\mathrm{Av_{additional}} = \frac{0.42 b_w S}{fy_t} = \frac{0.42 * 330 * S}{414} = 0.335S$$

$$\mathrm{Av_{total}} = \mathrm{Av_{additional}} + Av_{shear+torsion}$$

$$Av_{\text{total}} = 0.335S + 0.041S = 0.376S$$

0.376S=2*78

S=415mm

$$\beta_d = \frac{As_{cutoff}}{As_{total}} = \frac{1}{3}$$

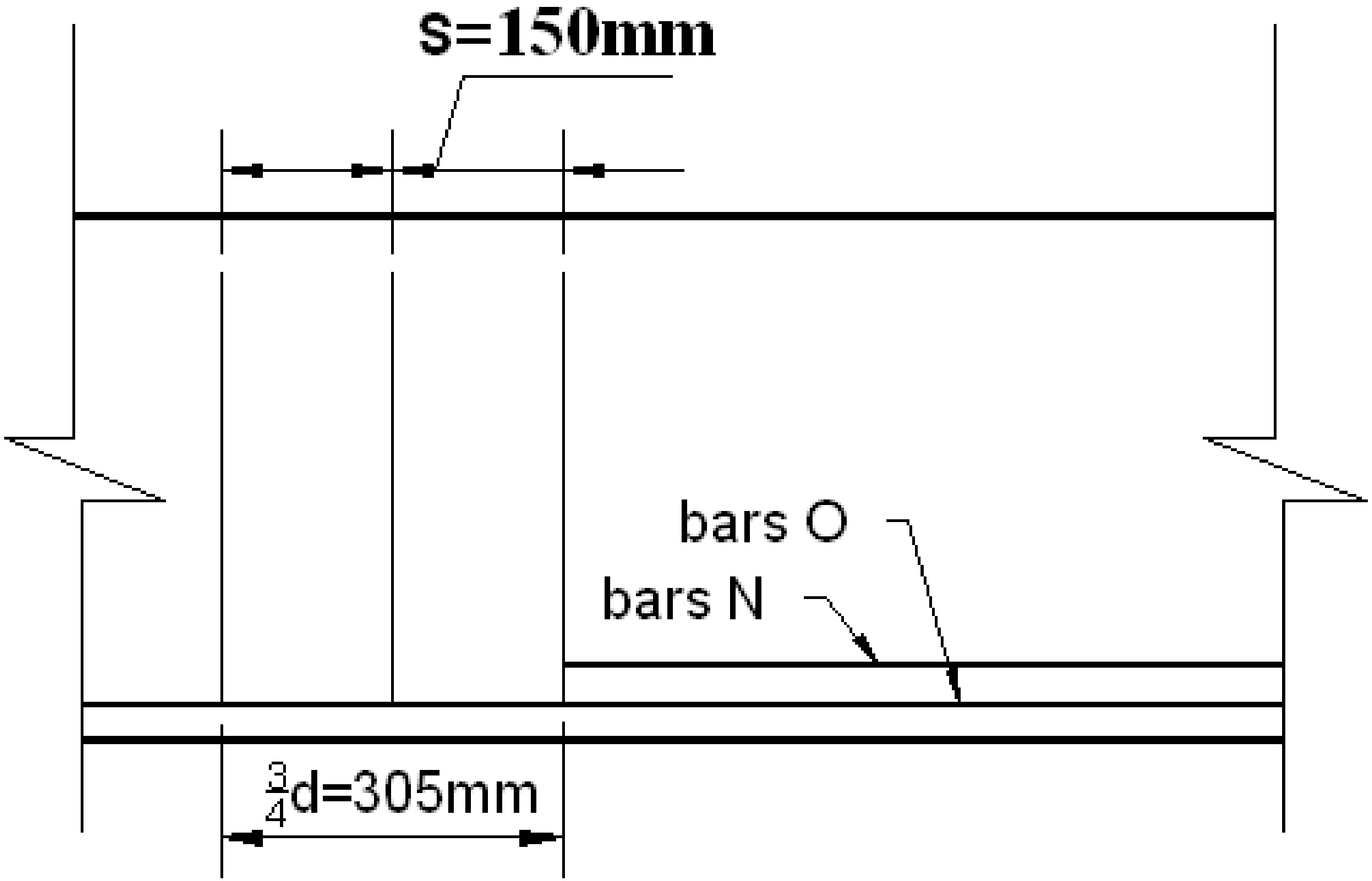
$$S_{max} = \frac{d}{8\,\beta_d} = \frac{406}{8 * \frac{1}{3}} = 152mm$$

$$S = 415mm > S_{max} = 152mm$$

$$\rightarrow use\ \emptyset 10@150mm \frac{c}{c} at\ \ distance \frac{3}{4} d = \frac{3}{4} * 406$$

$$= 305mm from\ termination\ point$$

No of stirrups=305/150=2+1=3



s=150mm
bars O
bars N
$\frac{3}{4}$d=305mm