ALMUSTAQBAL UNIVERSITY

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

EXAMPLES IN THE EQUIVALENT FRAME METHOD

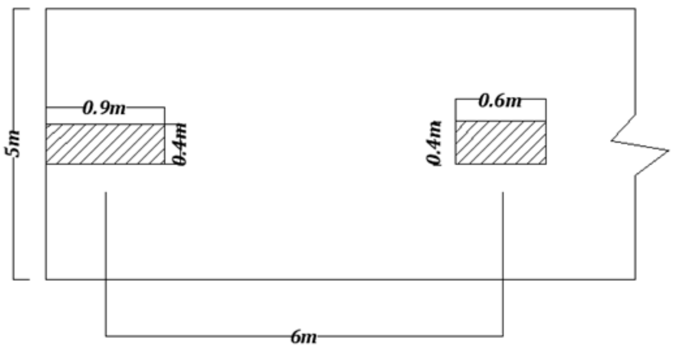
EXAMPLE2: For the flat plate slab shown below, calculate:

1. K_{sb} for joint A.
2. K_c for joint A.
3. K_t for joint A.
4. DF (distribution factor) for joint A.

Given: slab thickness is 160mm, column length is 3.2m and the column extends above and below the floor.

SOLUTION:

1. K_{sb} for joint A.



$\frac{C_1A}{l_1} = \frac{900}{6000} = 0.15, \frac{C_1B}{l_1} = 0.1$

$\therefore inverse\ the\ index.$

$k_{AB} = 4.36$

$$K_{sb} = k.\frac{E(I_s + I_b)}{l_1}$$

$$I_s = \frac{{h_s}^3 l_2}{12} = \frac{160^3 \times 5000}{12} = 1.707 \times 10^9 mm^4$$

$$K_{sb} = 4.36 \times \frac{E \times (1.707 \times 10^9 + 0)}{6000} = 1.24 \times 10^6 E\ N.mm$$

2. K_c for joint A.

$$\frac{C_1A}{l_c} = \frac{160}{3200} = 0.05$$

$\therefore\ K_{AB\ bottom} = 4.91, K_{BA\ top} = 4.21$

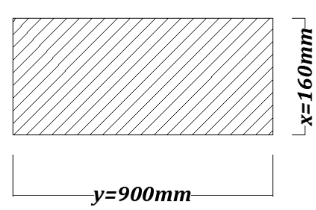
$$I_c = \frac{{C_1}^3 \times C_2}{12} = \frac{900^3 \times 400}{12} = 24.3 \times 10^9 mm^4$$

$$K_{cb} = 4.91 \times \frac{E \times 24.3 \times 10^9}{3200} = 37.285 \times 10^6 E\ N.mm$$

$$K_{ct} = 4.21 \times \frac{E \times 24.3 \times 10^9}{3200} = 31.97 \times 10^6 E\ N.mm$$

3. K_t for joint A.

$$K_t = \frac{9EC}{l_2(1 - \frac{C_2}{l_2})^3}\ , \qquad C = \sum(1 - 0.63\frac{x}{y})\frac{x^3 y}{3}$$



$$C = \left(1 - 0.63 \times \frac{160}{900}\right) \times \frac{160^3 \times 900}{3} = 1.09117 \times 10^9 mm^4$$

$$K_t = \frac{9E1.09117 \times 10^9}{5000 \times (1 - \frac{400}{5000})^3} = 2.522 \times 10^6 E \;\; N.mm$$

4. DF (distribution factor) for joint A.

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t}$$

$$\frac{1}{K_{ec}} = \frac{1}{(37.285 + 31.97) \times 10^6 E} + \frac{1}{2.522 \times 10^6 E} = 4.0979 \times 10^{-7} \times \frac{1}{E}$$

$$\therefore \; K_{ec} = 2.43 \times 10^6 E \;\; N.mm$$

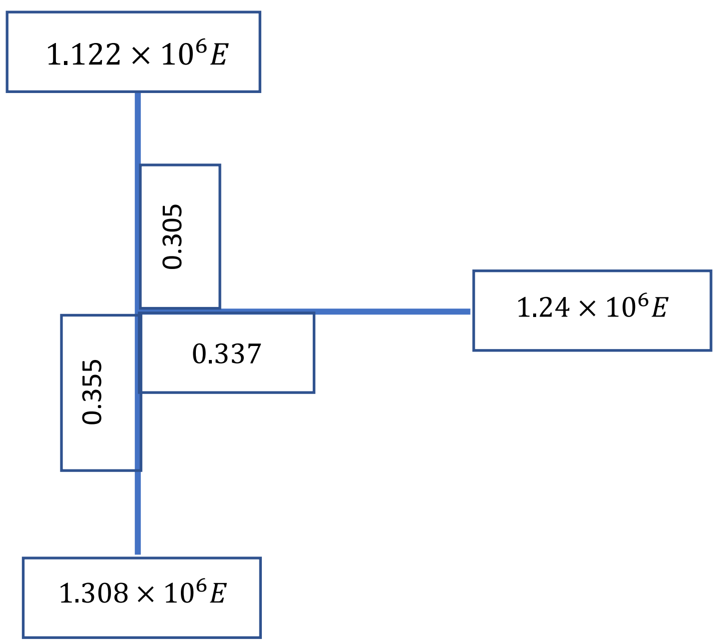
$$DF_{1-2} = \frac{K_{sb\,1-2}}{K_{sb1-2} + K_{ec}} = \frac{1.24 \times 10^6 E}{1.24 \times 10^6 E + 2.43 \times 10^6 E} = 0.337$$

$$K_{ect} = \frac{K_{ct}}{\sum K_c} \times K_{ec} = \frac{31.97 \times 10^6 E}{37.285 \times 10^6 E + 31.97 \times 10^6 E} \times 2.43 \times 10^6 E = 1.122 \times 10^6 E$$

$$K_{ecb} = \frac{K_{cb}}{\sum K_c} \times K_{ec} = \frac{37.285 \times 10^6 E}{37.285 \times 10^6 E + 31.97 \times 10^6 E} \times 2.43 \times 10^6 E = 1.308 \times 10^6 E$$

$$DF_{column\ top} = \frac{1.122 \times 10^6 E}{1.24 \times 10^6 E + 2.44 \times 10^6 E} = 0.305$$

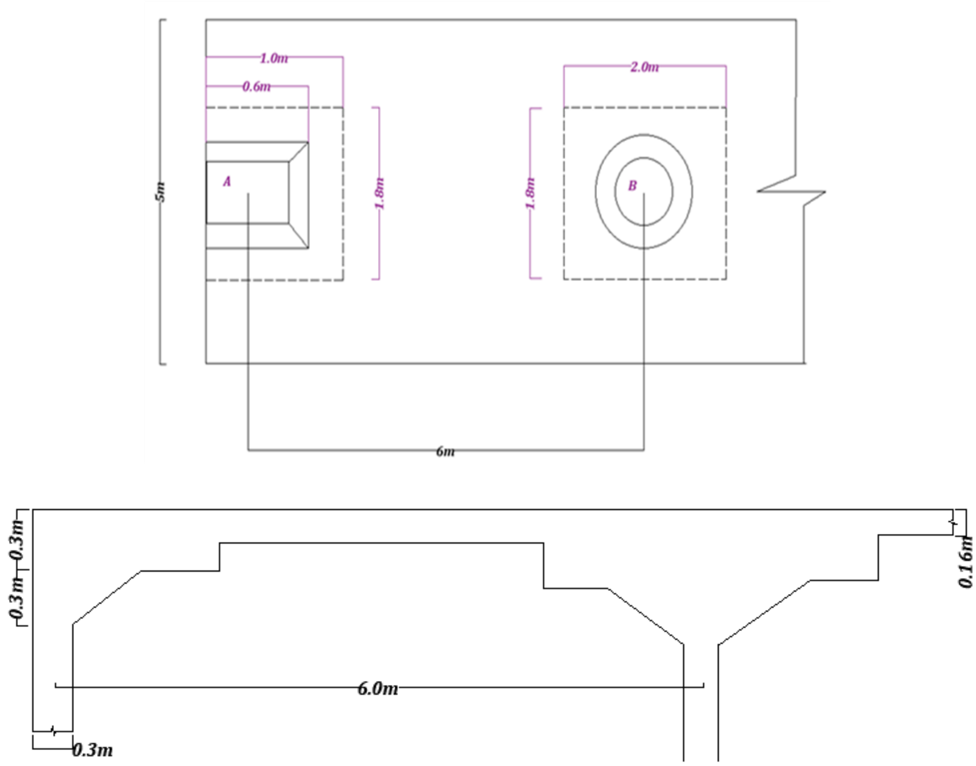
$$DF_{column\ bottom} = \frac{1.308 \times 10^6 E}{1.24 \times 10^6 E + 2.44 \times 10^6 E} = 0.355$$



EXAMPLE 3: for the slab below, calculate:

1. Stiffness of slab-beam (K_{sb}) for the interior panel, M_{AB}, M_{BA}, COF_{AB}, and COF_{BA}.
2. Stiffness of column (K_c), COF for joints A and B.

Given: column A has the size of ($300 \times 300mm$), column B has a diameter of (300mm), column capital has a diameter of (1000mm), column height (3m), slab thickness (160mm).



SOLUTION:

1. SLAB BEAM STIFFNESS:

$$\frac{C_1A}{l_1} = \frac{600}{6000} = 0.1$$

Equivalent square of column capital=$1000mm \times 0.89 = 890mm$.

$$\frac{C_1B}{l_1} = \frac{890}{6000} = 0.148 \cong 0.15$$

From Table A13.b

$K_{AB} = 5.03, \quad K_{BA} = 5.16$

$COF_{AB} = 0.566, \quad COF_{BA} = 0.551$

$M_{AB} = 0.088, \quad M_{BA} = 0.092$

$$K_{sb} = k.\frac{E(I_s + I_b)}{l_1}, \qquad I_s = \frac{l_2 \times h^3}{12} = \frac{5000 \times 160^3}{12} = 1.706 \times 10^9 mm^4, I_b = 0.$$

$$K_{sbAB} = 5.03 \times \frac{E(1.706 \times 10^6 + 0)}{6000} = 1.43 \times 10^6 E\ N.mm$$

$$K_{sbBA} = 5.16 \times \frac{E(1.706 \times 10^6 + 0)}{6000} = 1.467 \times 10^6 E\ \ N.mm$$

2. **Stiffness of column (K_c)**
JOINT A:

$$\frac{C_1 A}{l_c} = \frac{300 + 300}{3000} = 0.2$$

$$K_{AB} = 9.69, \ \ K_{BA} = 5$$

$$COF_{AB} = 0.425, \quad COF_{BA} = 0.875$$

$$K_c = k.\frac{EI_c}{l_c}, \ \ I_c = \frac{300^3 \times 300}{12} = 6.75 \times 10^9 mm^4$$

$$K_{col\ bot.AB} = 9.69 \times .\frac{E \times 6.75 \times 10^9}{3000} = 2.18 \times 10^6 E\ N.mm$$

$$K_{col\ top.BA} = 5 \times \frac{E \times 6.75 \times 10^9}{3000} = 1.125 \times 10^6 E\ N.mm$$

JOINT B:

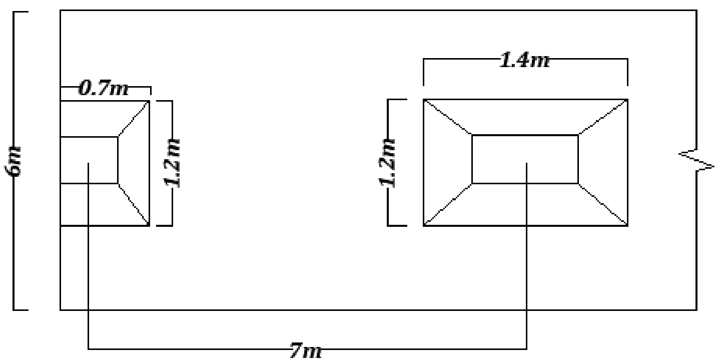
$$K_{col.Bot\ AB} = 9.69 \times \frac{E\frac{\pi 300^4}{64}}{3000} = 1284.273 \times 10^6 E\ \ N.mm$$

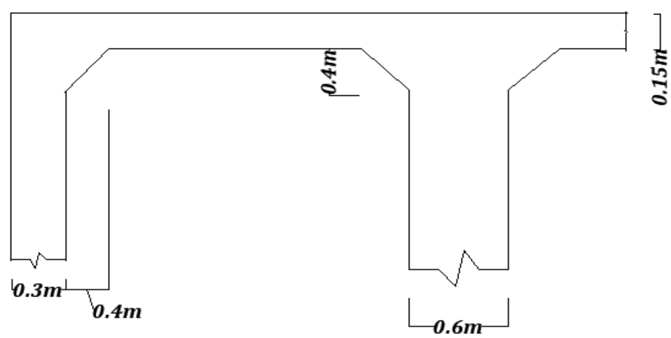
$$K_{col.Top\ AB} = 5 \times \frac{E\frac{\pi 300^4}{64}}{3000} = 662.67 \times 10^6 E\ \ N.mm$$

EXAMPLE 4: For the flat slab shown in the figure below, compute:

1. K_{sb} for joint A.
2. K_c for joint A.
3. K_t for joint A.
4. DF for joint A.

Given: slab thickness=150mm, column length=2.75m, column extends above and below





SOLUTION:

1. K_{sb} for joint A.

$$\frac{C_1A}{l_1} = \frac{700}{7000} = 0.1, \qquad \frac{C_1B}{l_1} = \frac{1400}{7000} = 0.2$$

$$k_{AB} = 4.27$$

$$K_{sb} = k.\frac{E(I_s + I_b)}{l_1}, \quad I_s = \frac{l_2 \times h^3}{12} = \frac{150^3 \times 6000}{12} = 1.6875 \times 10^9 mm^4$$

$$K_{sbAB} = 4.27 \times \frac{E(16875 \times 10^6 + 0)}{7000} = 1.03 \times 10^6 E \quad N.mm$$

2. **K_c for joint A.**

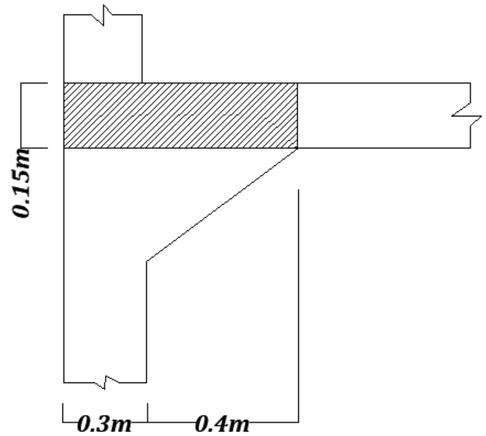
$$\frac{C_1A}{l_c} = \frac{0.55}{2.75} = 0.2, \quad k_{AB\,BOT} = 9.69, \quad k_{BA\,TOP} = 5$$

$$I_c = \frac{300^3 \times 400}{12} = 9 \times 10^9 mm^4$$

$$K_{c\,Bot} = 9.69 \times \frac{E \times 9 \times 10^9}{2750} = 3.17 \times 10^6\ N.mm$$

$$K_{c\,TOP} = 5 \times \frac{E \times 9 \times 10^9}{2750} = 1.636 \times 10^6\ N.mm$$

3. **K_t for joint A.**



$$K_t = \frac{9EC}{l_2(1-\frac{c_2}{l_2})^3} \quad , \quad C = \sum(1-0.63\frac{x}{y})\frac{x^3y}{3}$$

$$C = \left(1 - 0.63 \times \frac{150}{700}\right) \times \frac{150^3 \times 700}{3} = 6.81 \times 10^8 mm^4$$

$$K_t = \frac{9 \times E \times 6.81 \times 10^8}{6000 \times (1-\frac{1200}{6000})^3} = 3.99 \times 10^6 E\ \ N.mm$$

4. **DF for joint A.**

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t}$$

$$\frac{1}{K_{ec}} = \frac{1}{(3.17+1.636) \times 10^6 E} + \frac{1}{3.99 \times 10^6 E} = 0.459 \times 10^{-6} \times \frac{1}{E}$$

$$\therefore\ K_{ec} = 2.18 \times 10^6 E\ \ N.mm$$

$$DF_{1-2} = \frac{K_{sb\,1-2}}{K_{sb1-2} + K_{ec}} = \frac{1.03 \times 10^6 E}{1.03 \times 10^6 E + 2.18 \times 10^6 E} = 0.32$$

$$K_{ect} = \frac{K_{ct}}{\sum K_c} \times K_{ec} = \frac{1.636 \times 10^6 E}{1.636 \times 10^6 E + 3.17 \times 10^6 E} \times 2.18 \times 10^6 E = 0.74 \times 10^6 E$$

$$K_{ecb} = \frac{K_{cb}}{\sum K_c} \times K_{ec} = \frac{37.285 \times 10^6 E}{37.285 \times 10^6 E + 31.97 \times 10^6 E} \times 2.18 \times 10^6 E = 1.43 \times 10^6 E$$

$$DF = \frac{K_{ec\,top\,or\,bottom}}{k_{sb1-2} + K_{ec}}$$

$$DF_{column\ top} = \frac{0.74 \times 10^6 E}{1.03 \times 10^6 E + 2.18 \times 10^6 E} = 0.23$$

$$DF_{column\ bottom} = \frac{1.43 \times 10^6 E}{1.03 \times 10^6 E + 2.18 \times 10^6 E} = 0.455$$