Torsion in reinforced concrete beam

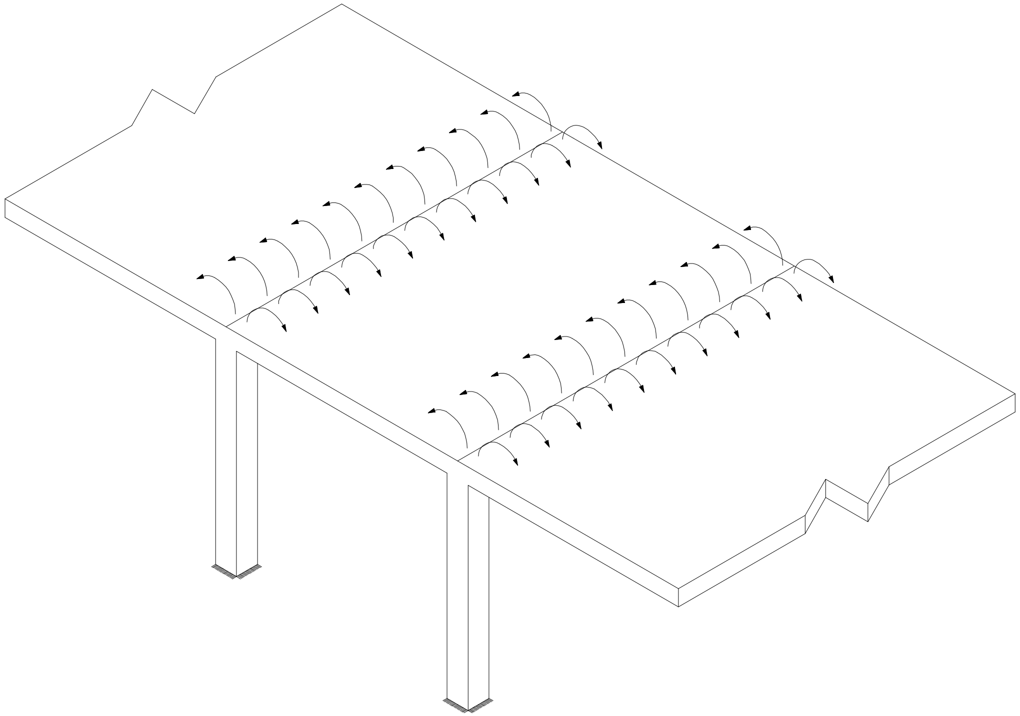
Torsional moment: moment tending to twist a member about longitudinal axis.

Typical cases: space frame, spandrel beams, beams supporting balconies or cantilever slabs, horizontally curve beams.

Type of torques:

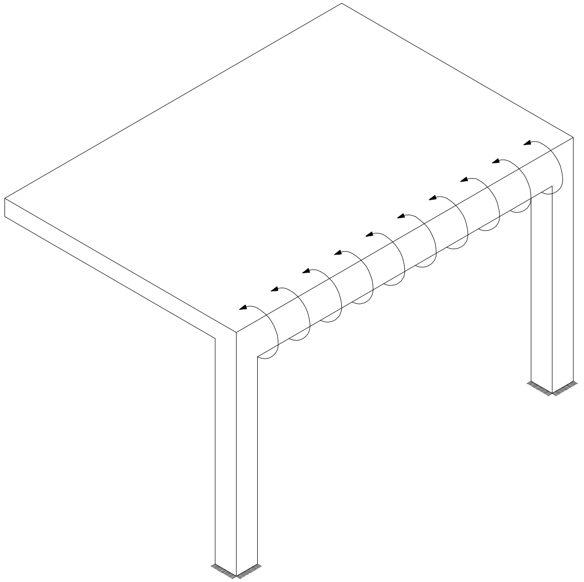
1. Compatibility torque

Redistribution of torsion occur



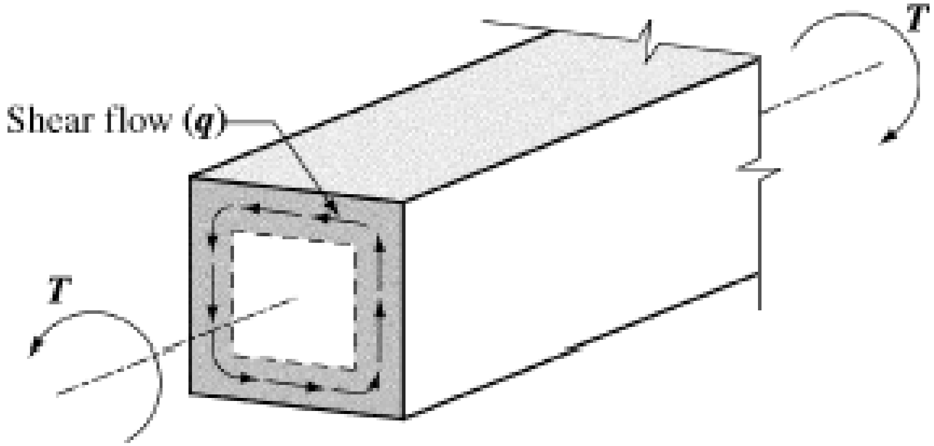
2. Equilibrium torque

No redistribution of torsion occur, torsion is very important in design

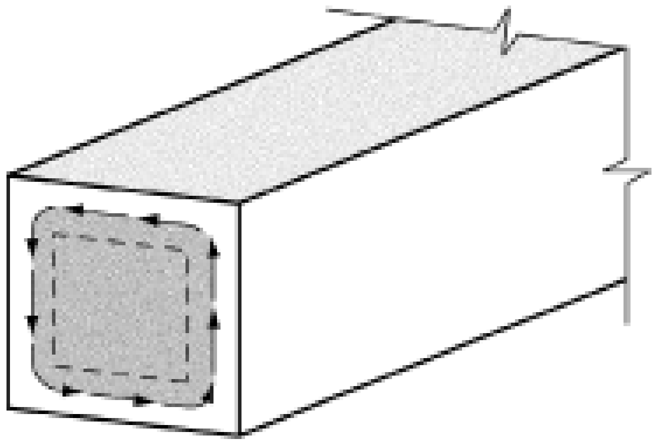


Design for torsion

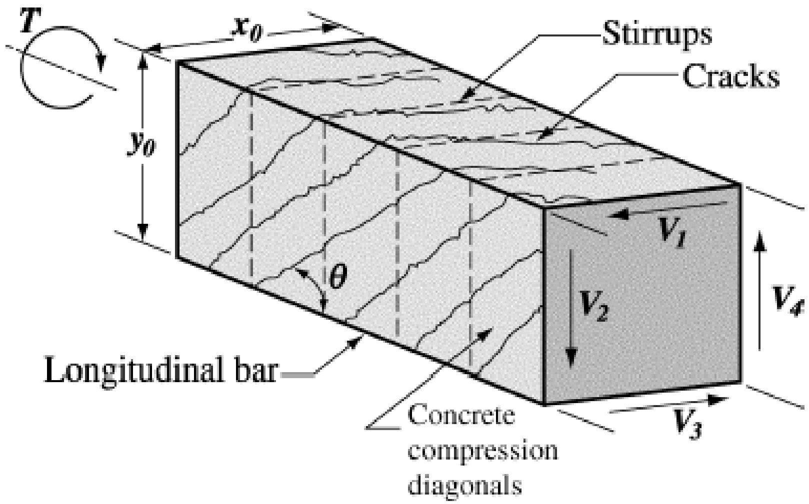
The design for torsion is based on a thin-walled tube, space truss analogy. A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam neglected as shown in Fig. (a). Once a reinforced concrete beam has cracked in torsion, its torsional resistance is provided primarily by closed stirrups and longitudinal bars located near the surface of the member. In the thin-walled tube analogy the resistance is assumed to be provided by the outer skin of the cross section roughly centered on the closed stirrups. Both hollow and solid sections are idealized as thin-walled tubes both before and after cracking.



(a) Thin-walled tube



(b) Area enclosed by shear flow path



Torsional effects may neglected when:

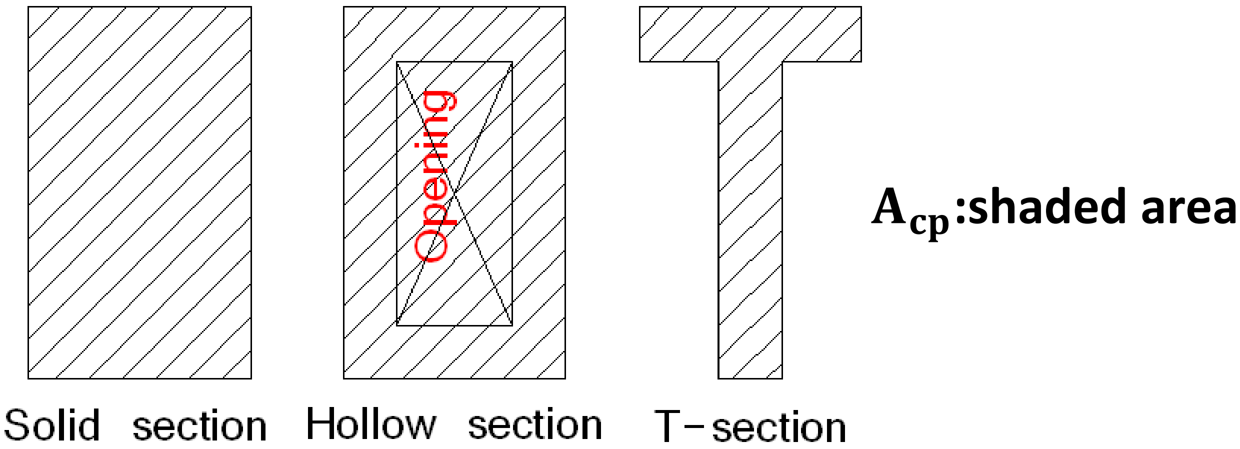
the factored torsional moment,

$$T_{u(ext)} < \emptyset \frac{\sqrt{fc'}}{12} \left(\frac{A_{Cp}^2}{P_{cp}} \right) \ldots \ldots \ldots \ldots .11.6.1$$

$\emptyset$: Reduction factor for torsion= 0.75

$\mathbf{A_{cp}}$:Area enclosed by outside perimeter of concrete cross-section

P_{cp}: Outside perimeter of (A_{cp})



Check cross-section dimensions for torsion:

The cross-section dimensions shall be such that:

1. For solid section:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7A_{oh}^2}\right)^2} \le \emptyset\left[\frac{V_c}{b_w d} + \frac{8\sqrt{fc'}}{12}\right] \dots\dots\dots\dots\dots 11 - 18$$

2. For hollow section

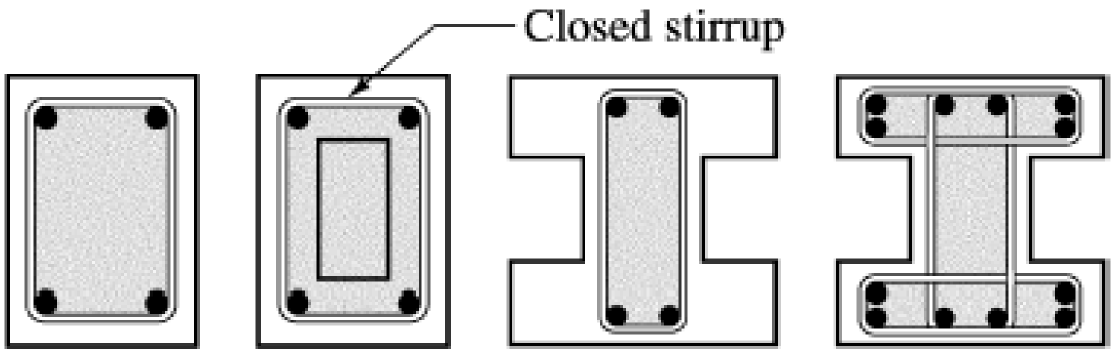
$$\left(\frac{V_u}{b_w d} + \frac{T_u P_h}{1.7A_{oh}^2}\right) \le \left[\frac{V_c}{b_w d} + \frac{8\sqrt{fc'}}{12}\right] \dots\dots\dots\dots\dots 11 - 19$$

A_{oh}: Area enclosed by C.L of out most closed transverse torsional reinforcement

P_h: Perimeter of C.L of out most closed transverse torsional reinforcement

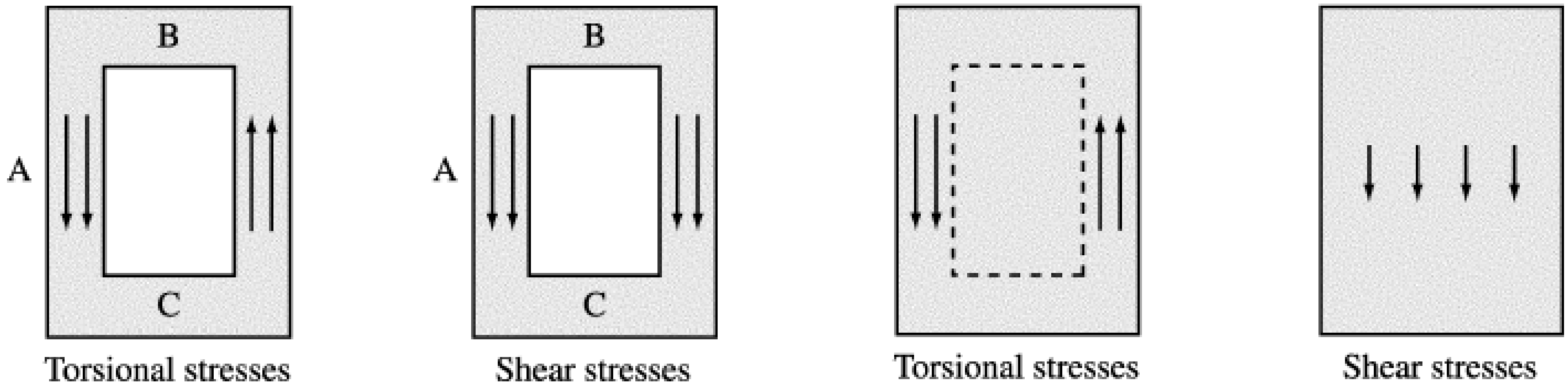
- If wall thickness less than A_{oh}/P_h, the **second** term in eq.11-19shall be taken as $T_u/(1.7\ A_{oh}t)$,

t: thickness of wall of the hollow section.



A_{oh} = shaded area

In a hollow section, the shear stresses due to shear and torsion both occur in the walls of the box as shown in Fig.(a) and hence are directly additive at point A as given in Eq. (11-19). In a solid section the shear stresses due to torsion act in the “tubular” outside section while the shear stresses due to Vu are spread across the width of the section as shown in Fig.(b). For this reason stresses are combined in Eq. (11-18) using the square root of the sum of the squares rather than by direct addition.



(a) Hollow section *(b) Solid section*

11.6.3.5 -Where $\mathbf{T_u}$ exceeds the threshold torsion, design of the cross section shall be based on:

$$T_u \leq \emptyset T_n \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \ldots 11-20$$

$$T_n = \frac{2A_o A_t \mathrm{f}_{yt}}{S} \cot(\theta) \ldots \ldots \ldots \ldots .11-21$$

$A_o = 0.85 * A_{oh}$

$\theta = 45^o$ for non-pre-stressed members.

$\boldsymbol{A_t}$: Area of **one** leg of **closed** stirrups resisting torsion.

$\mathbf{f}_{\boldsymbol{yt}}$: yield strength of closed stirrups

❖ **Longitudinal reinforcement for torsion** is required because, it achieves the following

a. It anchors the closed stirrups particularly at the corners in order to develop their (closed stirrups) fully yield strength.
b. It provides some resisting torque because of the dowel action which develops after cracking.
c. After cracking, the spiral cracks tend to widen. The longitudinal bars counteract this tendency and control crack width.

The additional area of longitudinal reinforcement (A_l) to resist torsion

$$A_l = \frac{A_t}{S} P_h \left(\frac{f_{yt}}{f_{yl}} \right) \cot^2(\theta) \ldots\ldots\ldots\ldots 11\text{-}22$$

f_{yl}: yield strength for longitudinal torsional reinforcement

❖ **11.6.3.8**-Reinforcement required for torsion (A_t, A_l), shall be added to that required for shear, moment and axial forces

$$\frac{A_{V+t}}{S} = \frac{A_V}{S} + \frac{2A_t}{S}$$

Note: If stirrups groups had four legs for shear, only the legs adjacent to sides of the beam would be include in this for torsion

Note: For hollow section in torsion, the distance from the centerline of the stirrups to the inside face of the wall of the hollow section shall not be less than 0.5 A_{oh}/P_h

Minimum torsion reinforcement:

$$\text{When}\,T_u > \emptyset \frac{\sqrt{fc'}}{12}\left(\frac{A_{cp}^2}{P_{cp}}\right)$$

$$(A_v + 2A_t)_{min} \geq max.\begin{bmatrix} 0.062\sqrt{fc'}\dfrac{b_w S}{f_{yt}} \\ 0.35\dfrac{b_w S}{f_{yt}} \end{bmatrix} \ldots$$ 11-23 for shear and torsion

and

$$A_{l_{min.}} \geq \frac{5}{12}\frac{\sqrt{fc'}A_{cp}}{f_y} - \underbrace{\left(\frac{A_t}{S}\right)}_{\geq 0.175\frac{b_w}{f_{yt}}} P_h \frac{f_{yt}}{f_{yl}} \ldots\ldots\ldots 11\text{-}24$$

$$\frac{A_t}{S} \geq 0.175\frac{b_w}{f_{yt}}$$

Spacing of torsion reinforcement:

1. **For closed stirrups: ACI 11.6.6.1**

$$S \leq min.\left(\frac{P_h}{8}, 300mm\right)$$

2. **For longitudinal reinforcement required for torsion A_l** ,shall be distributed around the perimeter of the closed stirrups with max spacing of 300 mm, the longitudinal bars shall be one bar in each corner of the stirrups.

Long bar diameter $\geq$ max $\left(\frac{1}{24}S, 10mm\right)$

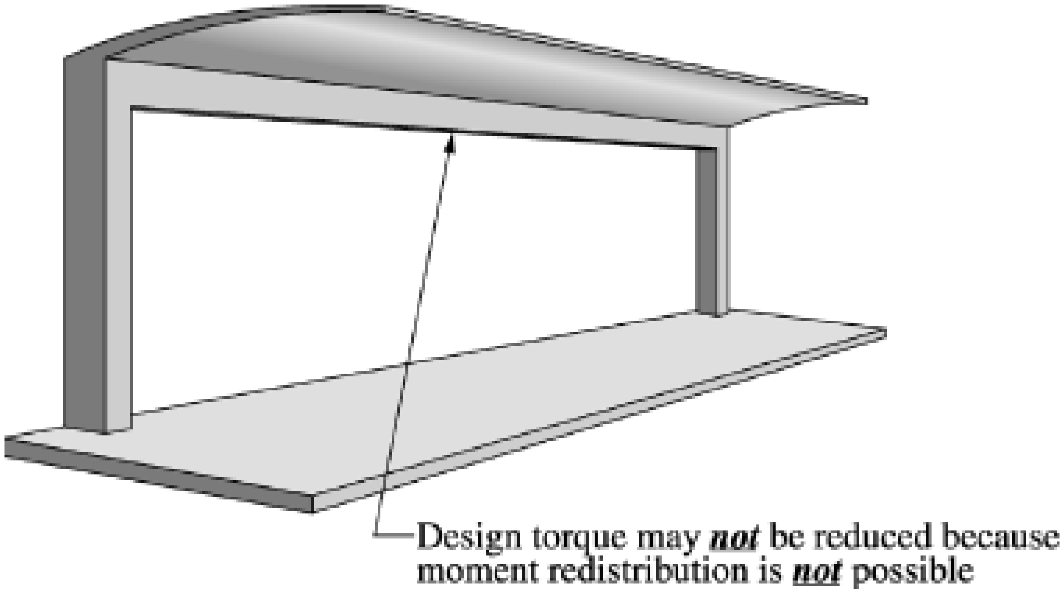
- Torsional reinforcement shall be provided for a distance of at least $(b_t + d)$ beyond the point required by analysis

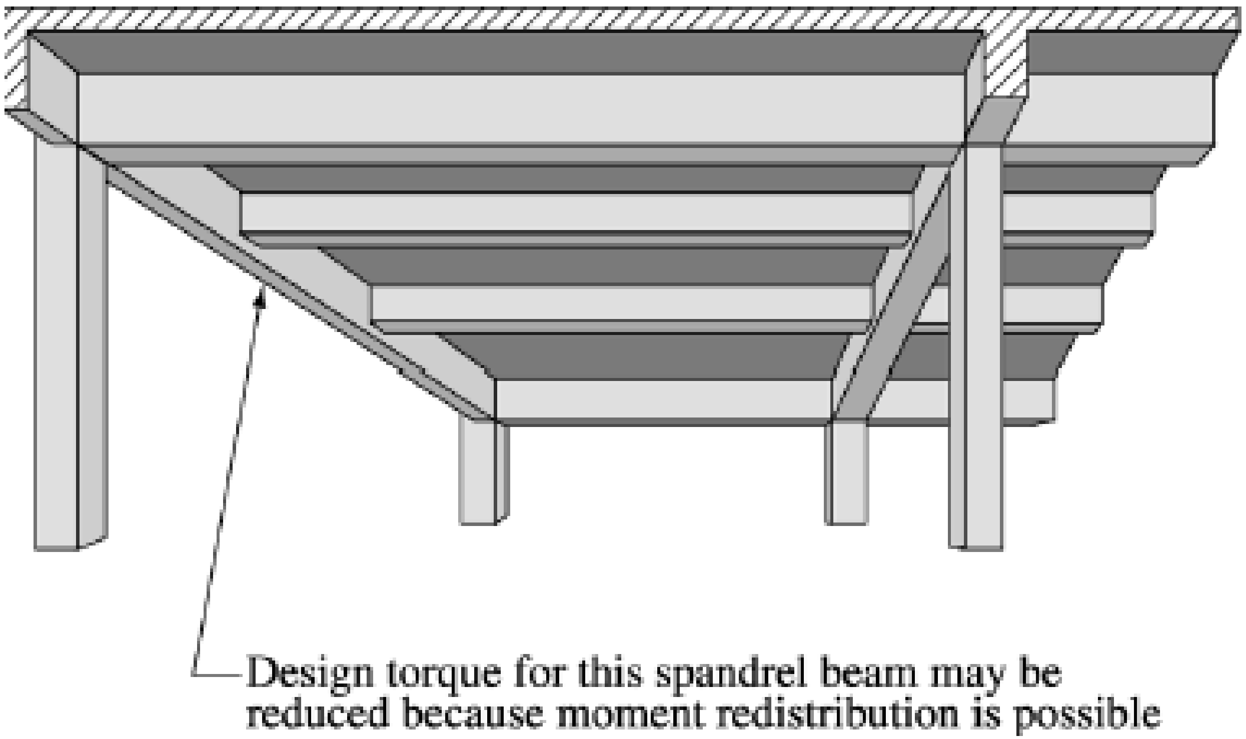
$\boldsymbol{b_t}$:is the width of that part of cross section containing the closed stirrups resisting torsion.

11.6.2.2-In a statically indeterminate structure where reduction of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the maximum $\mathbf{T_u}$ shall be permitted to be reduced to

$$\emptyset \frac{\sqrt{f_c'}}{3}\left(\frac{A_{cp}^2}{P_{cp}}\right) \ldots\ldots\ldots 11.6.2.2$$

$$f_{yl} \text{and} f_{yt} \leq 420\ MPa \ldots\ldots\ldots\ldots 11.6.3.4$$

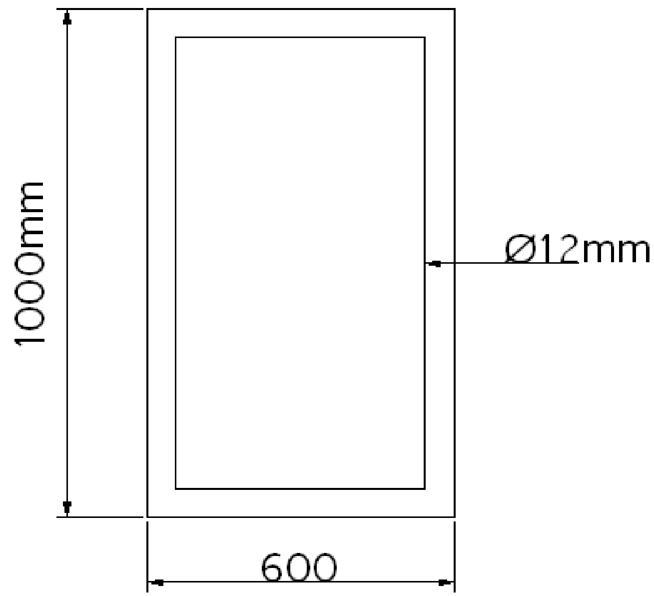

Design torque may ***not*** be reduced because
moment redistribution is ***not*** possible

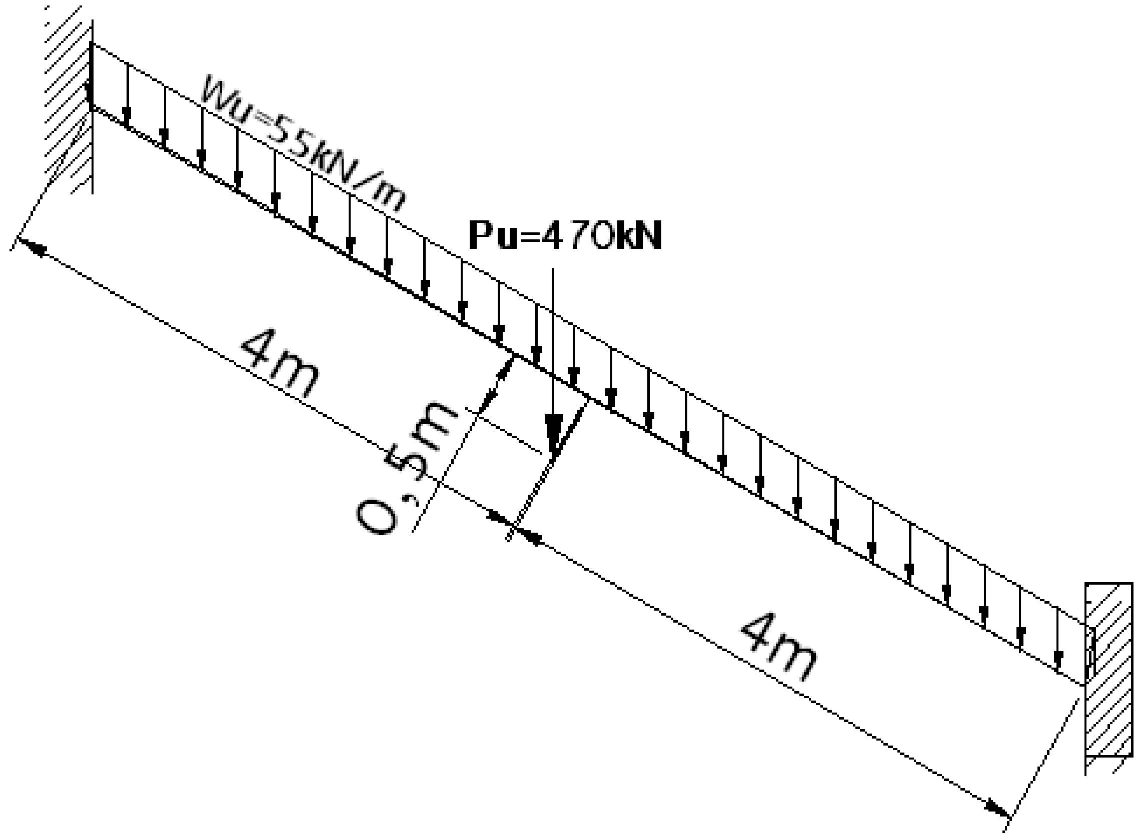

Design torque for this spandrel beam may be
reduced because moment redistribution is possible

Example:

$fc' = 28\ MPa$,$f_y = 400\ MPa$

Design critical section for shear and torsion, for the same critical section for shear, design the section for flexural





Solution:

Self $w_t = 0.6 * 1.0 * 24 = 14.4\ kN/m$

$w_u = 1.2 * 14.4 + 55 = 72.28\ kN/m$

$R_u = \frac{72.28*8}{2} + \frac{470}{2} = 524\ kN,$ due to symmetry of loads

d $= 1000 - 40 - 12 - (25/2) = 935mm$

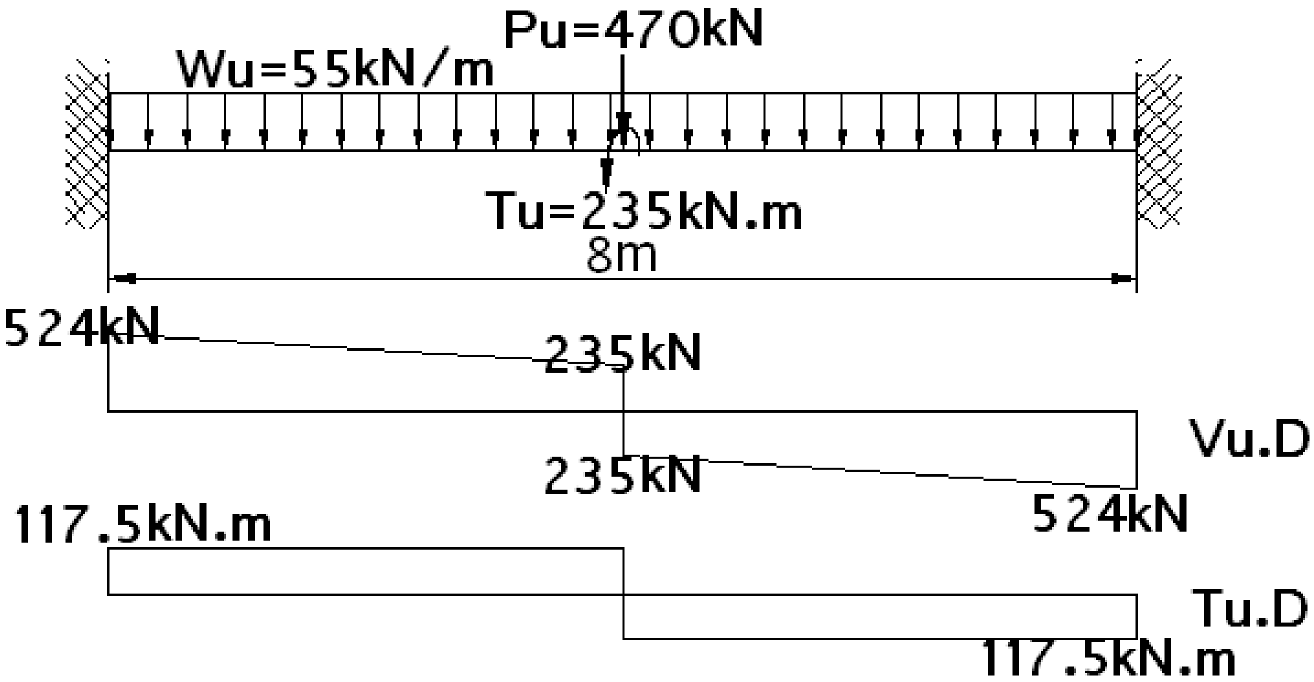
$V_{ud} = 524 - 72.28 * 0.935 = 456kN$

$T_u = 470 * 0.5 = 235\ kN.m$

$T_{ud} = 235/2 = 117.5\ kN.m$

$$M_u^- = \frac{W_u * l^2}{12} + \frac{P * l}{8} = \frac{72.28 * 8^2}{12} + \frac{470 * 8}{8} = 855\ kN.m$$

$$M_u^+ = \frac{W_u * l^2}{24} + \frac{P * l}{8} = \frac{72.28 * 8^2}{24} + \frac{470 * 8}{8} = 663\ kN.m$$


Pu=470kN
Wu=55kN/m
Tu=235kN.m
8m
524kN
235kN
Vu.D
235kN
524kN
117.5kN.m
Tu.D
117.5kN.m

Design for flexure

$$0.855 = 0.9\rho * 0.6 * 0.935^2 * 400(1 - 0.59\rho * 400/28) \rightarrow \rho^- = 0.0047$$

$$\rho_{max} = 0.85 * 0.85 * \frac{28}{400} * \frac{0.003}{0.007} = 0.0217$$

$$\rho_{min} = max\left(\frac{1.4}{f_y} = 0.0035, \frac{\sqrt{fc'}}{4f_y} = 0.0033\right) = 0.0035$$

$$\rho_{min} = 0.0035 < \rho^- = 0.0047 < \rho_{max} = 0.0217 \text{ O.K}$$

$$As^- = 0.0047 * 600 * 935 = 2637\ mm^2$$

$$0.663 = 0.9\rho * 0.6 * 0.935^2 * 400(1 - 0.59\rho * 400/28) \rightarrow \rho^+ = 0.00362$$

$$\rho_{min} = 0.0035 < \rho^+ = 0.0036 < \rho_{max} = 0.0217 \text{ O.K}$$

$$As^+ = 0.00362 * 600 * 935 = 2031\ mm^2$$

Design for torsion

use stirrups $\emptyset 12\ mm$

$$A_{cp} = 0.6 * 1.0 = 0.6\ m^2$$

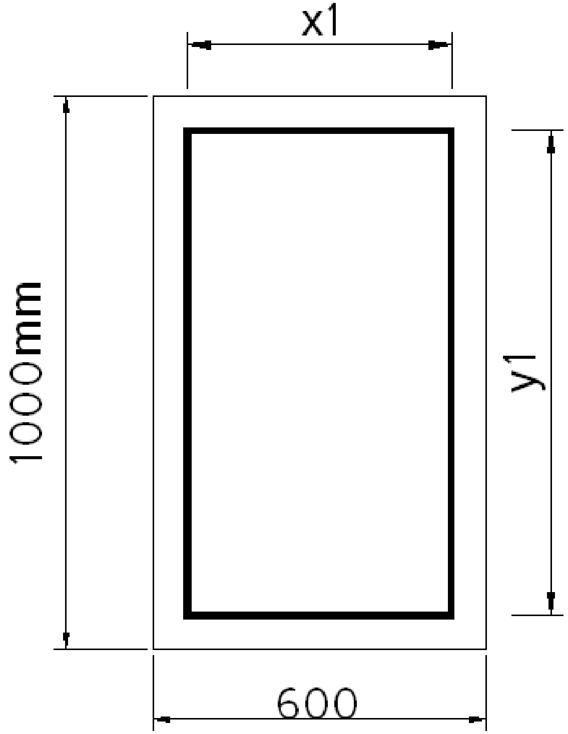
$$P_{cp} = (0.6 + 1.0) * 2 = 3.2\ m$$

$$T_{ud} = 117.5\ kN.m > \emptyset \frac{\sqrt{fc'}}{12}\left(\frac{A_{cp}^2}{P_{cp}}\right) = 0.75 \frac{\sqrt{28}}{12}\left(\frac{0.6^2}{3.2}\right) * 10^3$$

$$= 37.2\ kN.m$$

∴ Torsional effect shall be considered

Check dimensions



$$x_1 = 600 - 2 * 40 - 2 * (12/2)$$
$$= 508\, mm$$

$$y_1 = 1000 - 2 * 40 - 2 * (12/2)$$
$$= 908\, mm$$

$$A_{oh} = x1 * y1 = 0.508 * 0.908$$
$$= 0.461\, m^2$$

$$P_h = (x1 + y1) * 2$$
$$= (0.508 + 0.908) * 2 = 2.83\, m$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7 A_{oh}^2}\right)^2} \leq \emptyset\left[\frac{V_C}{b_w d} + \frac{8\sqrt{\mathrm{f}c'}}{12}\right]$$

$$\sqrt{\left(\frac{456}{0.6*0.935}\right)^2 + \left(\frac{117.5*2.83}{1.7*0.461^2}\right)^2} * 10^{-3} = 1.228\ MPa \leq 0.75\left(\frac{\sqrt{28}}{6} + \frac{8\sqrt{28}}{12}\right) = 3.307\ MPa$$ (else, increase section dimensions)

$$T_u = \emptyset T_n = \emptyset * \frac{2 \overbrace{A_o}^{=0.85A_{oh}} A_t f_{yt}}{S} \cot(\theta)$$

$$117.5 * 10^{-3} = \frac{0.75 * 2 * 0.85 * 0.461 * A_t * 400 * \cot(45)}{S}$$

$$\rightarrow A_t = 5 * 10^{-4} S$$

Design for shear

$$V_C = 0.17 * \sqrt{28} * 0.6 * 0.935 * 1000 = 504\ kN$$

$$V_u = \emptyset(V_c + V_s)$$

$$456 = 0.75 * (504 + V_s) \rightarrow V_s = 104\ kN$$

$$s = \frac{A_v f_y d}{V_s} = \frac{A_v * 400 * 0.935}{0.104} \rightarrow A_v = 2.78 * 10^{-4} S$$

$$(2A_t + A_v) = (2 * 5 + 2.78) * 10^{-4} S = 1.278 * 10^{-3}\ S$$

$$\geq max. \left[\begin{array}{c} 0.062\sqrt{fc'}\frac{b_w S}{f_{yt}} = 0.062 * \sqrt{28} * \frac{0.6 * S}{400} \\ = 4.92 * 10^{-4} S \\ 0.35\frac{b_w S}{f_{yt}} = \frac{0.35 * 0.6 * S}{400} = 5.25 * 10^{-4}\ S\ \ \textbf{control} \end{array} \right]$$

Area of two legs of Ø12mm closed stirrups=2*113=226mm^2

$$226 * 10^{-6} = 1.278 * 10^{-3}\, S \rightarrow S = 0.177\, m$$

$$S_{max} = \begin{bmatrix} \frac{P_h}{8} = \frac{2830}{8} = 254\, mm\, \boldsymbol{control} \\ 300\, mm \end{bmatrix}$$ for torsion

$$V_s = 104kN < 2V_c = 2 * 504 = 1008kN \rightarrow S_{max} = \begin{bmatrix} \frac{d}{2} = \frac{935}{2} = 467mm\, control \\ 600\, mm \end{bmatrix}$$ for shear

$S = 177\, mm < S_{max} = 254$mm O.K

$\therefore use\ \emptyset 12@175\, mm\ c/c\ closed\ stirrups$

Longitudinal reinforcement for torsion (A_l)

$$A_l = \frac{A_t}{S} P_h \left(\frac{f_{yt}}{f_{yl}}\right) \cot^2\theta = \frac{5 * 10^{-4} * S * 2.83}{S} = 1.415 * 10^{-3} m^2$$

$$= 1415 mm^2$$

$$A_{l_{min}} = \frac{5}{12} \frac{\sqrt{fc'} A_{cp}}{f_y} - \underbrace{\left(\frac{A_t}{S}\right)}_{\geq 0.175 \frac{b}{f_y}} P_h \frac{f_{yt}}{f_{yl}}$$

$$= \frac{5}{12} * \frac{\sqrt{28} * 0.6}{400} - \frac{5 * 10^{-4} * S * 2.83}{S} = 1.892 * 10^{-3} m^2$$

$$= 1892 mm^2$$

$$\frac{A_t}{S} = 5 * 10^{-4} \geq 0.175 \frac{b_w}{f_{yt}} = 0.175 * \frac{0.6}{400} = 2.62 * 10^{-4} \text{ O.K}$$

(else use $0.175 \frac{b_w}{f_{yt}}$ instead of $\frac{A_t}{S}$)

$$A_l < A_{l_{min}} \rightarrow A_l = 1892mm^2$$

$$\frac{A_l}{4} = \frac{1892}{4} = 473\ mm^2, S_{c/c}$$

$$= \frac{1000 - 2 * 40 - 2 * 12 - 25/2 - 18/2}{4 - 1} = 291\text{mm}$$

$$< \text{S}_{\text{max}} = 300\text{mm}$$

$$use\ d_b \geq max[\frac{1}{24} * s = \frac{177}{24} = 7.4mm, 10mm]$$

$$\therefore use\ 2\emptyset 18mm = 509\ mm^2 > 473\ mm^2$$

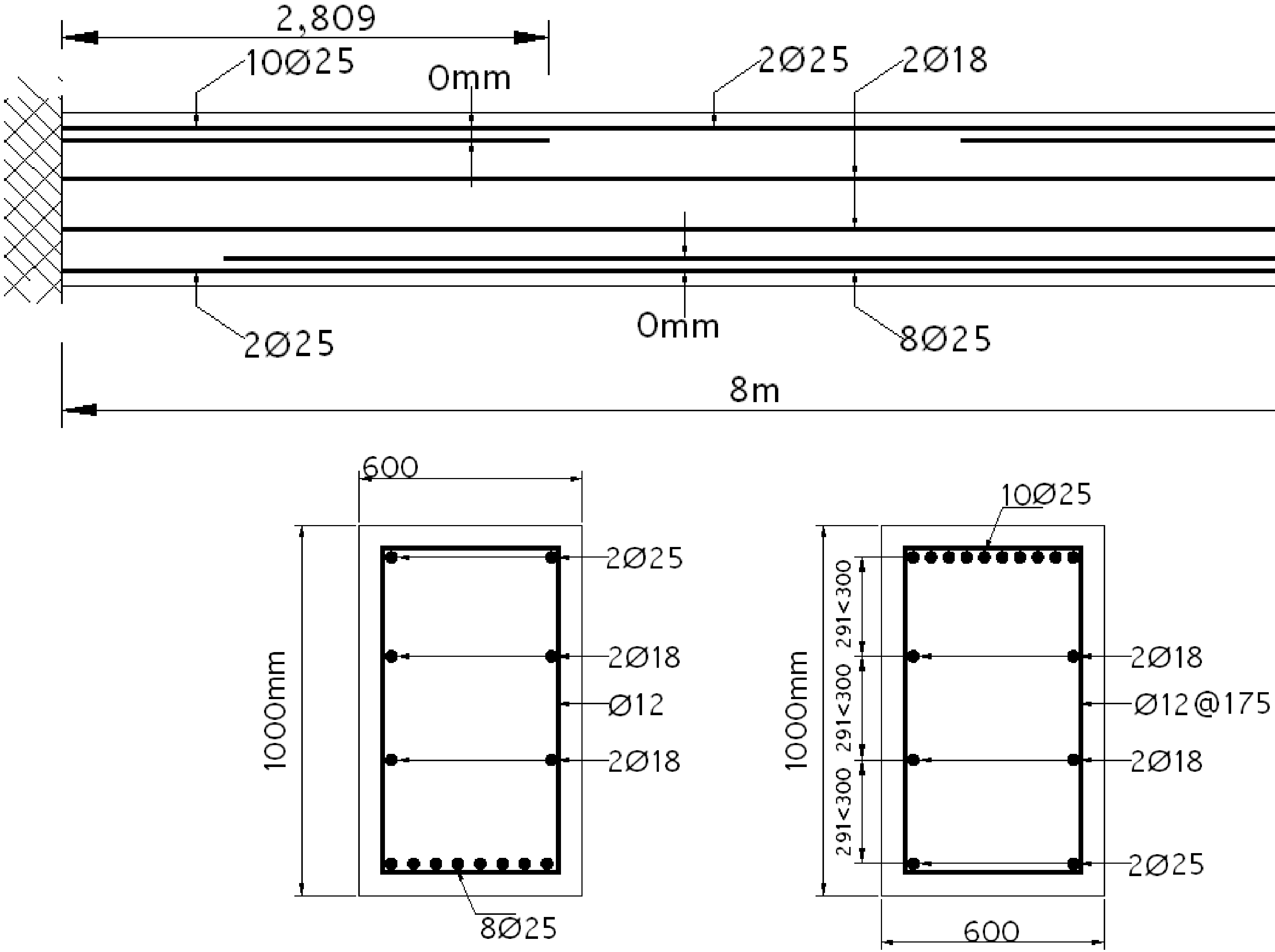
A_l, shall be added to that required for moment:

$As^- = 2673 + 1892 = 4565\ mm^2$, 10Ø25mm(one layer)

$$S_c = \frac{[600 - 2 * 40 * 2 * 12 - 2 * (25/2)]}{10 - 1} = 52mm$$

$$\geq \max \begin{bmatrix} d_b = 25mm \\ 25mm \end{bmatrix} O.K$$

$As^{+} = 2031 + 1892 = 3923\ mm^2$, 8Ø25mm(one layer)



Mispan section   Support section

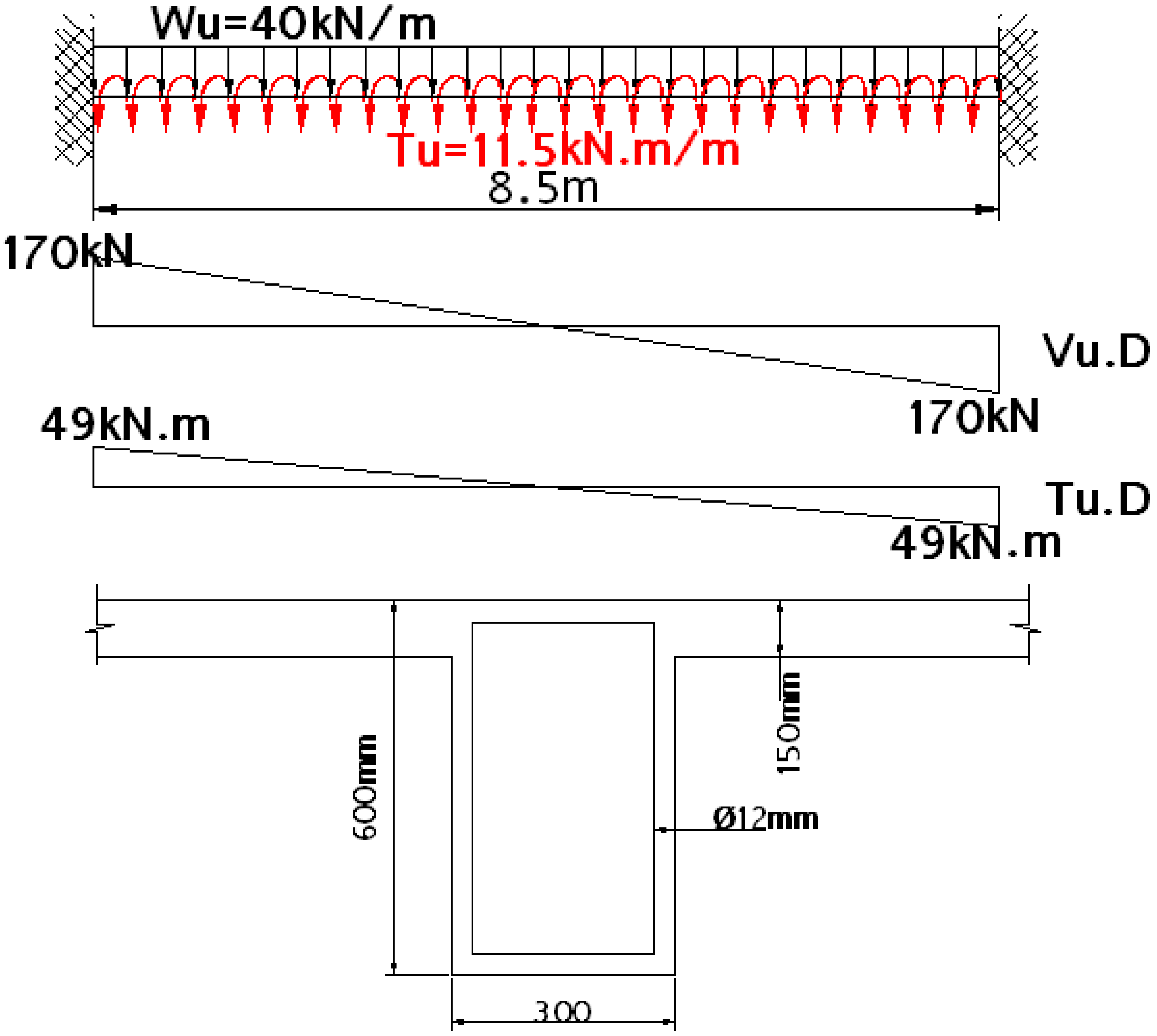
Example:

Wu=40kN/m along center line of the beam.

Tu=11.5kN.m/m.

fc'=34.5MPa, fy=414MPa.

Design beam for shear and torsion at critical section.


Wu=40kN/m
Tu=11.5kN.m/m
8.5m
170kN
Vu.D
170kN
49kN.m
Tu.D
49kN.m
600mm
150mm
Ø12mm
300

Solution

$R_u = \frac{40*8.5}{2} = 170\ kN,$ due to symmetry of loads

Assume $\emptyset 25\ mm$ bars and $\emptyset 12\ mm$ closed stirrup

$\text{d} = 600 - 40 - 12 - (25/2) = 535mm$

$V_{ud} = 170 - 40 * 0.535 = 149kN$

$$T_u\ at\ support = \frac{11.5 * 8.5}{2} = 49\ kN.m$$

$T_{u_d} = 49 - 11.5 * 0.535 = 43\ kN.m$

$$M^- = \frac{w_u l^2}{12} = \frac{40 * 8.5^2}{12} = 241\ kN.m$$

$$M^+ = \frac{w_u l^2}{24} = \frac{40 * 8.5^2}{24} = 120\ kN.m$$

Design for flexure

- **Negative moment(rectangular section, 300*600mm)**

$$0.241 = 0.9\rho * 0.3 * 0.535^2 * 414(1 - 0.59\rho * 414/34.5)$$

$$\rightarrow \rho^- = 0.008$$

$$fc' = 34.5 > 28MPa \rightarrow \beta_1 = 0.85 - \frac{fc' - 28}{7} * 0.05 = 0.80 > 0.65\ ok$$

$$\rho_{max} = 0.85 * 0.80 * \frac{34.5}{414} * \frac{0.003}{0.007} = 0.0243$$

$$\rho_{min} = max\left(\frac{1.4}{f_y} = 0.0034, \frac{\sqrt{fc'}}{4f_y} = 0.0035\right) = 0.0035$$

$$\rho_{min} = 0.0035 < \rho^- = 0.008 < \rho_{max} = 0.0243 \text{ O.K}$$

$$As^- = 0.008 * 300 * 535 = 1284\ mm^2$$

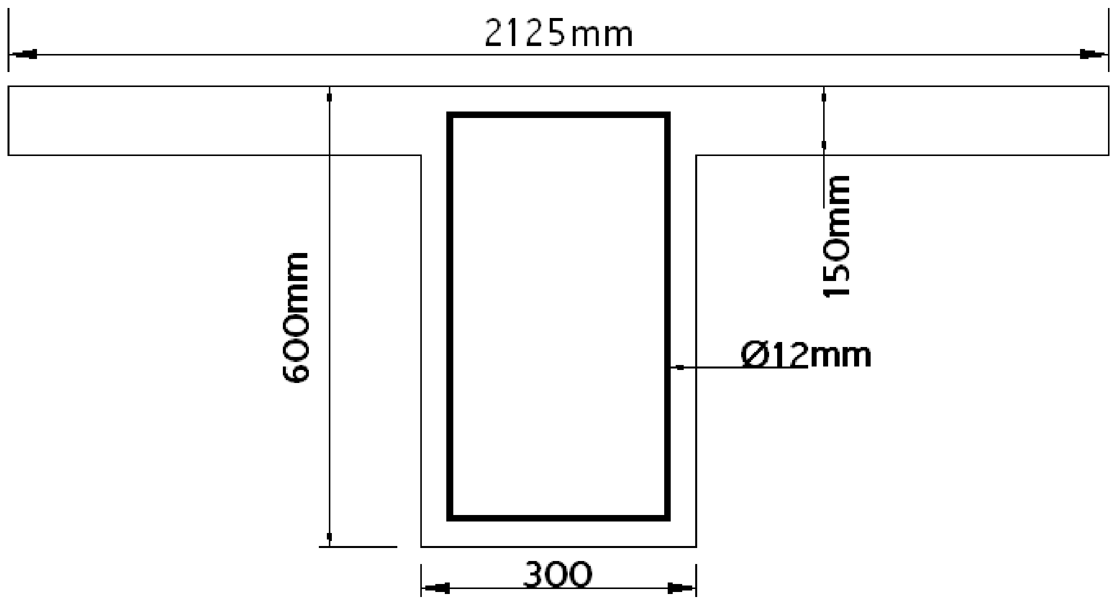
- **Positive moment** (T-section)

❖ $b \leq \frac{L}{4} = \frac{8500}{4} = 2125mm$

❖ $\frac{b-b_w}{2} \leq 8hf \rightarrow b = 2700mm$

❖ $\frac{b-b_w}{2} \leq \frac{1}{4}(lc_1 + lc_2) \rightarrow b = \; , lc_1 and\; lc_2,$ not available

choose min. value of b=2125mm(for flexure, ACI 8.10.2)



$$Mu_{ext} = 124\ kN.m$$

$$Mu_f = \emptyset 0.85 fc'bh_f \left(\text{d-} \frac{h_f}{2} \right)$$

$Let\ \emptyset = 0.9$ to be check later

$$Mu_f = 0.9 * 0.85 * 34.5 * 2.125 * 0.15 \left(0.535\text{-} \frac{0.15}{2} \right)$$

$$= 3.870MN.m > Mu_{ext} = 0.124MN.m \rightarrow a < h_f [RS\ (2.125 * 0.6m)]$$

$$Mu = \emptyset \rho b d^2 \text{fy} \left(1\text{-}0.59\ \rho \frac{\text{fy}}{\text{fc}'} \right)$$

$$0.124 = 0.9\rho * 2.125 * 0.535^2 * 414 \left(1 - 0.59\rho \frac{414}{34.5} \right) \rightarrow \rho^+ = 0.00055$$

$$\rho_t = 0.85\beta_1 \frac{\mathrm{fc}'}{\mathrm{fy}} \frac{0.003}{0.003 + 0.005} = 0.0212 > \rho \rightarrow \emptyset = 0.9\ \mathrm{O.K}$$

$$\rho_{max} = 0.85 * 0.80 * \frac{34.5}{414} * \frac{0.003}{0.007} = 0.0243$$

$$\rho_{min} = \max\left(\frac{1.4}{\mathrm{fy}} = 0.0034, \frac{\sqrt{\mathrm{fc}'}}{4\mathrm{fy}} = 0.0035\right)\frac{b_w}{b}$$

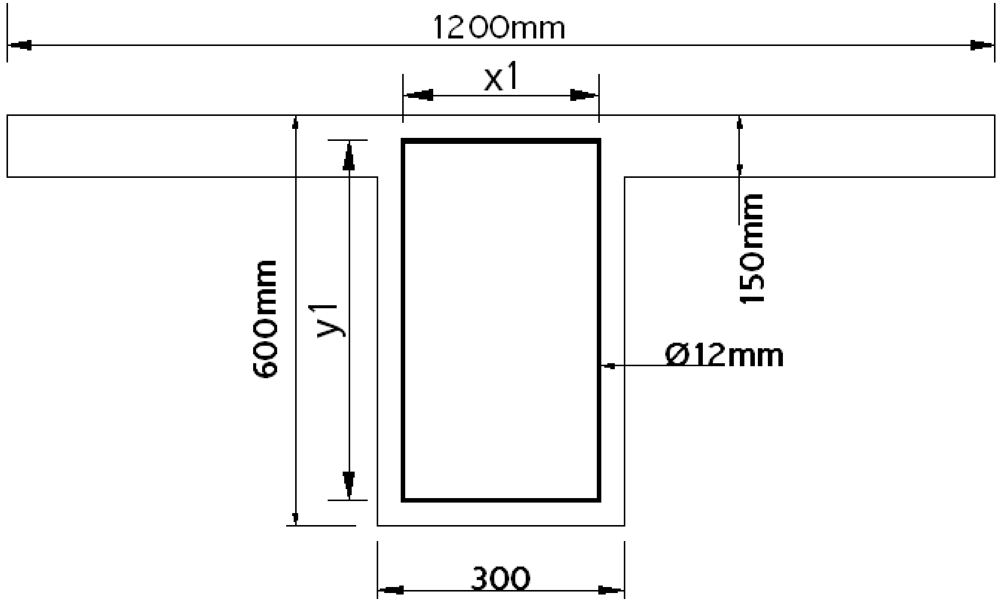
$$= 0.0005 < \rho^+ = 0.00055\ \ o.k$$

$$\rho_{min} < \rho^+ < \rho_{max}\ \ \therefore o.k$$

$$As^+ = 0.00055 * 300 * 2125 = 351\, mm^2$$

Design for torsion

- $\frac{b-b_w}{2} \leq \mathrm{h}_w \rightarrow \frac{b-300}{2} \leq (600-150) \rightarrow b = 1200mm$
- $\frac{b-b_w}{2} \leq 4\mathrm{h}_f \rightarrow \frac{b-300}{2} \leq 4*150 \rightarrow b = 1500mm$



Choose minimum value of b=1200mm (for torsion, ACI 11.6.1.1& 13.2.4)

$A_{cp} = (0.6-0.15)*0.3+0.15*1.2 = 0.315\ m^2$

$P_{cp} = 1.2*2+0.6*2 = 3.6\ m$

$$T_{u_d} = 43\ kN.m > \emptyset \frac{\sqrt{fc'}}{12} * \left(\frac{{A_{cp}}^2}{P_{cp}}\right) * 10^3$$

$$= 0.75 \frac{\sqrt{34.5}}{12} \left(\frac{0.315^2}{3.6}\right) * 10^3 = 10.1\ kN.m$$

∴Torsional effect should be considered

Check section dimensions

$$y_1 = 600 - 2*40 - 2*\frac{12}{2}$$

$$= 508\,mm$$

$$x_1 = 300 - 2*40 - 2*\frac{12}{12}$$

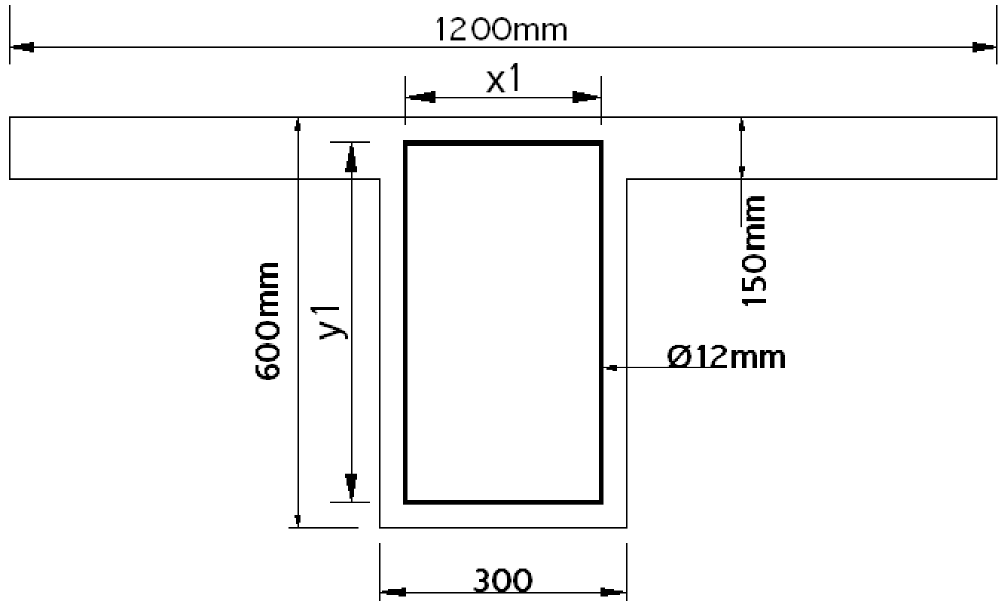
$$= 208\,mm$$

$$P_h = (0.508 + 0.208)*2$$

$$= 1.432\,m$$

$$A_{oh} = 0.508*0.208$$

$$= 0.1056\,m^2$$



$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7A_{oh}^2}\right)^2} \leq \emptyset\left[\frac{V_C}{b_w d} + \frac{8\sqrt{\mathrm{f}c'}}{12}\right]$$

$$\sqrt{\left(\frac{149}{0.3 * 0.535}\right)^2 + \left(\frac{43 * 1.432}{1.7 * 0.1056^2}\right)^2}\, 10^{-3} = 3.38\ MPa$$

$$< 0.75\left(\frac{0.17\sqrt{34.5} * b_w d}{b_w d} + \frac{8}{12}\sqrt{34.5}\right) = 3.68\ MPa$$

$$\therefore o.k$$

$$T_u = \emptyset T_n = \emptyset * \frac{2 \overbrace{A_o}^{=0.85A_{oh}} A_t \mathrm{f}_{yt}}{S} \cot(\theta)$$

$$43 = \frac{0.75 * 2 * 0.85 * 0.1056 * A_t * 414 * 10^3 * \cot(45)}{S}$$

$$A_t = 7.714 * 10^{-4} S$$

Design for shear

$$V_c = 0.17 * \sqrt{34.5} * 0.3 * 0.535 * 1000 = 160\ kN$$

$$V_{ud} = \emptyset(V_c + V_s)$$

$$149 = 0.75 * (160 + V_s) \rightarrow V_s = 39\ kN < 4\ V_c$$

$$= 640\ kN\ \ o.k$$

$$S = \frac{A_v \mathrm{f}_y d}{V_s} \rightarrow A_v = \frac{0.039 * S}{414 * 0.535} = 1.761 * 10^{-4} S$$

$$A_v + 2A_t = (1.761 * 10^{-4} + 2 * 7.714 * 10^{-4})S = 1.728 *$$

$$10^{-3} S \geq max. \begin{bmatrix} 0.062\sqrt{fc'}\frac{b_w S}{\mathrm{f}_{yt}} = 2.639 * 10^{-4} S\ \textcolor{red}{control} \\ \\ 0.35\frac{b_w S}{\mathrm{f}_{yt}} = 2.53 * 10^{-4}\ S \end{bmatrix} o.k$$

Area of two legs of Ø12 $mm = 2 * 113 = 226\ mm^2$

$1.728 * 10^{-3} S = 226 * 10^{-6} \rightarrow S = 0.131\ m$

$S_{max} = \begin{bmatrix} \frac{P_h}{8} = \frac{1432}{8} = 179\ mm\ \textbf{control} \\ 300\ mm \end{bmatrix}$ for torsion

$V_u = 149 kN < 2V_c = 320\ kN$

$S_{max} = \begin{bmatrix} \frac{d}{2} = \frac{535}{2} = 267\ mm \\ 600\ mm \end{bmatrix}$ for shear

$S = 131\ mm < S_{max} = 179$mm O.K

use Ø12@130 *mm c/c closed stirrups*

$$A_l = \frac{A_t}{S} P_h \left(\frac{f_{yt}}{f_{yl}}\right) \cot^2\theta = \frac{7.714 * 10^{-4} * S}{S} * 1.432$$

$$= 1.105 * 10^{-3} m^2 = 1105\, mm^2$$

$$A_{l_{min}} = \frac{5}{12}\frac{\sqrt{fc'}A_{cp}}{f_y} - \underbrace{\left(\frac{A_t}{S}\right)}_{\geq 0.175\frac{b_w}{f_{yt}}} P_h \frac{f_{yt}}{f_{yl}}$$

$$= \frac{5}{12} * \frac{\sqrt{34.5} * 0.315}{414} - \frac{7.714 * 10^{-4} * S}{S} * 1.432$$

$$= 7.57 * 10^{-4} m^2 = 757 mm^2$$

$$\frac{A_t}{S} = 7.714 * 10^{-4} \geq 0.175\frac{b_w}{f_{yt}} = 0.175 * \frac{0.3}{414}$$

$$= 1.288 * 10^{-4}\ o.k$$

$$A_l > A_{l_{min}}\ o.k$$

$$\frac{A_l}{3} = \frac{1105}{3} = 368\ mm^2\ , S_{c/c}$$

$$= \frac{600 - 2*40 - 2*12 - 25/2 - 16/2}{3-1} = 238\text{mm}$$

$$< \text{S}_{\text{max}} = 300\text{mm}$$

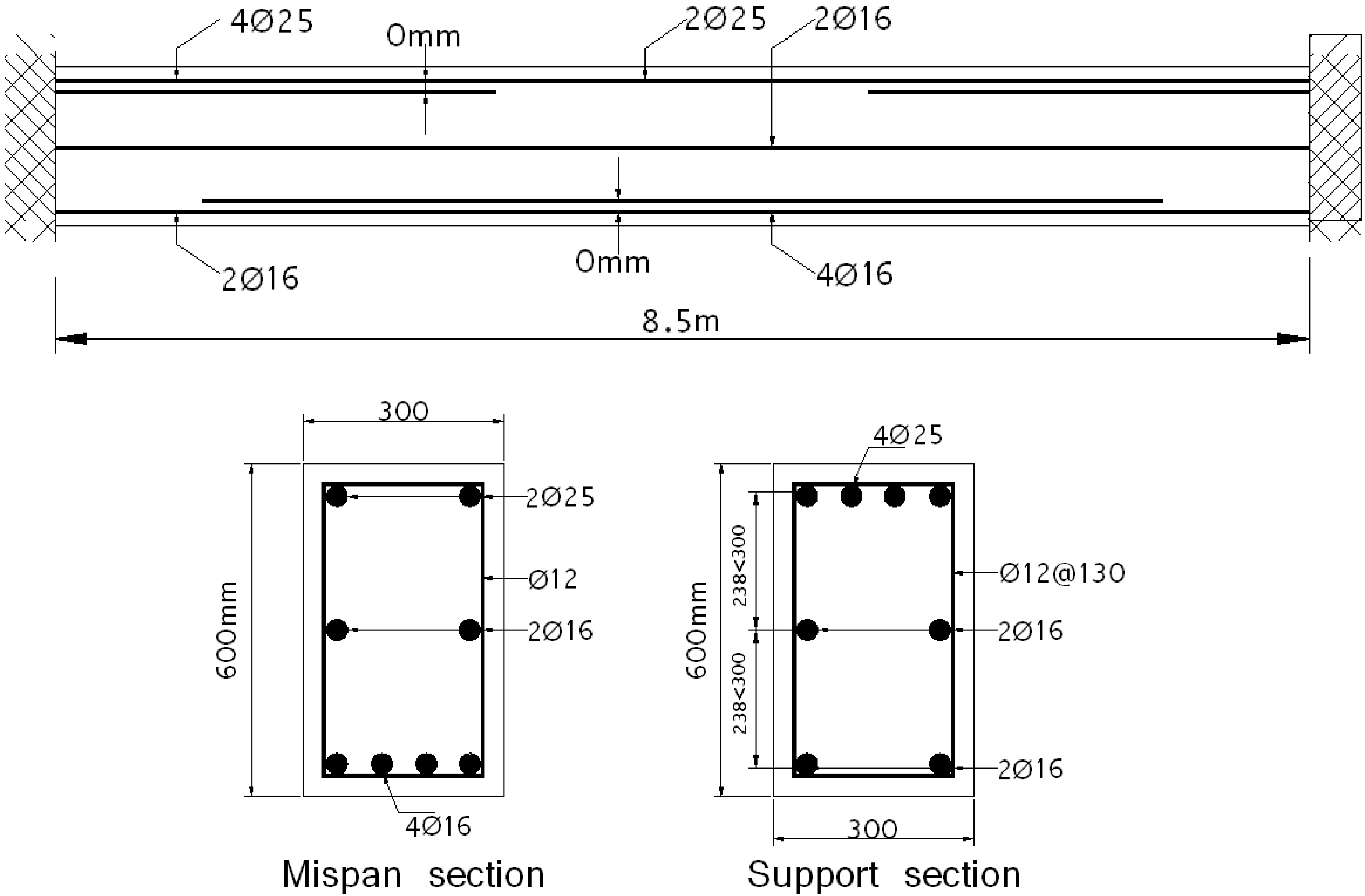
$$use\ d_b \geq max[\frac{1}{24} * s = \frac{131}{24} = 5.4mm, 10mm]$$

$$\therefore use\ 2Ø16mm = 402\ mm^2 > 368\ mm^2$$

A_l, shall be added to that required for moment:

$$As^- = 1284 + 368 = 1652\ mm^2\ use\ 4Ø25 = 1964\ mm^2$$

$$As^+ = 351 + 368 = 719\ mm^2\ use\ 4Ø16 = 804\ mm^2$$


4Ø25
0mm
2Ø25
2Ø16
2Ø16
0mm
4Ø16
8.5m
300
600mm
2Ø25
Ø12
2Ø16
4Ø16
Mispan section
4Ø25
238<300
238<300
600mm
Ø12@130
2Ø16
2Ø16
300
Support section