(c) $$Fr_2 = \frac{v_2}{\sqrt{gy_2}}$$
$$= \frac{2.89}{\sqrt{9.81 * 3.46}}$$
$$= \underline{0.496}$$

(d) $$h_f = \frac{(y_2 - y_1)^3}{4y_1y_2}$$
$$= \frac{(3.46 - 1.25)^3}{4 * 3.46 * 1.25}$$
$$= \underline{0.625\ \text{m}}$$

(e) $$E_1 = \frac{v_1^2}{2g} + y_1$$
$$= \frac{8^2}{2 * 9.81} + 1.25 \quad \text{m}$$
$$= \underline{4.51\ \text{m}}$$

percentage loss $$= \frac{h_f}{E_1} * 100\%$$
$$= \frac{0.625}{4.51} * 100\%$$
$$= \underline{14\ \%}$$

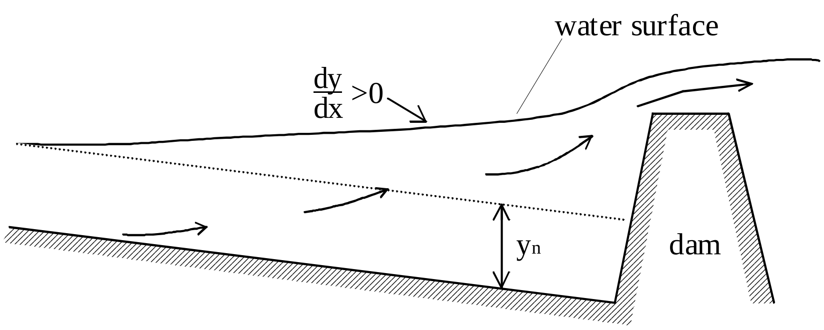
8.6 Gradually Varied Flow

- It is not always possible to have uniform depth across the flow i.e. normal flow with normal depth.

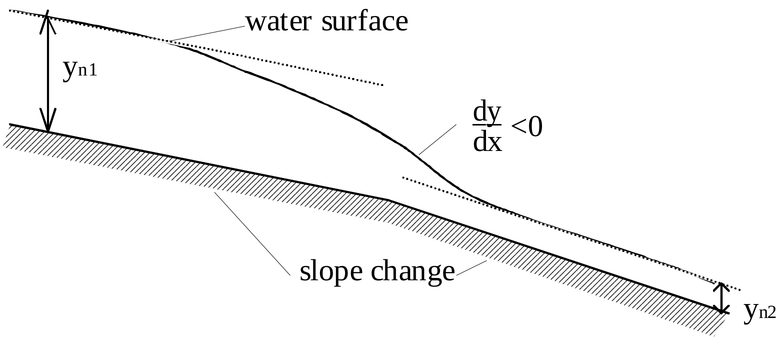
- The depth of flow can be changed by the conditions along the channel.

- Examples of Gradually Varied Flow are:

  – backwater curve

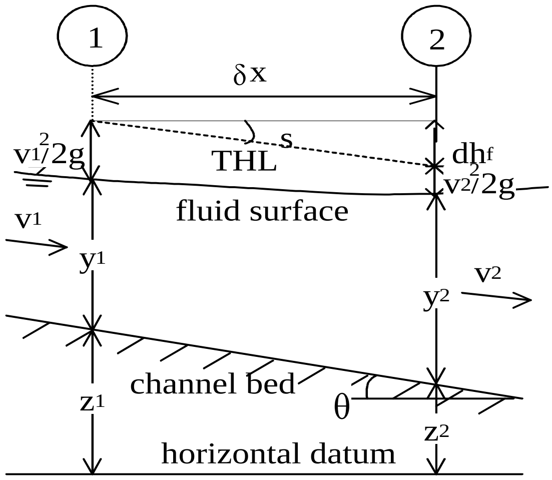


  – Downdrop curve



- In a **uniform flow**, the body weight effect in balanced out by the wall friction.

- In **gradually varied flow**, the weight and the friction effects are unable to make the flow uniform.



- Basic assumptions are
  - slowly changing bottom slope
  - slowly changing water depth (no hydraulic jump)
  - slowly changing cross section
  - one dimensional velocity distribution
  - pressure distribution approximately hydrostatic

- Denoting

v_1	$= v;$	v_2	$= v + dv$
z_1	$= z,$	z_2	$= z + dz$
y_1	$= y,$	y_2	$= y + dy$
p_1	$= p,$	p_2	$= p$

Apply Bernoulli's equation between section 1 and 2,

$$\frac{p}{\gamma} + \frac{v^2}{2g} + y + z = \frac{p}{\gamma} + \frac{(v+dv)^2}{2g} + (y+dy) + (z+dz) + dh_f$$

Neglecting higher order terms,

$$dh_f + dy + dz + \frac{v}{g}du = 0$$

When lim $dx \to 0$,

$$\frac{dh_f}{dx} + \frac{dy}{dx} + \frac{dz}{dx} + \frac{v}{g}\frac{dv}{dx} = 0 \qquad (8.30)$$

- In (8.30), the four terms are

$\frac{dh_f}{dx}$ - rate of head loss along the channel

= S (head loss gradient in Manning equation)

$\frac{dy}{dx}$ - rate of change of water depth

- water surface profile's gradient

$\frac{dz}{dx}$ - rate of vertical change along channel

$= -\sin\theta$

$\frac{v}{g}\frac{dv}{dx}$ - rate of change of velocity head along the channel

- By Continuity equation

$$v*A = \text{constant}$$

i.e.

$$A\frac{dv}{dx} + v\frac{dA}{dx} = 0$$

$$A\frac{dv}{dx} + v\frac{dA}{dy}\frac{dy}{dx} = 0$$

$$A\frac{dv}{dx} + vB\frac{dy}{dx} = 0$$

or

$$\frac{dv}{dx} = -\frac{vB}{A}\frac{dy}{dx}$$

$$= -\frac{v}{y}\frac{dy}{dx}$$

Therefore

$$\frac{v}{g}\frac{dv}{dx} = -\frac{v^2}{gy}\frac{dy}{dx}$$

$$= -Fr^2\frac{dy}{dx} \qquad (8.31)$$

- Hence (8.30) becomes

$$s + \frac{dy}{dx} - \sin\theta - Fr^2\frac{dy}{dx} = 0$$

or

$$\frac{dy}{dx} = [\frac{\sin\theta - S}{1 - Fr^2}] \qquad (8.32)$$

- general equation of gradually varied flow.

- (8.32) is a 1st order non-linear differential equation. Numerical method is used to solve the equation.

  The equation is rewritten as

$$\frac{dx}{dy} = [\frac{1-Fr^2}{\sin\theta - S}]$$

$$\int_{x_1}^{x_2} dx = \int_{y_1}^{y_2} (\frac{1-Fr^2}{\sin\theta - S})dy$$

$$x_2 = x_1 + \int_{y_1}^{y_2} (\frac{1-Fr^2}{\sin\theta - S})dy \qquad (8.33)$$

  The simplest solution is the direct mid-point solution of the integral.

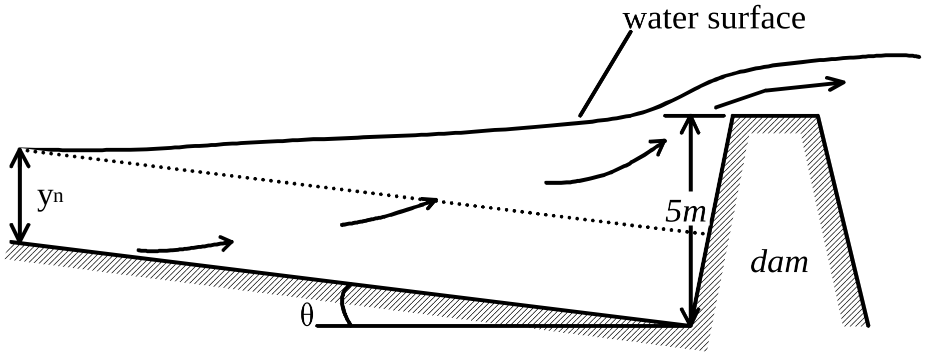
  i.e. $$x_2 = x_1 + (\frac{1-Fr^2}{\sin\theta - S})_{(\frac{y_1+y_2}{2})} *(y_2-y_1) \qquad (8.34)$$

- (8.34) may be used to calculate the water profile in a step-by-step sequence from a known (x_1, y_1) value.

Worked example:

Determine the upstream profile of a backwater curve given:

$Q = 10\ m^3/s$, $b = 3m$, $\sin\theta = 0.001$, $n = 0.022$.



Answer

For normal flow, ($S \to \sin\theta$)

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

i.e. $$10 = \frac{(3y_n)}{0.022} * \left(\frac{3y_n}{3 + 2y_n}\right) * \sqrt{0.001}$$

$$y_n = 2.44\ m$$

The water profile is from 2.44 m to 5 m along the channel.

From Manning equation,

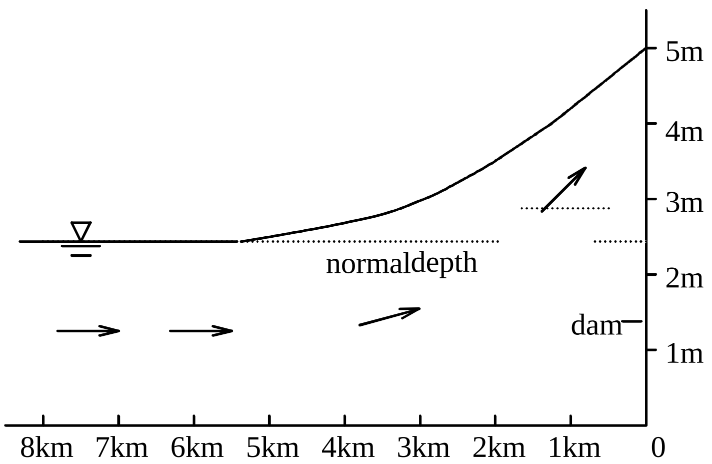
$$v = \frac{1}{n} * R^{2/3} * S^{1/2}$$

$$S = \frac{n^2 v^2}{R^{4/3}}$$

Hence $$x_2 = x_1 + \left(\frac{1 - Fr^2}{0.001 - \frac{n^2 v^2}{R^{4/3}}}\right) * (y_2 - y_1)$$

section, I	yi (m)	dy (m)	yave (m)	v (m/s)	Fr	1-Fr*Fr	R (m)	So - Sf	dx (m)	x (m)
1	5									0
		0.25	4.875	0.684	0.099	0.990	1.147	0.000812	305	
2	4.75									305
		0.25	4.625	0.721	0.107	0.989	1.133	0.000787	314	
3	4.5									619
		0.25	4.375	0.762	0.116	0.986	1.117	0.000758	326	
4	4.25									945
		0.25	4.125	0.808	0.127	0.984	1.100	0.000722	341	
5	4									1285
		0.25	3.875	0.860	0.140	0.981	1.081	0.000677	362	
6	3.75									1647
		0.25	3.625	0.920	0.154	0.976	1.061	0.000622	392	
7	3.5									2040
		0.25	3.375	0.988	0.172	0.971	1.038	0.000551	440	
8	3.25									2480
		0.25	3.125	1.067	0.193	0.963	1.014	0.000459	524	
9	3									3004
		0.25	2.875	1.159	0.218	0.952	0.986	0.000337	707	
10	2.75									3711
		0.31	2.595	1.285	0.255	0.935	0.951	0.000146	1992	
11	2.44									5703

From the table, the water level is not affected by the dam at 5.7 km upstream.



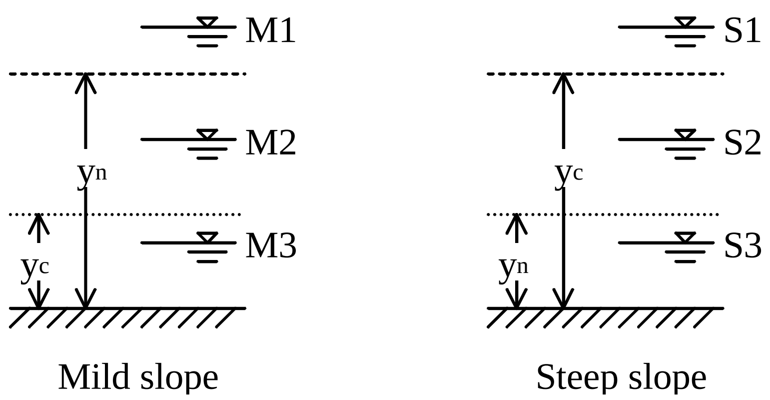
From the graph, the water depth at any location can be obtained.

8.6.1 Classifications of Surface Profile of Gradually Varied Flow

◆ It is customary to compare the actual channel slope, sinθ or S_o with the critical slope S_c for the same Q.

◆ There are **five** classes of channel slope giving rise to twelve distinct types of solution curves.
- $S_o > S_c$ - Steep (S)
- $S_o = S_c$ - Critical (C)
- $S_o < S_c$ - Mild (M)
- $S_o = 0$ - Horizontal (H)
- $S_o < 0$ - Adverse (A)

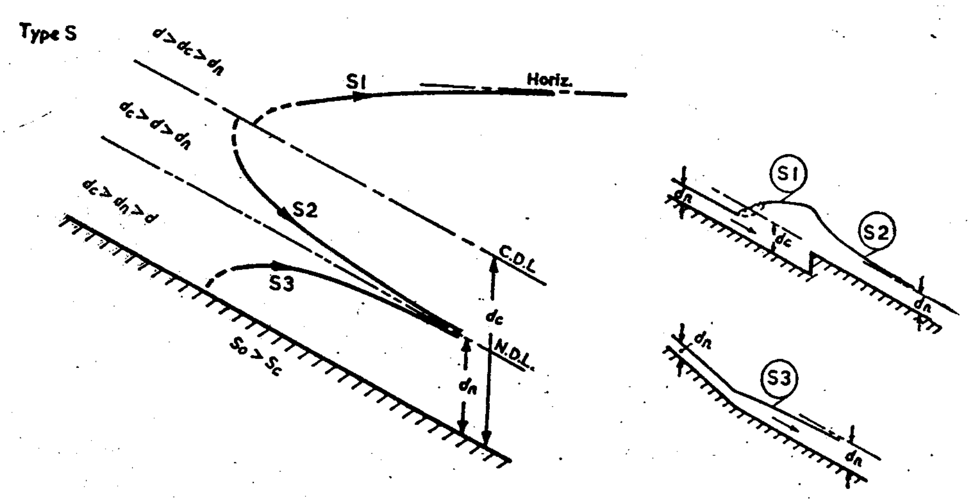
◆ There are **three** number designators for the type of profile relates to the position of the actual water surface in relation to the position of the water for normal and critical flow in a channel.
- 1 the surface of stream lies above both normal and critical depth
- 2 the surface of stream lies between normal and critical depth
- 3 the surface of stream lies below both normal and critical depth



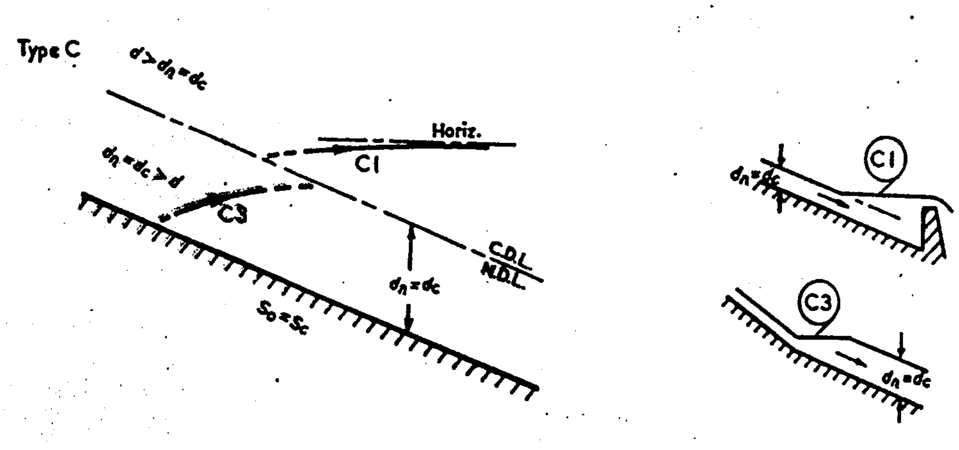
- Combining the two designators, we have

Slope class	Slope notation	Depth class	Froude number	Actual depth	Profile
$\sin\theta > S$	Steep (S)	$y_c > y_n$	Fr < 1	$y > y_n$; $y > y_c$	S1
			Fr > 1	$y_c > y > y_n$	S2
			Fr > 1	$y < y_n$; $y < y_c$	S3
$\sin\theta = S$	Critical (C)	$y_c = y_n$	Fr < 1	$y > y_n = y_c$	C1
			Fr > 1	$y < y_n = y_c$	C3
$\sin\theta < S$	Mild (M)	$y_c < y_n$	Fr < 1	$y > y_n$; $y > y_c$	M1
			Fr < 1	$y_n > y > y_c$	M2
			Fr > 1	$y < y_n$; $y < y_c$	M3
$\sin\theta = 0$	Horizontal (H)	$y_n = \infty$	Fr < 1	$y > y_c$	H2
			Fr > 1	$y < y_n$; $y < y_c$	H3
$\sin\theta < 0$	Adverse (A)	$y_n = \text{Im}$	Fr < 1	$y > y_c$	A2
			Fr > 1	$y < y_c$	A3

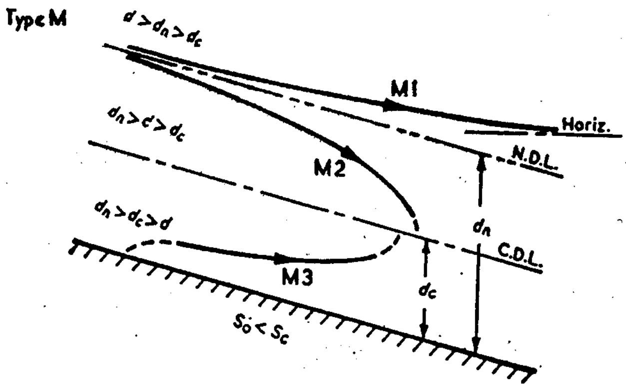
- For type S

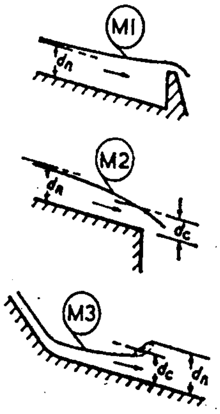


- For type C

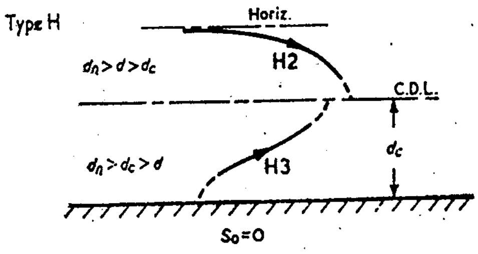


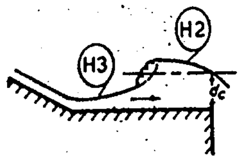
◆ For type M



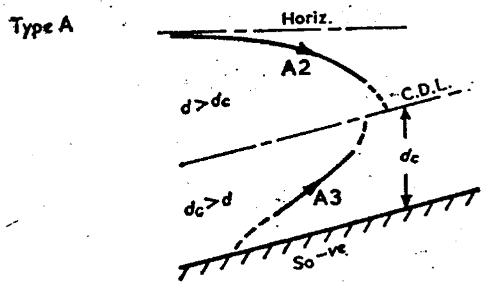


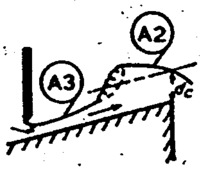
◆ For type H





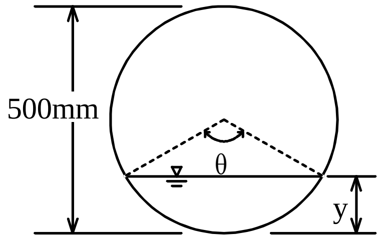
◆ For type A





Class Exercise 8.1:

A 500 mm-diameter concrete pipe on a 1:500 slope is to carry water at a velocity of 0.18 m/s. Find the depth of the flow. (n=0.013)



(y = 18 mm)

Class Exercise 8.2:

What are the dimensions for an optimum rectangular brick channel ($n = 0.015$) designed to carry 5 m^3/s of water in uniform flow with $s = 0.001$? What will be the percentage increase in flow rate if the channel is a semi-circle but retained the same sectional area? (increase = 8.4%)

Class Exercise 8.3:

A trapezoidal channel has a bottom width of 6.0 m and side slopes of 1:1. The depth of flow is 1.5 m at a discharge of 15 m^3/s. Determine the specific energy and alternate depth. (E = 1.59 m, y = 0.497 m)

Class Exercise 8.4:

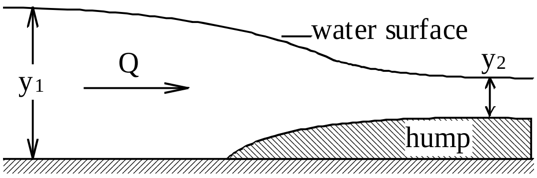
A triangular channel has an apex angle of 60° and carries a flow with a velocity of 2.0 m/s and depth of 1.25 m.

(a) Is the flow subcritical or supercritical?
(b) What is the critical depth?
(c) What is the specific energy?
(d) What is the alternate depth possible for this specific energy?

(y_c = 1.148 m, E = 1.454 m, y = 1.06 m)

Class Exercise 8.5:

A rectangular channel is 4.0 m wide and carries a discharge of 20 m^3/s at a depth of 2.0 m. At a certain section it is proposed to build a hump. Calculate the water surface elevations at upstream of the hump and over the hump if the hump height is 0.33 m. (Assume no loss of energy at the hump.)

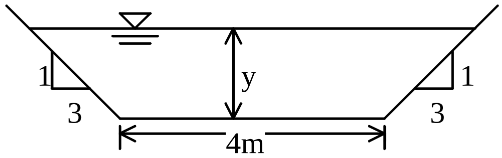


Class Exercise 8.6:

In a hydraulic jump occurring in a horizontal, rectangular channel it is desired to have an energy head loss equal to 6 times the supercritical flow depth. Calculate the Froude number of the flow necessary to have this jump. (Fr_1 = 4.822)

Tutorial – Open Channel Flow

1. Calculate the normal depth in a concrete trapezoidal channel with side slope of 1 to 3, a bed slope of 0.00033, a bottom width of 4.0 m and a water discharge of 39 m^3/s. Manning coefficient is 0.013.



2. Determine the critical depth of the trapezoidal channel for a discharge of 15 m^3/s. The width of the channel bottom, b = 6 m, and the side slope is 45°.

3. Consider a flow in a wide channel over a bump with an approaching velocity, v_1 at the upstream is 1 m/s and the depth, y_1 is 1 m. If the ~~maximum~~ bump height is 15 cm, determine

   (a) the Froude number over the top of the bump, and
   (b) the depression in the water surface. ($y_2 > 0.5$ m)

4. Water flows in a trapezoidal channel at a rate of 8.5 m^3/s. The channel has a bottom width of 3 m and side slope of 1:1. If a hydraulic jump is forced to occur where the upstream depth is 0.3 m, what will be the downstream depth and velocity? What are the values of Fr_1 and Fr_2?

5. A wide canal has a bed slope of 1 in 1000 and conveys water at a normal depth of 1.2 m. A weir is to be constructed at one point to increase the depth of flow to 2.4 m. How far upstream of the weir will the depth be 1.35 m? (Take n in the Manning equation as 0.013)